on the problems in "7 Inverse hyperbolic functions/7.3 Inverse hyperbolic tangent"
Test results for the 71 problems in "7.3.2 (d x) ^m (a+b arctanh (c $x^{\wedge} n$ ) )^p.txt"
Problem 3: Result more than twice size of optimal antiderivative.

$$
\int \frac{a+b \operatorname{arctanh}(c x)}{x} \mathrm{~d} x
$$

Optimal(type 4, 22 leaves, 1 step):

$$
a \ln (x)-\frac{b \operatorname{poly} \log (2,-c x)}{2}+\frac{b \operatorname{polylog}(2, c x)}{2}
$$

Result(type 4, 46 leaves):

$$
a \ln (c x)+b \ln (c x) \operatorname{arctanh}(c x)-\frac{b \operatorname{dilog}(c x+1)}{2}-\frac{b \ln (c x) \ln (c x+1)}{2}-\frac{b \operatorname{dilog}(c x)}{2}
$$

Problem 5: Result more than twice size of optimal antiderivative.

$$
\int x^{3}(a+b \operatorname{arctanh}(c x))^{2} \mathrm{~d} x
$$

Optimal(type 3, 99 leaves, 11 steps):

$$
\frac{a b x}{2 c^{3}}+\frac{b^{2} x^{2}}{12 c^{2}}+\frac{b^{2} x \operatorname{arctanh}(c x)}{2 c^{3}}+\frac{b x^{3}(a+b \operatorname{arctanh}(c x))}{6 c}-\frac{(a+b \operatorname{arctanh}(c x))^{2}}{4 c^{4}}+\frac{x^{4}(a+b \operatorname{arctanh}(c x))^{2}}{4}+\frac{b^{2} \ln \left(-c^{2} x^{2}+1\right)}{3 c^{4}}
$$

Result(type 3, 277 leaves):
$\frac{x^{4} a^{2}}{4}+\frac{b^{2} x^{4} \operatorname{arctanh}(c x)^{2}}{4}+\frac{b^{2} \operatorname{arctanh}(c x) x^{3}}{6 c}+\frac{b^{2} x \operatorname{arctanh}(c x)}{2 c^{3}}+\frac{b^{2} \operatorname{arctanh}(c x) \ln (c x-1)}{4 c^{4}}-\frac{b^{2} \operatorname{arctanh}(c x) \ln (c x+1)}{4 c^{4}}+\frac{b^{2} \ln (c x-1)}{3 c^{4}}$
$+\frac{b^{2} \ln (c x+1)}{3 c^{4}}+\frac{b^{2} x^{2}}{12 c^{2}}+\frac{b^{2} \ln (c x-1)^{2}}{16 c^{4}}-\frac{b^{2} \ln (c x-1) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{8 c^{4}}+\frac{b^{2} \ln (c x+1)^{2}}{16 c^{4}}-\frac{b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{8 c^{4}}$

$$
+\frac{b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{8 c^{4}}+\frac{a b x^{4} \operatorname{arctanh}(c x)}{2}+\frac{x^{3} a b}{6 c}+\frac{a b x}{2 c^{3}}+\frac{a b \ln (c x-1)}{4 c^{4}}-\frac{a b \ln (c x+1)}{4 c^{4}}
$$

Problem 6: Result more than twice size of optimal antiderivative.

$$
\int x^{2}(a+b \operatorname{arctanh}(c x))^{2} \mathrm{~d} x
$$

Optimal(type 4, 116 leaves, 9 steps):
$\frac{b^{2} x}{3 c^{2}}-\frac{b^{2} \operatorname{arctanh}(c x)}{3 c^{3}}+\frac{b x^{2}(a+b \operatorname{arctanh}(c x))}{3 c}+\frac{(a+b \operatorname{arctanh}(c x))^{2}}{3 c^{3}}+\frac{x^{3}(a+b \operatorname{arctanh}(c x))^{2}}{3}-\frac{2 b(a+b \operatorname{arctanh}(c x)) \ln \left(\frac{2}{-c x+1}\right)}{3 c^{3}}$

$$
-\frac{b^{2} \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{3 c^{3}}
$$

Result(type 4, 269 leaves):
$\frac{x^{3} a^{2}}{3}+\frac{x^{3} b^{2} \operatorname{arctanh}(c x)^{2}}{3}+\frac{b^{2} \operatorname{arctanh}(c x) x^{2}}{3 c}+\frac{b^{2} \operatorname{arctanh}(c x) \ln (c x-1)}{3 c^{3}}+\frac{b^{2} \operatorname{arctanh}(c x) \ln (c x+1)}{3 c^{3}}+\frac{b^{2} x}{3 c^{2}}+\frac{b^{2} \ln (c x-1)}{6 c^{3}}-\frac{b^{2} \ln (c x+1)}{6 c^{3}}$

$$
\begin{aligned}
& +\frac{b^{2} \ln (c x-1)^{2}}{12 c^{3}}-\frac{b^{2} \operatorname{dilog}\left(\frac{c x}{2}+\frac{1}{2}\right)}{3 c^{3}}-\frac{b^{2} \ln (c x-1) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{6 c^{3}}-\frac{b^{2} \ln (c x+1)^{2}}{12 c^{3}}+\frac{b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{6 c^{3}} \\
& -\frac{b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{6 c^{3}}+\frac{2 x^{3} a b \operatorname{arctanh}(c x)}{3}+\frac{a b x^{2}}{3 c}+\frac{a b \ln (c x-1)}{3 c^{3}}+\frac{a b \ln (c x+1)}{3 c^{3}}
\end{aligned}
$$

Problem 7: Result more than twice size of optimal antiderivative.

$$
\int x(a+b \operatorname{arctanh}(c x))^{2} \mathrm{~d} x
$$

Optimal(type 3, 69 leaves, 6 steps):

$$
\frac{a b x}{c}+\frac{b^{2} x \operatorname{arctanh}(c x)}{c}-\frac{(a+b \operatorname{arctanh}(c x))^{2}}{2 c^{2}}+\frac{x^{2}(a+b \operatorname{arctanh}(c x))^{2}}{2}+\frac{b^{2} \ln \left(-c^{2} x^{2}+1\right)}{2 c^{2}}
$$

Result(type 3, 238 leaves):

$$
\begin{aligned}
\frac{x^{2} a^{2}}{2} & +\frac{b^{2} x^{2} \operatorname{arctanh}(c x)^{2}}{2}+\frac{b^{2} x \operatorname{arctanh}(c x)}{c}+\frac{b^{2} \operatorname{arctanh}(c x) \ln (c x-1)}{2 c^{2}}-\frac{b^{2} \operatorname{arctanh}(c x) \ln (c x+1)}{2 c^{2}}+\frac{b^{2} \ln (c x-1)^{2}}{8 c^{2}} \\
& -\frac{b^{2} \ln (c x-1) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{4 c^{2}}+\frac{b^{2} \ln (c x-1)}{2 c^{2}}+\frac{b^{2} \ln (c x+1)}{2 c^{2}}+\frac{b^{2} \ln (c x+1)^{2}}{8 c^{2}}-\frac{b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{4 c^{2}} \\
& +\frac{b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{4 c^{2}}+x^{2} a b \operatorname{arctanh}(c x)+\frac{a b x}{c}+\frac{a b \ln (c x-1)}{2 c^{2}}-\frac{a b \ln (c x+1)}{2 c^{2}}
\end{aligned}
$$

Problem 9: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))^{2}}{x} \mathrm{~d} x
$$

Optimal(type 4, 113 leaves, 6 steps):
$-2(a+b \operatorname{arctanh}(c x))^{2} \operatorname{arctanh}\left(-1+\frac{2}{-c x+1}\right)-b(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)+b(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,-1+\frac{2}{-c x+1}\right)$

$$
+\frac{b^{2} \operatorname{poly} \log \left(3,1-\frac{2}{-c x+1}\right)}{2}-\frac{b^{2} \text { polylog }\left(3,-1+\frac{2}{-c x+1}\right)}{2}
$$

Result(type 4, 700 leaves):
$a^{2} \ln (c x)+b^{2} \ln (c x) \operatorname{arctanh}(c x)^{2}-b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)+\frac{b^{2} \operatorname{polylog}\left(3,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{2}-b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)$ $+b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(1+\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)+2 b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2,-\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)-2 b^{2} \operatorname{polylog}\left(3,-\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)+b^{2} \operatorname{arctanh}(c x)^{2} \ln (1$ $\left.-\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)+2 b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2, \frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)-2 b^{2} \operatorname{polylog}\left(3, \frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)$ $-\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{2} \operatorname{arctanh}(c x)^{2}}{2}+\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{3} \operatorname{arctanh}(c x)^{2}}{2}$
$+\frac{I b^{2} \pi \operatorname{csgn}\left(I\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{arctanh}(c x)^{2}}{2}$
$-\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{2} \operatorname{arctanh}(c x)^{2}}{2}$
$-a b \operatorname{dilog}(c x)$

Problem 10: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))^{2}}{x^{5}} \mathrm{~d} x
$$

Optimal(type 3, 103 leaves, 13 steps):
$-\frac{b^{2} c^{2}}{12 x^{2}}-\frac{b c(a+b \operatorname{arctanh}(c x))}{6 x^{3}}-\frac{b c^{3}(a+b \operatorname{arctanh}(c x))}{2 x}+\frac{c^{4}(a+b \operatorname{arctanh}(c x))^{2}}{4}-\frac{(a+b \operatorname{arctanh}(c x))^{2}}{4 x^{4}}+\frac{2 b^{2} c^{4} \ln (x)}{3}-\frac{b^{2} c^{4} \ln \left(-c^{2} x^{2}+1\right)}{3}$
Result(type 3, 289 leaves):
$-\frac{a^{2}}{4 x^{4}}-\frac{b^{2} \operatorname{arctanh}(c x)^{2}}{4 x^{4}}-\frac{c b^{2} \operatorname{arctanh}(c x)}{6 x^{3}}-\frac{c^{3} b^{2} \operatorname{arctanh}(c x)}{2 x}+\frac{c^{4} b^{2} \operatorname{arctanh}(c x) \ln (c x+1)}{4}-\frac{c^{4} b^{2} \operatorname{arctanh}(c x) \ln (c x-1)}{4}-\frac{b^{2} c^{2}}{12 x^{2}}$

$$
\begin{aligned}
& +\frac{2 c^{4} b^{2} \ln (c x)}{3}-\frac{c^{4} b^{2} \ln (c x+1)}{3}-\frac{c^{4} b^{2} \ln (c x-1)}{3}-\frac{c^{4} b^{2} \ln (c x-1)^{2}}{16}+\frac{c^{4} b^{2} \ln (c x-1) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{8}-\frac{c^{4} b^{2} \ln (c x+1)^{2}}{16} \\
& +\frac{c^{4} b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{8}-\frac{c^{4} b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{8}-\frac{a b \operatorname{arctanh}(c x)}{2 x^{4}}-\frac{c a b}{6 x^{3}}-\frac{c^{3} a b}{2 x}+\frac{c^{4} a b \ln (c x+1)}{4} \\
& -\frac{c^{4} a b \ln (c x-1)}{4}
\end{aligned}
$$

Problem 11: Result more than twice size of optimal antiderivative.

$$
\int(a+b \operatorname{arctanh}(c x))^{3} \mathrm{~d} x
$$

Optimal(type 4, 106 leaves, 5 steps):
$\frac{(a+b \operatorname{arctanh}(c x))^{3}}{c}+x(a+b \operatorname{arctanh}(c x))^{3}-\frac{3 b(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{-c x+1}\right)}{c}-\frac{3 b^{2}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{c}$

$$
+\frac{3 b^{3} \text { polylog }\left(3,1-\frac{2}{-c x+1}\right)}{2 c}
$$

Result(type 4, 260 leaves):
$a^{3} x+b^{3} \operatorname{arctanh}(c x)^{3} x+\frac{b^{3} \operatorname{arctanh}(c x)^{3}}{c}-\frac{3 b^{3} \operatorname{arctanh}(c x)^{2} \ln \left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{c}-\frac{3 b^{3} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{c}$

$$
\begin{aligned}
& +\frac{3 b^{3} \text { polylog }\left(3,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{2 c}+3 \operatorname{arctanh}(c x)^{2} x a b^{2}+\frac{3 a b^{2} \operatorname{arctanh}(c x)^{2}}{c}-\frac{6 \operatorname{arctanh}(c x) \ln \left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right) a b^{2}}{c} \\
& -\frac{3 \text { polylog }\left(2,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right) a b^{2}}{c}+3 a^{2} b \operatorname{arctanh}(c x) x+\frac{3 a^{2} b \ln \left(-c^{2} x^{2}+1\right)}{2 c}
\end{aligned}
$$

Problem 15: Unable to integrate problem.

$$
\int(d x)^{m}(a+b \operatorname{arctanh}(c x)) \mathrm{d} x
$$

Optimal(type 5, 70 leaves, 2 steps):

$$
\frac{(d x)^{1+m}(a+b \operatorname{arctanh}(c x))}{d(1+m)}-\frac{b c(d x)^{2+m} \text { hypergeom }\left(\left[1,1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right], c^{2} x^{2}\right)}{d^{2}(1+m)(2+m)}
$$

Result(type 8, 16 leaves):

$$
\int(d x)^{m}(a+b \operatorname{arctanh}(c x)) \mathrm{d} x
$$

Problem 20: Result more than twice size of optimal antiderivative.

$$
\int x^{7}\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 3, 111 leaves, 12 steps):

$$
\frac{a b x^{2}}{4 c^{3}}+\frac{b^{2} x^{4}}{24 c^{2}}+\frac{b^{2} x^{2} \operatorname{arctanh}\left(c x^{2}\right)}{4 c^{3}}+\frac{b x^{6}\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)}{12 c}-\frac{\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2}}{8 c^{4}}+\frac{x^{8}\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2}}{8}+\frac{b^{2} \ln \left(-c^{2} x^{4}+1\right)}{6 c^{4}}
$$

Result(type 3, 297 leaves):
$\frac{b^{2}\left(x^{8} c^{4}-1\right) \ln \left(c x^{2}+1\right)^{2}}{32 c^{4}}+\frac{b\left(-3 x^{8} b \ln \left(-c x^{2}+1\right) c^{4}+6 a c^{4} x^{8}+2 b c^{3} x^{6}+6 b c x^{2}+3 b \ln \left(-c x^{2}+1\right)\right) \ln \left(c x^{2}+1\right)}{48 c^{4}}+\frac{b^{2} x^{8} \ln \left(-c x^{2}+1\right)^{2}}{32}$

$$
-\frac{a b x^{8} \ln \left(-c x^{2}+1\right)}{8}+\frac{a^{2} x^{8}}{8}-\frac{b^{2} x^{6} \ln \left(-c x^{2}+1\right)}{24 c}+\frac{a b x^{6}}{12 c}+\frac{b^{2} x^{4}}{24 c^{2}}-\frac{b^{2} x^{2} \ln \left(-c x^{2}+1\right)}{8 c^{3}}+\frac{a b x^{2}}{4 c^{3}}-\frac{b^{2} \ln \left(-c x^{2}+1\right)^{2}}{32 c^{4}}+\frac{b \ln \left(-c x^{2}+1\right) a}{8 c^{4}}
$$

$$
+\frac{b^{2} \ln \left(-c x^{2}+1\right)}{6 c^{4}}-\frac{b \ln \left(-c x^{2}-1\right) a}{8 c^{4}}+\frac{b^{2} \ln \left(-c x^{2}-1\right)}{6 c^{4}}
$$

Problem 21: Unable to integrate problem.

$$
\int x^{5}\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 4, 132 leaves, 10 steps):

$$
\begin{aligned}
& \frac{b^{2} x^{2}}{6 c^{2}}-\frac{b^{2} \operatorname{arctanh}\left(c x^{2}\right)}{6 c^{3}}+\frac{b x^{4}\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)}{6 c}+\frac{\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2}}{6 c^{3}}+\frac{x^{6}\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2}}{6}-\frac{b\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right) \ln \left(\frac{2}{-c x^{2}+1}\right)}{3 c^{3}} \\
& \quad-\frac{b^{2} \operatorname{polylog}\left(2,1-\frac{2}{-c x^{2}+1}\right)}{6 c^{3}} \\
& \text { Result (type 8, 18 leaves) : }
\end{aligned}
$$

Problem 22: Result more than twice size of optimal antiderivative.

$$
\int x^{3}\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 3, 81 leaves, 7 steps):

$$
\frac{a b x^{2}}{2 c}+\frac{b^{2} x^{2} \operatorname{arctanh}\left(c x^{2}\right)}{2 c}-\frac{\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2}}{4 c^{2}}+\frac{x^{4}\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2}}{4}+\frac{b^{2} \ln \left(-c^{2} x^{4}+1\right)}{4 c^{2}}
$$

Result (type 3, 246 leaves):

$$
\begin{aligned}
& \frac{b^{2}\left(c^{2} x^{4}-1\right) \ln \left(c x^{2}+1\right)^{2}}{16 c^{2}}+\frac{b\left(-b x^{4} \ln \left(-c x^{2}+1\right) c^{2}+2 a c^{2} x^{4}+2 b c x^{2}+b \ln \left(-c x^{2}+1\right)\right) \ln \left(c x^{2}+1\right)}{8 c^{2}}+\frac{b^{2} x^{4} \ln \left(-c x^{2}+1\right)^{2}}{16}-\frac{a b x^{4} \ln \left(-c x^{2}+1\right)}{4} \\
& \quad+\frac{x^{4} a^{2}}{4}-\frac{b^{2} x^{2} \ln \left(-c x^{2}+1\right)}{4 c}+\frac{a b x^{2}}{2 c}-\frac{b^{2} \ln \left(-c x^{2}+1\right)^{2}}{16 c^{2}}+\frac{b \ln \left(-c x^{2}+1\right) a}{4 c^{2}}+\frac{b^{2} \ln \left(-c x^{2}+1\right)}{4 c^{2}}-\frac{b \ln \left(-c x^{2}-1\right) a}{4 c^{2}}+\frac{b^{2} \ln \left(-c x^{2}-1\right)}{4 c^{2}}
\end{aligned}
$$

Problem 23: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2}}{x^{5}} \mathrm{~d} x
$$

Optimal(type 3, 80 leaves, 9 steps):

$$
-\frac{b c\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)}{2 x^{2}}+\frac{c^{2}\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2}}{4}-\frac{\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2}}{4 x^{4}}+b^{2} c^{2} \ln (x)-\frac{b^{2} c^{2} \ln \left(-c^{2} x^{4}+1\right)}{4}
$$

Result(type 3, 256 leaves):
$\frac{b^{2}\left(c^{2} x^{4}-1\right) \ln \left(c x^{2}+1\right)^{2}}{16 x^{4}}-\frac{b\left(b x^{4} \ln \left(-c x^{2}+1\right) c^{2}+2 b c x^{2}-b \ln \left(-c x^{2}+1\right)+2 a\right) \ln \left(c x^{2}+1\right)}{8 x^{4}}-\frac{1}{16 x^{4}}\left(-b^{2} c^{2} x^{4} \ln \left(-c x^{2}+1\right)^{2}+4 b c^{2} \ln \left(c x^{2}\right.\right.$
$-1) x^{4} a+4 b^{2} c^{2} \ln \left(c x^{2}-1\right) x^{4}-4 b c^{2} \ln \left(c x^{2}+1\right) x^{4} a+4 b^{2} c^{2} \ln \left(c x^{2}+1\right) x^{4}-16 b^{2} c^{2} \ln (x) x^{4}-4 b^{2} c x^{2} \ln \left(-c x^{2}+1\right)+8 a b c x^{2}+b^{2} \ln \left(-c x^{2}\right.$ $\left.+1)^{2}-4 b \ln \left(-c x^{2}+1\right) a+4 a^{2}\right)$

Problem 24: Unable to integrate problem.

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{3}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 4, 115 leaves, 6 steps):
$\frac{c\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{3}}{2}-\frac{\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{3}}{2 x^{2}}+\frac{3 b c\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2} \ln \left(2-\frac{2}{c x^{2}+1}\right)}{2}$
$-\frac{3 b^{2} c\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right) \operatorname{polylog}\left(2,-1+\frac{2}{c x^{2}+1}\right)}{2}-\frac{3 b^{3} c \operatorname{polylog}\left(3,-1+\frac{2}{c x^{2}+1}\right)}{4}$
Result(type 8, 18 leaves):

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{3}}{x^{3}} \mathrm{~d} x
$$

Problem 25: Unable to integrate problem.

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{3}}{x^{5}} \mathrm{~d} x
$$

Optimal(type 4, 127 leaves, 8 steps):
$\frac{3 b c^{2}\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2}}{4}-\frac{3 b c\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2}}{4 x^{2}}+\frac{c^{2}\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{3}}{4}-\frac{\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{3}}{4 x^{4}}$

$$
+\frac{3 b^{2} c^{2}\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right) \ln \left(2-\frac{2}{c x^{2}+1}\right)}{2}-\frac{3 b^{3} c^{2} \operatorname{polylog}\left(2,-1+\frac{2}{c x^{2}+1}\right)}{4}
$$

Result(type 8, 18 leaves):

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{3}}{x^{5}} \mathrm{~d} x
$$

Problem 34: Result more than twice size of optimal antiderivative.

$$
\int x^{11}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2} \mathrm{~d} x
$$

Optimal (type 3, 111 leaves, 12 steps):

$$
\frac{a b x^{3}}{6 c^{3}}+\frac{b^{2} x^{6}}{36 c^{2}}+\frac{b^{2} x^{3} \operatorname{arctanh}\left(c x^{3}\right)}{6 c^{3}}+\frac{b x^{9}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)}{18 c}-\frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{12 c^{4}}+\frac{x^{12}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{12}+\frac{b^{2} \ln \left(-c^{2} x^{6}+1\right)}{9 c^{4}}
$$

Result(type 3, 297 leaves):

$$
\begin{aligned}
& \frac{b^{2}\left(x^{12} c^{4}-1\right) \ln \left(c x^{3}+1\right)^{2}}{48 c^{4}}+\frac{b\left(-3 x^{12} b \ln \left(-c x^{3}+1\right) c^{4}+6 a c^{4} x^{12}+2 b c^{3} x^{9}+6 b c x^{3}+3 b \ln \left(-c x^{3}+1\right)\right) \ln \left(c x^{3}+1\right)}{72 c^{4}}+\frac{b^{2} x^{12} \ln \left(-c x^{3}+1\right)^{2}}{48} \\
& \quad-\frac{a b x^{12} \ln \left(-c x^{3}+1\right)}{12}+\frac{a^{2} x^{12}}{12}-\frac{b^{2} x^{9} \ln \left(-c x^{3}+1\right)}{36 c}+\frac{a b x^{9}}{18 c}+\frac{b^{2} x^{6}}{36 c^{2}}-\frac{b^{2} x^{3} \ln \left(-c x^{3}+1\right)}{12 c^{3}}+\frac{a b x^{3}}{6 c^{3}}-\frac{b^{2} \ln \left(-c x^{3}+1\right)^{2}}{48 c^{4}}-\frac{b \ln \left(-c x^{3}-1\right) a}{12 c^{4}} \\
& \quad+\frac{b^{2} \ln \left(-c x^{3}-1\right)}{9 c^{4}}+\frac{b \ln \left(-c x^{3}+1\right) a}{12 c^{4}}+\frac{b^{2} \ln \left(-c x^{3}+1\right)}{9 c^{4}}
\end{aligned}
$$

Problem 35: Unable to integrate problem.

$$
\int x^{8}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 4, 132 leaves, 10 steps):

$$
\begin{aligned}
& \frac{b^{2} x^{3}}{9 c^{2}}-\frac{b^{2} \operatorname{arctanh}\left(c x^{3}\right)}{9 c^{3}}+\frac{b x^{6}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)}{9 c}+\frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{9 c^{3}}+\frac{x^{9}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{9}-\frac{2 b\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right) \ln \left(\frac{2}{-c x^{3}+1}\right)}{9 c^{3}} \\
& \quad-\frac{b^{2} \operatorname{polylog}\left(2,1-\frac{2}{-c x^{3}+1}\right)}{9 c^{3}}
\end{aligned}
$$

Result(type 8, 18 leaves):

$$
\int x^{8}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2} \mathrm{~d} x
$$

Problem 37: Unable to integrate problem.

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{x^{4}} \mathrm{~d} x
$$

Optimal(type 4, 82 leaves, 5 steps):

$$
\frac{c\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{3}-\frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{3 x^{3}}+\frac{2 b c\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right) \ln \left(2-\frac{2}{c x^{3}+1}\right)}{3}-\frac{b^{2} c \operatorname{poly} \log \left(2,-1+\frac{2}{c x^{3}+1}\right)}{3}
$$

Result(type 8, 18 leaves):

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{x^{4}} \mathrm{~d} x
$$

Problem 38: Unable to integrate problem.

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{x^{10}} \mathrm{~d} x
$$

Optimal(type 4, 130 leaves, 9 steps):

$$
\begin{aligned}
& -\frac{b^{2} c^{2}}{9 x^{3}}+\frac{b^{2} c^{3} \operatorname{arctanh}\left(c x^{3}\right)}{9}-\frac{b c\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)}{9 x^{6}}+\frac{c^{3}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{9}-\frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{9 x^{9}} \\
& \quad+\frac{2 b c^{3}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right) \ln \left(2-\frac{2}{c x^{3}+1}\right)}{9}-\frac{b^{2} c^{3} \operatorname{polylog}\left(2,-1+\frac{2}{c x^{3}+1}\right)}{9}
\end{aligned}
$$

Result(type 8, 18 leaves):

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{x^{10}} \mathrm{~d} x
$$

Problem 39: Unable to integrate problem.

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 1, 1 leaves, 47 steps):
Result(type 8, 18 leaves):

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{x^{2}} \mathrm{~d} x
$$

Problem 40: Unable to integrate problem.

$$
\int x^{5}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{3} \mathrm{~d} x
$$

Optimal(type 4, 129 leaves, 9 steps):
$\frac{b\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{2 c^{2}}+\frac{b x^{3}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{2}}{2 c}-\frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{3}}{6 c^{2}}+\frac{x^{6}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{3}}{6}-\frac{b^{2}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right) \ln \left(\frac{2}{-c x^{3}+1}\right)}{c^{2}}$
$-\frac{b^{3} \operatorname{poly} \log \left(2,1-\frac{2}{-c x^{3}+1}\right)}{2 c^{2}}$
Result(type 8, 18 leaves):

$$
\int x^{5}\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{3} \mathrm{~d} x
$$

Problem 41: Unable to integrate problem.

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{3}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 1, 1 leaves, 47 steps):
0
Result(type 8, 18 leaves):

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(c x^{3}\right)\right)^{3}}{x^{2}} \mathrm{~d} x
$$

Problem 44: Result more than twice size of optimal antiderivative.

$$
\int \frac{a+b \operatorname{arctanh}\left(\frac{c}{x}\right)}{x} \mathrm{~d} x
$$

Optimal(type 4, 26 leaves, 2 steps):

$$
a \ln (x)+\frac{b \text { polylog }\left(2,-\frac{c}{x}\right)}{2}-\frac{b \text { polylog }\left(2, \frac{c}{x}\right)}{2}
$$

Result(type 4, 62 leaves):

$$
-a \ln \left(\frac{c}{x}\right)-b \ln \left(\frac{c}{x}\right) \operatorname{arctanh}\left(\frac{c}{x}\right)+\frac{b \operatorname{dilog}\left(1+\frac{c}{x}\right)}{2}+\frac{b \ln \left(\frac{c}{x}\right) \ln \left(1+\frac{c}{x}\right)}{2}+\frac{b \operatorname{dilog}\left(\frac{c}{x}\right)}{2}
$$

Problem 46: Result more than twice size of optimal antiderivative.

$$
\int x^{3}\left(a+b \operatorname{arctanh}\left(\frac{c}{x}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 3, 109 leaves, 14 steps):

$$
\begin{aligned}
& \frac{b^{2} c^{2} x^{2}}{12}+\frac{b c^{3} x\left(a+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right)}{2}+\frac{b c x^{3}\left(a+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right)}{6}-\frac{c^{4}\left(a+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right)^{2}}{4}+\frac{x^{4}\left(a+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right)^{2}}{4}+\frac{b^{2} c^{4} \ln \left(1-\frac{c^{2}}{x^{2}}\right)}{3} \\
& \quad+\frac{2 b^{2} c^{4} \ln (x)}{3}
\end{aligned}
$$

Result(type 3, 327 leaves):

$$
\begin{aligned}
\frac{x^{4} a^{2}}{4} & +\frac{b^{2} x^{4} \operatorname{arctanh}\left(\frac{c}{x}\right)^{2}}{4}+\frac{c b^{2} \operatorname{arctanh}\left(\frac{c}{x}\right) x^{3}}{6}+\frac{c^{3} b^{2} \operatorname{arctanh}\left(\frac{c}{x}\right) x}{2}-\frac{c^{4} b^{2} \operatorname{arctanh}\left(\frac{c}{x}\right) \ln \left(1+\frac{c}{x}\right)}{4}+\frac{c^{4} b^{2} \operatorname{arctanh}\left(\frac{c}{x}\right) \ln \left(\frac{c}{x}-1\right)}{4}+\frac{b^{2} c^{2} x^{2}}{12} \\
& -\frac{2 c^{4} b^{2} \ln \left(\frac{c}{x}\right)}{3}+\frac{c^{4} b^{2} \ln \left(1+\frac{c}{x}\right)}{3}+\frac{c^{4} b^{2} \ln \left(\frac{c}{x}-1\right)}{3}+\frac{c^{4} b^{2} \ln \left(\frac{c}{x}-1\right)^{2}}{16}-\frac{c^{4} b^{2} \ln \left(\frac{c}{x}-1\right) \ln \left(\frac{c}{2 x}+\frac{1}{2}\right)}{8}+\frac{c^{4} b^{2} \ln \left(1+\frac{c}{x}\right)^{2}}{16} \\
& -\frac{c^{4} b^{2} \ln \left(-\frac{c}{2 x}+\frac{1}{2}\right) \ln \left(1+\frac{c}{x}\right)}{8}+\frac{c^{4} b^{2} \ln \left(-\frac{c}{2 x}+\frac{1}{2}\right) \ln \left(\frac{c}{2 x}+\frac{1}{2}\right)}{8}+\frac{a b x^{4} \operatorname{arctanh}\left(\frac{c}{x}\right)}{2}+\frac{a b c x^{3}}{6}+\frac{c^{3} a b x}{2}-\frac{c^{4} a b \ln \left(1+\frac{c}{x}\right)}{4} \\
& +\frac{c^{4} a b \ln \left(\frac{c}{x}-1\right)}{4}
\end{aligned}
$$

Problem 47: Result more than twice size of optimal antiderivative.

$$
\int x\left(a+b \operatorname{arctanh}\left(\frac{c}{x}\right)\right)^{3} \mathrm{~d} x
$$

Optimal(type 4, 125 leaves, 8 steps):
$-\frac{3 b c^{2}\left(a+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right)^{2}}{2}+\frac{3 b c x\left(a+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right)^{2}}{2}-\frac{c^{2}\left(a+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right)^{3}}{2}+\frac{x^{2}\left(a+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right)^{3}}{2}-3 b^{2} c^{2}(a$
$\left.+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right) \ln \left(2-\frac{2}{1+\frac{c}{x}}\right)+\frac{3 b^{3} c^{2} \text { polylog }\left(2,-1+\frac{2}{1+\frac{c}{x}}\right)}{2}$
Result(type ?, 5589 leaves): Display of huge result suppressed!
Problem 48: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(\frac{c}{x}\right)\right)^{3}}{x} \mathrm{~d} x
$$

Optimal(type 4, 196 leaves, 9 steps):

$$
\begin{aligned}
& \left.+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right)^{3} \operatorname{arctanh}\left(-1+\frac{2}{1-\frac{c}{x}}\right)+\frac{3 b\left(a+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right)^{2} \operatorname{polylog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)}{2} \\
& -\frac{3 b\left(a+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right)^{2} \operatorname{polylog}\left(2,-1+\frac{2}{1-\frac{c}{x}}\right)}{2}-\frac{3 b^{2}\left(a+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right) \operatorname{polylog}\left(3,1-\frac{2}{1-\frac{c}{x}}\right)}{\left.1-\frac{c}{x}\right)} \\
& +\frac{3 b^{2}\left(a+b \operatorname{arccoth}\left(\frac{x}{c}\right)\right) \operatorname{polylog}\left(3,-1+\frac{2}{1-\frac{c}{x}}\right)}{2}-\frac{3 b^{3} \operatorname{polylog}\left(4,1-\frac{2}{1-\frac{c}{x}}\right)}{3 b^{3} \operatorname{polylog}\left(4,-1+\frac{2}{1-\frac{c}{x}}\right)}
\end{aligned}
$$

Result(type 4, 1630 leaves):

$$
\begin{aligned}
& -b^{3} \ln \left(\frac{c}{x}\right) \operatorname{arctanh}\left(\frac{c}{x}\right)^{3}+b^{3} \operatorname{arctanh}\left(\frac{c}{x}\right)^{3} \ln \left(\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}-1\right)+\frac{3 b^{3} \operatorname{arctanh}\left(\frac{c}{x}\right)^{2} \operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}\right)}{2} \\
& -\frac{3 b^{3} \operatorname{arctanh}\left(\frac{c}{x}\right) \operatorname{polylog}\left(3,-\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}\right)}{2}-b^{3} \operatorname{arctanh}\left(\frac{c}{x}\right)^{3} \ln \left(1+\frac{1+\frac{c}{x}}{\sqrt{1-\frac{c^{2}}{x^{2}}}}\right)-3 b^{3} \operatorname{arctanh}\left(\frac{c}{x}\right)^{2} \operatorname{polylog}\left(2,-\frac{1+\frac{c}{x}}{\sqrt{1-\frac{c^{2}}{x^{2}}}}\right)
\end{aligned}
$$

$+6 b^{3} \operatorname{arctanh}\left(\frac{c}{x}\right) \operatorname{polylog}\left(3,-\frac{1+\frac{c}{x}}{\sqrt{1-\frac{c^{2}}{x^{2}}}}\right)-b^{3} \operatorname{arctanh}\left(\frac{c}{x}\right)^{3} \ln \left(1-\frac{1+\frac{c}{x}}{\sqrt{1-\frac{c^{2}}{x^{2}}}}\right)-3 b^{3} \operatorname{arctanh}\left(\frac{c}{x}\right)^{2} \operatorname{poly} \log \left(2, \frac{1+\frac{c}{x}}{\sqrt{1-\frac{c^{2}}{x^{2}}}}\right)$
$+6 b^{3} \operatorname{arctanh}\left(\frac{c}{x}\right)$ polylog $\left(3, \frac{1+\frac{c}{x}}{\sqrt{1-\frac{c^{2}}{x^{2}}}}\right)-\frac{3 a b^{2} \text { polylog }\left(3,-\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}\right)}{2}+6 a b^{2} \operatorname{polylog}\left(3,-\frac{1+\frac{c}{x}}{\left.\sqrt{1-\frac{c^{2}}{x^{2}}}\right)}+6 a b\right.$
$+\frac{3 a^{2} b \operatorname{dilog}\left(1+\frac{c}{x}\right)}{2}+\frac{3 a^{2} b \operatorname{dilog}\left(\frac{c}{x}\right)}{2}-3 a b^{2} \ln \left(\frac{c}{x}\right) \operatorname{arctanh}\left(\frac{c}{x}\right)^{2}+3 a b^{2} \operatorname{arctanh}\left(\frac{c}{x}\right) \operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}\right)$
$+3 a b^{2} \operatorname{arctanh}\left(\frac{c}{x}\right)^{2} \ln \left(\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}-1\right)-3 a b^{2} \operatorname{arctanh}\left(\frac{c}{x}\right)^{2} \ln \left(1+\frac{1+\frac{c}{x}}{\sqrt{1-\frac{c^{2}}{x^{2}}}}\right)-6 a b^{2} \operatorname{arctanh}\left(\frac{c}{x}\right) \operatorname{polylog}\left(2,-\frac{1+\frac{c}{x}}{\sqrt{1-\frac{c^{2}}{x^{2}}}}\right)$
$-3 a b^{2} \operatorname{arctanh}\left(\frac{c}{x}\right)^{2} \ln \left(1-\frac{1+\frac{c}{x}}{\sqrt{1-\frac{c^{2}}{x^{2}}}}\right)-6 a b^{2} \operatorname{arctanh}\left(\frac{c}{x}\right) \operatorname{polylog}\left(2, \frac{1+\frac{c}{x}}{\sqrt{1-\frac{c^{2}}{x^{2}}}}\right)-3 a^{2} b \ln \left(\frac{c}{x}\right) \operatorname{arctanh}\left(\frac{c}{x}\right)+\frac{3 a^{2} b \ln \left(\frac{c}{x}\right) \ln \left(1+\frac{c}{x}\right)}{2}$
$\left.-\frac{\mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\left(\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}-1\right)}{1+\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}}\right) \operatorname{arctanh}\left(\frac{c}{x}\right)^{3}}{2}\right)$

$$
\begin{aligned}
& \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}-1\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}-1\right)}{2}\right) \operatorname{arctanh}\left(\frac{c}{x}\right)^{3}\right. \\
& +\frac{\left.3 \mathrm{I} a b^{2} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}-1\right)\right) \operatorname{csgn}\left(\frac{\left(\mathrm{I}\left(\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}-1\right)\right.}{1+\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}}\right)\right)^{2} \operatorname{arctanh}\left(\frac{c}{x}\right)^{2}}{2} \\
& +\frac{\left.3 \mathrm{I} a b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}-1\right)}{1+\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}}\right)\right)^{2} \operatorname{arctanh}\left(\frac{c}{x}\right)^{2}}{2}+ \\
& +\frac{3 b^{3} \text { polylog }\left(4,-\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}\right)}{4}-6 b^{3} \text { polylog }\left(4,-\frac{1+\frac{c}{x}}{\sqrt{1-\frac{c^{2}}{x^{2}}}}\right) \\
& -6 b^{3} \text { polylog }\left(4, \frac{1+\frac{c}{x}}{\sqrt{1-\frac{c^{2}}{x^{2}}}}\right)-a^{3} \ln \left(\frac{c}{x}\right)
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{3 \mathrm{I} a b^{2} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}-1\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}-1\right)}{2}\right) \operatorname{arctanh}\left(\frac{c}{x}\right)^{2}\right)}{\left.1+\frac{\left.1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}\right)} \\
& I b^{3} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}-1\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}-1\right)}{1+\frac{\left(1+\frac{c}{x}\right)^{2}}{1-\frac{c^{2}}{x^{2}}}}\right)^{2} \operatorname{arctanh}\left(\frac{c}{x}\right)^{3}\right.
\end{aligned}
$$

Problem 49: Attempted integration timed out after 120 seconds.

$$
\int \frac{\left(a+b \operatorname{arctanh}\left(\frac{c}{x^{2}}\right)\right)^{2}}{x^{5}} \mathrm{~d} x
$$

Optimal(type 3, 87 leaves, 7 steps):

$$
-\frac{a b}{2 c x^{2}}-\frac{b^{2} \operatorname{arccoth}\left(\frac{x^{2}}{c}\right)}{2 c x^{2}}+\frac{\left(a+b \operatorname{arccoth}\left(\frac{x^{2}}{c}\right)\right)^{2}}{4 c^{2}}-\frac{\left(a+b \operatorname{arccoth}\left(\frac{x^{2}}{c}\right)\right)^{2}}{4 x^{4}}-\frac{b^{2} \ln \left(1-\frac{c^{2}}{x^{4}}\right)}{4 c^{2}}
$$

## Result(type 1, 1 leaves):???

Problem 50: Unable to integrate problem.

$$
\int x^{2}\left(a+b \operatorname{arctanh}\left(\frac{c}{x^{2}}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 4, 895 leaves, 80 steps):

$$
\frac{\mathrm{I} b^{2} c^{3 / 2} \text { polylog }\left(2,-1+\frac{2 \sqrt{c}}{-\mathrm{I} x+\sqrt{c}}\right)}{3}+\frac{\mathrm{I} b^{2} c^{3 / 2} \operatorname{polylog}\left(2,1-\frac{(1+\mathrm{I})(-x+\sqrt{c})}{-\mathrm{I} x+\sqrt{c}}\right)}{6}+\frac{2 b^{2} c x \ln \left(1+\frac{c}{x^{2}}\right)}{3}+\frac{a b x^{3} \ln \left(1+\frac{c}{x^{2}}\right)}{3}
$$

$$
\begin{aligned}
& -\frac{b^{2} c^{3 / 2} \arctan \left(\frac{x}{\sqrt{c}}\right) \ln \left(1+\frac{c}{x^{2}}\right)}{3}-\frac{b^{2} c^{3 / 2} \operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right) \ln \left(1+\frac{c}{x^{2}}\right)}{3}-\frac{b^{2} x^{3} \ln \left(1-\frac{c}{x^{2}}\right) \ln \left(1+\frac{c}{x^{2}}\right)}{6} \\
& +\frac{2 b^{2} c^{3 / 2} \arctan \left(\frac{x}{\sqrt{c}}\right) \ln \left(\frac{2 \sqrt{c}}{-\mathrm{I} x+\sqrt{c}}\right)}{3}-\frac{b^{2} c^{3 / 2} \arctan \left(\frac{x}{\sqrt{c}}\right) \ln \left(\frac{(1+\mathrm{I})(-x+\sqrt{c})}{-\mathrm{I} x+\sqrt{c}}\right)}{3}-\frac{2 b^{2} c^{3 / 2} \operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right) \ln \left(\frac{2 \sqrt{c}}{x+\sqrt{c}}\right)}{3}
\end{aligned}
$$

$$
+\frac{b^{2} c^{3 / 2} \operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right) \ln \left(\frac{2(-x+\sqrt{-c}) \sqrt{c}}{(\sqrt{-c}-\sqrt{c})(x+\sqrt{c})}\right)}{3}-\frac{b^{2} c^{3 / 2} \arctan \left(\frac{x}{\sqrt{c}}\right) \ln \left(\frac{(1-\mathrm{I})(x+\sqrt{c})}{-\mathrm{I} x+\sqrt{c}}\right)}{3}
$$

$$
+\frac{b^{2} c^{3 / 2} \operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right) \ln \left(\frac{2(x+\sqrt{-c}) \sqrt{c}}{(x+\sqrt{c})(\sqrt{-c}+\sqrt{c})}\right)}{3}-\frac{2 b^{2} c^{3 / 2} \arctan \left(\frac{x}{\sqrt{c}}\right) \ln \left(2-\frac{2 \sqrt{c}}{-\mathrm{I} x+\sqrt{c}}\right)}{3}
$$

$$
+\frac{2 b^{2} c^{3 / 2} \operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right) \ln \left(2-\frac{2 \sqrt{c}}{x+\sqrt{c}}\right)}{3}-\frac{\mathrm{I} b^{2} c^{3 / 2} \operatorname{polylog}\left(2, \frac{\mathrm{I} x}{\sqrt{c}}\right)}{3}-\frac{\mathrm{I} b^{2} c^{3 / 2} \operatorname{polylog}\left(2,1-\frac{2 \sqrt{c}}{-\mathrm{I} x+\sqrt{c}}\right)}{3}-\frac{2 a b c^{3 / 2} \arctan \left(\frac{x}{\sqrt{c}}\right)}{3}
$$

$$
-\frac{2 b^{2} c x \ln \left(1-\frac{c}{x^{2}}\right)}{3}+\frac{b^{2} c^{3 / 2} \arctan \left(\frac{x}{\sqrt{c}}\right) \ln \left(1-\frac{c}{x^{2}}\right)}{3}-\frac{b c^{3 / 2} \operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\left(2 a-b \ln \left(1-\frac{c}{x^{2}}\right)\right)}{3}+\frac{4 b^{2} c^{3 / 2} \arctan \left(\frac{x}{\sqrt{c}}\right)}{3}
$$

$$
-\frac{4 b^{2} c^{3 / 2} \operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)}{3}+\frac{b^{2} c^{3 / 2} \operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)^{2}}{3}+\frac{b^{2} x^{3} \ln \left(1+\frac{c}{x^{2}}\right)^{2}}{12}+\frac{b^{2} c^{3 / 2} \operatorname{polylog}\left(2,-\frac{x}{\sqrt{c}}\right)}{3}-\frac{b^{2} c^{3 / 2} \operatorname{polylog}\left(2, \frac{x}{\sqrt{c}}\right)}{3}
$$

$$
+\frac{b^{2} c^{3 / 2} \operatorname{polylog}\left(2,1-\frac{2 \sqrt{c}}{x+\sqrt{c}}\right)}{3}-\frac{b^{2} c^{3 / 2} \operatorname{polylog}\left(2,-1+\frac{2 \sqrt{c}}{x+\sqrt{c}}\right)}{3}-\frac{b^{2} c^{3 / 2} \operatorname{polylog}\left(2,1-\frac{2(-x+\sqrt{-c}) \sqrt{c}}{(\sqrt{-c}-\sqrt{c})(x+\sqrt{c})}\right)}{6}
$$

$$
-\frac{b^{2} c^{3 / 2} \operatorname{polylog}\left(2,1-\frac{2(x+\sqrt{-c}) \sqrt{c}}{(x+\sqrt{c})(\sqrt{-c}+\sqrt{c})}\right)}{6}+\frac{x^{3}\left(2 a-b \ln \left(1-\frac{c}{x^{2}}\right)\right)^{2}}{12}+\frac{\mathrm{I} b^{2} c^{3 / 2} \operatorname{polylog}\left(2,1+\frac{(-1+\mathrm{I})(x+\sqrt{c})}{-\mathrm{I} x+\sqrt{c}}\right)}{6}
$$

$$
+\frac{\mathrm{I} b^{2} c^{3 / 2} \arctan \left(\frac{x}{\sqrt{c}}\right)^{2}}{3}+\frac{\mathrm{I} b^{2} c^{3 / 2} \operatorname{polylog}\left(2, \frac{-\mathrm{I} x}{\sqrt{c}}\right)}{3}+\frac{4 a b c x}{3}
$$

Result(type 8, 18 leaves):

$$
\int x^{2}\left(a+b \operatorname{arctanh}\left(\frac{c}{x^{2}}\right)\right)^{2} \mathrm{~d} x
$$

Problem 51: Unable to integrate problem.

$$
\int(d x)^{m}\left(a+b \operatorname{arctanh}\left(\frac{c}{x^{2}}\right)\right) \mathrm{d} x
$$

Optimal(type 5, 69 leaves, 3 steps):

$$
\frac{(d x)^{1+m}\left(a+b \operatorname{arctanh}\left(\frac{c}{x^{2}}\right)\right)}{d(1+m)}-\frac{2 b c d(d x)^{-1+m} \text { hypergeom }\left(\left[1, \frac{1}{4}-\frac{m}{4}\right],\left[\frac{5}{4}-\frac{m}{4}\right], \frac{c^{2}}{x^{4}}\right)}{-m^{2}+1}
$$

Result(type 8, 18 leaves):

$$
\int(d x)^{m}\left(a+b \operatorname{arctanh}\left(\frac{c}{x^{2}}\right)\right) \mathrm{d} x
$$

Problem 55: Result more than twice size of optimal antiderivative.

$$
\int x^{3}(a+b \operatorname{arctanh}(c \sqrt{x}))^{2} \mathrm{~d} x
$$

Optimal(type 3, 167 leaves, 22 steps):

$$
\begin{aligned}
& \frac{71 b^{2} x}{420 c^{6}}+\frac{3 b^{2} x^{2}}{70 c^{4}}+\frac{b^{2} x^{3}}{84 c^{2}}+\frac{b x^{3 / 2}(a+b \operatorname{arctanh}(c \sqrt{x}))}{6 c^{5}}+\frac{b x^{5 / 2}(a+b \operatorname{arctanh}(c \sqrt{x}))}{10 c^{3}}+\frac{b x^{7 / 2}(a+b \operatorname{arctanh}(c \sqrt{x}))}{14 c} \\
& -\frac{(a+b \operatorname{arctanh}(c \sqrt{x}))^{2}}{4 c^{8}}+\frac{x^{4}(a+b \operatorname{arctanh}(c \sqrt{x}))^{2}}{4}+\frac{44 b^{2} \ln \left(-c^{2} x+1\right)}{105 c^{8}}+\frac{a b \sqrt{x}}{2 c^{7}}+\frac{b^{2} \operatorname{arctanh}(c \sqrt{x}) \sqrt{x}}{2 c^{7}}
\end{aligned}
$$

Result(type 3, 395 leaves):

$$
\begin{aligned}
& \frac{x^{4} a^{2}}{4}+\frac{a b \sqrt{x}}{2 c^{7}}+\frac{b^{2} \operatorname{arctanh}(c \sqrt{x}) \sqrt{x}}{2 c^{7}}+\frac{71 b^{2} x}{420 c^{6}}+\frac{3 b^{2} x^{2}}{70 c^{4}}+\frac{b^{2} x^{3}}{84 c^{2}}+\frac{x^{3 / 2} a b}{6 c^{5}}+\frac{b^{2} \operatorname{arctanh}(c \sqrt{x}) x^{3} / 2}{6 c^{5}}+\frac{b^{2} x^{7} / 2 \operatorname{arctanh}(c \sqrt{x})}{14 c} \\
& \quad+\frac{b^{2} \operatorname{arctanh}(c \sqrt{x}) x^{5 / 2}}{10 c^{3}}+\frac{a b \ln (c \sqrt{x}-1)}{4 c^{8}}-\frac{a b \ln (1+c \sqrt{x})}{4 c^{8}}-\frac{b^{2} \operatorname{arctanh}(c \sqrt{x}) \ln (1+c \sqrt{x})}{4 c^{8}}-\frac{b^{2} \ln (c \sqrt{x}-1) \ln \left(\frac{c \sqrt{x}}{2}+\frac{1}{2}\right)}{8 c^{8}} \\
& \quad-\frac{b^{2} \ln \left(-\frac{c \sqrt{x}}{2}+\frac{1}{2}\right) \ln (1+c \sqrt{x})}{8 c^{8}}+\frac{b^{2} \ln \left(-\frac{c \sqrt{x}}{2}+\frac{1}{2}\right) \ln \left(\frac{c \sqrt{x}}{2}+\frac{1}{2}\right)}{8 c^{8}}+\frac{b^{2} \operatorname{arctanh}(c \sqrt{x}) \ln (c \sqrt{x}-1)}{4 c^{8}}+\frac{a b x^{5 / 2}}{10 c^{3}}+\frac{x^{7 / 2} a b}{14 c}
\end{aligned}
$$

$$
+\frac{a b x^{4} \operatorname{arctanh}(c \sqrt{x})}{2}+\frac{b^{2} x^{4} \operatorname{arctanh}(c \sqrt{x})^{2}}{4}+\frac{44 b^{2} \ln (c \sqrt{x}-1)}{105 c^{8}}+\frac{44 b^{2} \ln (1+c \sqrt{x})}{105 c^{8}}+\frac{b^{2} \ln (c \sqrt{x}-1)^{2}}{16 c^{8}}+\frac{b^{2} \ln (1+c \sqrt{x})^{2}}{16 c^{8}}
$$

Problem 56: Result more than twice size of optimal antiderivative.

$$
\int x(a+b \operatorname{arctanh}(c \sqrt{x}))^{3} \mathrm{~d} x
$$

Optimal(type 4, 194 leaves, 19 steps):

$$
\begin{aligned}
& -\frac{b^{3} \operatorname{arctanh}(c \sqrt{x})}{2 c^{4}}+\frac{b^{2} x(a+b \operatorname{arctanh}(c \sqrt{x}))}{2 c^{2}}+\frac{2 b(a+b \operatorname{arctanh}(c \sqrt{x}))^{2}}{c^{4}}+\frac{b x^{3} / 2(a+b \operatorname{arctanh}(c \sqrt{x}))^{2}}{2 c}-\frac{(a+b \operatorname{arctanh}(c \sqrt{x}))^{3}}{2 c^{4}} \\
& \quad+\frac{x^{2}(a+b \operatorname{arctanh}(c \sqrt{x}))^{3}}{2}-\frac{4 b^{2}(a+b \operatorname{arctanh}(c \sqrt{x})) \ln \left(\frac{2}{1-c \sqrt{x}}\right)}{c^{4}}-\frac{2 b^{3} \operatorname{polylog}\left(2,1-\frac{2}{1-c \sqrt{x}}\right)}{c^{4}}+\frac{b^{3} \sqrt{x}}{2 c^{3}} \\
& \quad+\frac{3 b(a+b \operatorname{arctanh}(c \sqrt{x}))^{2} \sqrt{x}}{2 c^{3}}
\end{aligned}
$$

Result(type 4, 1347 leaves)

$$
-\frac{3 a b^{2} \ln (c \sqrt{x}-1) \ln \left(\frac{c \sqrt{x}}{2}+\frac{1}{2}\right)}{4 c^{4}}+\frac{3 a b^{2} \ln \left(-\frac{c \sqrt{x}}{2}+\frac{1}{2}\right) \ln \left(\frac{c \sqrt{x}}{2}+\frac{1}{2}\right)}{4 c^{4}}-\frac{3 a b^{2} \ln \left(-\frac{c \sqrt{x}}{2}+\frac{1}{2}\right) \ln (1+c \sqrt{x})}{4 c^{4}}+\frac{3 a b^{2} \operatorname{arctanh}(c \sqrt{x}) \sqrt{x}}{c^{3}}
$$

$$
+\frac{a b^{2} x^{3 / 2} \operatorname{arctanh}(c \sqrt{x})}{c}-\frac{3 I b^{3} \pi \operatorname{arctanh}(c \sqrt{x})^{2}}{4 c^{4}}+\frac{3 a b^{2} \operatorname{arctanh}(c \sqrt{x}) \ln (c \sqrt{x}-1)}{2 c^{4}}-\frac{3 a b^{2} \operatorname{arctanh}(c \sqrt{x}) \ln (1+c \sqrt{x})}{2 c^{4}}
$$

$$
-\frac{b^{3} \operatorname{arctanh}(c \sqrt{x})^{3}}{2 c^{4}}-\frac{4 b^{3} \operatorname{dilog}\left(1-\frac{\mathrm{I}(1+c \sqrt{x})}{\sqrt{-c^{2} x+1}}\right)}{c^{4}}-\frac{4 b^{3} \operatorname{dilog}\left(1+\frac{\mathrm{I}(1+c \sqrt{x})}{\sqrt{-c^{2} x+1}}\right)}{c^{4}}+\frac{2 b^{3} \operatorname{arctanh}(c \sqrt{x})^{2}}{c^{4}}+\frac{b^{3} x^{2} \operatorname{arctanh}(c \sqrt{x})^{3}}{2}
$$

$$
+\frac{b^{3} \sqrt{x}}{2 c^{3}}-\frac{b^{3} \operatorname{arctanh}(c \sqrt{x})}{2 c^{4}}-\frac{b^{3}}{2 c^{4}}
$$

$$
-\frac{3 \operatorname{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{\left(-c^{2} x+1\right)\left(1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)}\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{\left.1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)}\right) \operatorname{arctanh}(c \sqrt{x})^{2}}{8 c^{4}}-\frac{3 a^{2} b \ln (1+c \sqrt{x})}{4 c^{4}}
$$

$$
+\frac{3 a b^{2} \ln (c \sqrt{x}-1)^{2}}{8 c^{4}}+\frac{2 a b^{2} \ln (1+c \sqrt{x})}{c^{4}}+\frac{a^{2} b x^{3 / 2}}{2 c}+\frac{3 a^{2} b \sqrt{x}}{2 c^{3}}+\frac{3 b^{3} \sqrt{x} \operatorname{arctanh}(c \sqrt{x})^{2}}{2 c^{3}}+\frac{b^{3} x^{3 / 2} \operatorname{arctanh}(c \sqrt{x})^{2}}{2 c}
$$

$$
+\frac{b^{3} \operatorname{arctanh}(c \sqrt{x}) x}{2 c^{2}}+\frac{a b^{2} x}{2 c^{2}}+\frac{3 a^{2} b x^{2} \operatorname{arctanh}(c \sqrt{x})}{2}+\frac{3 a b^{2} x^{2} \operatorname{arctanh}(c \sqrt{x})^{2}}{2}+\frac{2 a b^{2} \ln (c \sqrt{x}-1)}{c^{4}}+\frac{3 a b^{2} \ln (1+c \sqrt{x})^{2}}{8 c^{4}}
$$

$$
\begin{aligned}
& +\frac{3 a^{2} b \ln (c \sqrt{x}-1)}{4 c^{4}}-\frac{4 b^{3} \operatorname{arctanh}(c \sqrt{x}) \ln \left(1-\frac{\mathrm{I}(1+c \sqrt{x})}{\sqrt{-c^{2} x+1}}\right)}{c^{4}}-\frac{4 b^{3} \operatorname{arctanh}(c \sqrt{x}) \ln \left(1+\frac{\mathrm{I}(1+c \sqrt{x})}{\sqrt{-c^{2} x+1}}\right)}{c^{4}} \\
& +\frac{3 b^{3} \operatorname{arctanh}(c \sqrt{x})^{2} \ln \left(\frac{1+c \sqrt{x}}{\sqrt{-c^{2} x+1}}\right)}{2 c^{4}}-\frac{3 b^{3} \operatorname{arctanh}(c \sqrt{x})^{2} \ln (1+c \sqrt{x})}{4 c^{4}}+\frac{3 b^{3} \operatorname{arctanh}(c \sqrt{x})^{2} \ln (c \sqrt{x}-1)}{4 c^{4}}+\frac{a^{3} x^{2}}{2} \\
& +\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{\left(-c^{2} x+1\right)\left(1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}}\right) \operatorname{arctanh}(c \sqrt{x})^{2}}{8 c^{4}} \\
& +\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})}{\sqrt{-c^{2} x+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)^{2} \operatorname{arctanh}(c \sqrt{x})^{2}}{4 c^{4}}-\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})}{\sqrt{-c^{2} x+1}}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right) \operatorname{arctanh}(c \sqrt{x})^{2}}{8 c^{4}} \\
& +\underline{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{\left(-c^{2} x+1\right)\left(1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)}\right)^{2} \operatorname{arctanh}(c \sqrt{x})^{2}} \\
& 8 c^{4} \\
& -\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{\left(-c^{2} x+1\right)\left(1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)}\right)^{3} \operatorname{arctanh}(c \sqrt{x})^{2}}{8 c^{4}}-\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}}\right)^{3} \operatorname{arctanh}(c \sqrt{x})^{2}}{4 c^{4}} \\
& +\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}}\right)^{2} \operatorname{arctanh}(c \sqrt{x})^{2}}{4 c^{4}}-\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)^{3} \operatorname{arctanh}(c \sqrt{x})^{2}}{8 c^{4}}
\end{aligned}
$$

Problem 57: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c \sqrt{x}))^{3}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 4, 126 leaves, 8 steps):
$3 b c^{2}(a+b \operatorname{arctanh}(c \sqrt{x}))^{2}+c^{2}(a+b \operatorname{arctanh}(c \sqrt{x}))^{3}-\frac{(a+b \operatorname{arctanh}(c \sqrt{x}))^{3}}{x}+6 b^{2} c^{2}(a+b \operatorname{arctanh}(c \sqrt{x})) \ln \left(2-\frac{2}{1+c \sqrt{x}}\right)$

$$
-3 b^{3} c^{2} \text { polylog }\left(2,-1+\frac{2}{1+c \sqrt{x}}\right)-\frac{3 b c(a+b \operatorname{arctanh}(c \sqrt{x}))^{2}}{\sqrt{x}}
$$

Result(type ?, 5252 leaves): Display of huge result suppressed!
Problem 58: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c \sqrt{x}))^{3}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 4, 194 leaves, 17 steps):

$$
\frac{b^{3} c^{4} \operatorname{arctanh}(c \sqrt{x})}{2}-\frac{b^{2} c^{2}(a+b \operatorname{arctanh}(c \sqrt{x}))}{2 x}+2 b c^{4}(a+b \operatorname{arctanh}(c \sqrt{x}))^{2}-\frac{b c(a+b \operatorname{arctanh}(c \sqrt{x}))^{2}}{2 x^{3 / 2}}+\frac{c^{4}(a+b \operatorname{arctanh}(c \sqrt{x}))^{3}}{2}
$$

$$
-\frac{(a+b \operatorname{arctanh}(c \sqrt{x}))^{3}}{2 x^{2}}+4 b^{2} c^{4}(a+b \operatorname{arctanh}(c \sqrt{x})) \ln \left(2-\frac{2}{1+c \sqrt{x}}\right)-2 b^{3} c^{4} \operatorname{polylog}\left(2,-1+\frac{2}{1+c \sqrt{x}}\right)-\frac{b^{3} c^{3}}{2 \sqrt{x}}
$$

$$
-\frac{3 b c^{3}(a+b \operatorname{arctanh}(c \sqrt{x}))^{2}}{2 \sqrt{x}}
$$

Result(type 4, 1373 leaves):

$$
\begin{aligned}
& \frac{b^{3} c^{4} \operatorname{arctanh}(c \sqrt{x})}{2}-\frac{a^{3}}{2 x^{2}}-\frac{b^{3} \operatorname{arctanh}(c \sqrt{x})^{3}}{2 x^{2}}+\frac{c^{4} b^{3} \operatorname{arctanh}(c \sqrt{x})^{3}}{2}-2 c^{4} b^{3} \operatorname{arctanh}(c \sqrt{x})^{2}+4 c^{4} b^{3} \operatorname{dilog}\left(1+\frac{1+c \sqrt{x}}{\sqrt{-c^{2} x+1}}\right) \\
& -4 c^{4} b^{3} \operatorname{dilog}\left(\frac{1+c \sqrt{x}}{\sqrt{-c^{2} x+1}}\right) \\
& +\frac{3 \mathrm{I} c^{4} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{\left(-c^{2} x+1\right)\left(1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)}\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}}\right) \operatorname{arctanh}(c \sqrt{x})^{2}}{8}+\frac{3 c^{4} a^{2} b \ln (1+c \sqrt{x})}{4} \\
& -\frac{3 c^{4} a b^{2} \ln (c \sqrt{x}-1)^{2}}{8}-2 c^{4} a b^{2} \ln (1+c \sqrt{x})-\frac{3 c^{3} b^{3} \operatorname{arctanh}(c \sqrt{x})^{2}}{2 \sqrt{x}}-\frac{3 c^{3} a^{2} b}{2 \sqrt{x}}-\frac{c^{4} b^{3} \sqrt{-c^{2} x+1}}{2\left(-\sqrt{-c^{2} x+1}+c \sqrt{x}+1\right)} \\
& +\frac{c^{4} b^{3} \sqrt{-c^{2} x+1}}{2\left(\sqrt{-c^{2} x+1}+c \sqrt{x}+1\right)}-\frac{c a^{2} b}{2 x^{3 / 2}}-\frac{c b^{3} \operatorname{arctanh}(c \sqrt{x})^{2}}{2 x^{3 / 2}}-\frac{c^{2} b^{3} \operatorname{arctanh}(c \sqrt{x})}{2 x}-\frac{3 a^{2} b \operatorname{arctanh}(c \sqrt{x})}{2 x^{2}}-\frac{3 a b^{2} \operatorname{arctanh}(c \sqrt{x})^{2}}{2 x^{2}}-\frac{c^{2} a b^{2}}{2 x} \\
& +4 c^{4} b^{3} \operatorname{arctanh}(c \sqrt{x}) \ln \left(1+\frac{1+c \sqrt{x}}{\sqrt{-c^{2} x+1}}\right)+4 c^{4} a b^{2} \ln (c \sqrt{x})-2 c^{4} a b^{2} \ln (c \sqrt{x}-1)-\frac{3 c^{4} a b^{2} \ln (1+c \sqrt{x})^{2}}{8}-\frac{3 c^{4} a^{2} b \ln (c \sqrt{x}-1)}{4}
\end{aligned}
$$

$-\frac{3 c^{4} b^{3} \operatorname{arctanh}(c \sqrt{x})^{2} \ln \left(\frac{1+c \sqrt{x}}{\sqrt{-c^{2} x+1}}\right)}{2}+\frac{3 c^{4} b^{3} \operatorname{arctanh}(c \sqrt{x})^{2} \ln (1+c \sqrt{x})}{4}-\frac{3 c^{4} b^{3} \operatorname{arctanh}(c \sqrt{x})^{2} \ln (c \sqrt{x}-1)}{4}$
$-\frac{3 I c^{4} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{\left(-c^{2} x+1\right)\left(1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}}\right) \operatorname{arctanh}(c \sqrt{x})^{2}}{8}$
$-\frac{3 I c^{4} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})}{\sqrt{-c^{2} x+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)^{2} \operatorname{arctanh}(c \sqrt{x})^{2}}{4}-\frac{3 c^{3} a b^{2} \operatorname{arctanh}(c \sqrt{x})}{\sqrt{x}}+\frac{3 c^{4} a b^{2} \operatorname{arctanh}(c \sqrt{x}) \ln (1+c \sqrt{x})}{2}$
$-\frac{3 c^{4} a b^{2} \operatorname{arctanh}(c \sqrt{x}) \ln (c \sqrt{x}-1)}{2}+\frac{3 c^{4} a b^{2} \ln (c \sqrt{x}-1) \ln \left(\frac{c \sqrt{x}}{2}+\frac{1}{2}\right)}{4}-\frac{3 c^{4} a b^{2} \ln \left(-\frac{c \sqrt{x}}{2}+\frac{1}{2}\right) \ln \left(\frac{c \sqrt{x}}{2}+\frac{1}{2}\right)}{4}$
$+\frac{3 c^{4} a b^{2} \ln \left(-\frac{c \sqrt{x}}{2}+\frac{1}{2}\right) \ln (1+c \sqrt{x})}{4}-\frac{c a b^{2} \operatorname{arctanh}(c \sqrt{x})}{x^{3 / 2}}+\frac{3 I c^{4} b^{3} \pi \operatorname{arctanh}(c \sqrt{x})^{2}}{4}$
$+\frac{3 I c^{4} b^{3} \pi \operatorname{csgn}\left(\frac{1}{1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}}\right)^{3} \operatorname{arctanh}(c \sqrt{x})^{2}}{4}-\frac{3 I c^{4} b^{3} \pi \operatorname{csgn}\left(\frac{1}{1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}}\right)^{2} \operatorname{arctanh}(c \sqrt{x})^{2}}{4}$
$+\frac{3 I c^{4} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)^{3} \operatorname{arctanh}(c \sqrt{x})^{2}}{8}+\frac{3 I c^{4} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{\left(-c^{2} x+1\right)\left(1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)}\right)^{3} \operatorname{arctanh}(c \sqrt{x})^{2}}{8}$
$+\frac{31 c^{4} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})}{\sqrt{-c^{2} x+1}}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right) \operatorname{arctanh}(c \sqrt{x})^{2}}{8}$
${ }^{3 \mathrm{I} c^{4} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(1+c \sqrt{x})^{2}}{\left(-c^{2} x+1\right)\left(1+\frac{(1+c \sqrt{x})^{2}}{-c^{2} x+1}\right)}\right)^{2} \operatorname{arctanh}(c \sqrt{x})^{2}}$

Problem 61: Result more than twice size of optimal antiderivative.

$$
\int \frac{a+b \operatorname{arctanh}\left(c x^{3 / 2}\right)}{x} \mathrm{~d} x
$$

Optimal(type 4, 26 leaves, 2 steps):

$$
a \ln (x)-\frac{b \operatorname{poly} \log \left(2,-c x^{3 / 2}\right)}{3}+\frac{b \operatorname{poly} \log \left(2, c x^{3 / 2}\right)}{3}
$$

Result(type 4, 62 leaves):

$$
\frac{2 a \ln \left(c x^{3 / 2}\right)}{3}+\frac{2 b \ln \left(c x^{3 / 2}\right) \operatorname{arctanh}\left(c x^{3 / 2}\right)}{3}-\frac{b \operatorname{dilog}\left(1+c x^{3 / 2}\right)}{3}-\frac{b \ln \left(c x^{3 / 2}\right) \ln \left(1+c x^{3 / 2}\right)}{3}-\frac{b \operatorname{dilog}\left(c x^{3 / 2}\right)}{3}
$$

Problem 64: Unable to integrate problem.

$$
\int \operatorname{arctanh}\left(c x^{3 / 2}\right)^{2} \mathrm{~d} x
$$

Optimal(type 4, 2622 leaves, 200 steps):

$$
\begin{aligned}
& -\frac{(-1)^{2 / 3} \ln \left(\frac{(-1)^{2 / 3}\left(1+c^{1 / 3} \sqrt{x}\right)}{1+(-1)^{2 / 3}}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}-\frac{(-1)^{2 / 3} \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{1}{2}+\frac{(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2 c^{2 / 3}} \\
& +\frac{(-1)^{2 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}-\frac{(-1)^{2 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}} \\
& -\frac{(-1)^{2 / 3} \ln \left(\frac{(-1)^{2 / 3}\left(1-c^{1 / 3} \sqrt{x}\right)}{1+(-1)^{2 / 3}}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}+\frac{(-1)^{2 / 3} \ln \left(-\frac{(-1)^{2 / 3}\left(1+c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{2 / 3}}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}} \\
& -\frac{(-1)^{2 / 3} \ln \left(-(-1)^{2 / 3}+c^{1 / 3} \sqrt{x}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}-\frac{(-1)^{2 / 3} \ln \left(\frac{1}{2}-\frac{(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{2}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}} \\
& +\frac{(-1)^{2 / 3} \ln \left(\frac{(-1)^{1 / 3}-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}} \\
& -\frac{(-1)^{2 / 3} \ln \left(\frac{(-1)^{1 / 3}-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right) \ln \left(\frac{1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2 c^{2 / 3}} \\
& -\frac{(-1)^{1 / 3} \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(-\frac{(-1)^{2 / 3}\left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}
\end{aligned}
$$

$+\frac{(-1)^{2 / 3} \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{(-1)^{1 / 3}+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}$
$-\frac{(-1)^{2 / 3} \ln \left(\frac{1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right) \ln \left(\frac{(-1)^{1 / 3}+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}-\frac{(-1)^{1 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}$
$+\frac{(-1)^{1 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}+\frac{(-1)^{1 / 3} \ln \left(-\frac{(-1)^{1 / 3}\left(1-c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{1 / 3}}\right) \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}$
$-\frac{(-1)^{1 / 3} \ln \left(\frac{(-1)^{1 / 3}\left(1+c^{1 / 3} \sqrt{x}\right)}{1+(-1)^{1 / 3}}\right) \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}+\frac{(-1)^{1 / 3} \ln \left((-1)^{1 / 3}+c^{1 / 3} \sqrt{x}\right) \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}$
$+\frac{(-1)^{1 / 3} \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{1}{2}+\frac{(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2 c^{2 / 3}}+\frac{(-1)^{1 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}$
$-\frac{(-1)^{1 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}-\frac{(-1)^{1 / 3} \ln \left(\frac{(-1)^{1 / 3}\left(1-c^{1 / 3} \sqrt{x}\right)}{1+(-1)^{1 / 3}}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}$
$+\frac{(-1)^{1 / 3} \ln \left((-1)^{1 / 3}-c^{1 / 3} \sqrt{x}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}+\frac{(-1)^{1 / 3} \ln \left(-\frac{(-1)^{1 / 3}\left(1+c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{1 / 3}}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}$
$-\frac{(-1)^{1 / 3} \ln \left(-\frac{(-1)^{1 / 3}\left((-1)^{1 / 3}+c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{2 / 3}}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}$
$+\frac{(-1)^{1 / 3} \ln \left(\frac{1}{2}-\frac{(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{2}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}-\frac{(-1)^{2 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}$
$+\frac{(-1)^{2 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}+\frac{(-1)^{2 / 3} \ln \left(-\frac{(-1)^{2 / 3}\left(1-c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{2 / 3}}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}$
$-\frac{(-1)^{2 / 3} \ln \left(-(-1)^{2 / 3}-c^{1 / 3} \sqrt{x}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}+\frac{(-1)^{1 / 3} \operatorname{polylog}\left(2, \frac{1}{2}-\frac{(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2 c^{2 / 3}}$
$+\frac{(-1)^{1 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}-\frac{(-1)^{1 / 3} \operatorname{poly} \log \left(2, \frac{1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}$
$-\frac{(-1)^{1 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}+\frac{(-1)^{1 / 3} \operatorname{polylog}\left(2, \frac{1}{2}+\frac{(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2 c^{2 / 3}}$
$+\frac{(-1)^{1 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}-\frac{(-1)^{1 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}$
$-\frac{(-1)^{1 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}-\frac{(-1)^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{2}-\frac{(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2 c^{2 / 3}}$
$+\frac{(-1)^{2 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}-\frac{(-1)^{2 / 3} \operatorname{poly} \log \left(2, \frac{1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}$
$-\frac{(-1)^{2 / 3} \operatorname{polylog}\left(2, \frac{(-1)^{1 / 3}-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}-\frac{(-1)^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{2}+\frac{(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2 c^{2 / 3}}$
$-\frac{(-1)^{2 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}+\frac{(-1)^{2 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}$
$-\frac{(-1)^{2 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}-\frac{(-1)^{2 / 3} \operatorname{polylog}\left(2, \frac{(-1)^{1 / 3}+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}$
$-\frac{\ln \left(1-c x^{3 / 2}\right) \ln \left(1-c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}+\frac{\ln \left(1+c x^{3 / 2}\right) \ln \left(1-c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}-\frac{\ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{-(-1)^{1 / 3}+c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}$
$+\frac{\ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{-(-1)^{2 / 3}+c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}-\frac{\ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{1}{2}+\frac{c^{1 / 3} \sqrt{x}}{2}\right)}{2 c^{2 / 3}}+\frac{\ln \left(1-c x^{3 / 2}\right) \ln \left(1+c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}$
$-\frac{\ln \left(1+c x^{3 / 2}\right) \ln \left(1+c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}-\frac{\ln \left(\frac{1}{2}-\frac{c^{1 / 3} \sqrt{x}}{2}\right) \ln \left(1+c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}+\frac{\ln \left(\frac{(-1)^{1 / 3}-c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right) \ln \left(1+c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}$

$$
\begin{aligned}
& -\frac{\ln \left(\frac{(-1)^{2 / 3}-c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right) \ln \left(1+c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}-\frac{\ln \left(1+c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{-(-1)^{1 / 3}-c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}+\frac{\ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{(-1)^{1 / 3}+c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2 c^{2 / 3}} \\
& +\frac{\ln \left(1+c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{-(-1)^{2 / 3}-c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}-\frac{\ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{(-1)^{2 / 3}+c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}-\frac{(-1)^{1 / 3} \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)^{2}}{4 c^{2 / 3}} \\
& -\frac{(-1)^{1 / 3} \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)^{2}}{4 c^{2 / 3}}+\frac{(-1)^{2 / 3} \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)^{2}}{4 c^{2 / 3}}+\frac{(-1)^{2 / 3} \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)^{2}}{4 c^{2 / 3}} \\
& +\frac{(-1)^{1 / 3} \operatorname{polylog}\left(2, \frac{1}{1+(-1)^{2 / 3}}-c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}-\frac{(-1)^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{1-(-1)^{1 / 3}}+c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}} \\
& +\frac{(-1)^{1 / 3} \text { polylog }\left(2, \frac{1}{1+(-1)^{2 / 3}}+c^{1 / 3} \sqrt{x}\right)}{2 c^{2 / 3}}+\frac{x \ln \left(1-c x^{3 / 2}\right)^{2}}{4}+\frac{x \ln \left(1+c x^{3 / 2}\right)^{2}}{4}+\frac{\ln \left(1-c^{1 / 3} \sqrt{x}\right)^{2}}{4 c^{2 / 3}}+\frac{\ln \left(1+c^{1 / 3} \sqrt{x}\right)^{2}}{4 c^{2 / 3}} \\
& -\frac{\operatorname{polylog}\left(2, \frac{1}{2}-\frac{c^{1 / 3} \sqrt{x}}{2}\right)}{2 c^{2 / 3}}-\frac{\operatorname{polylog}\left(2, \frac{1-c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}+\frac{\operatorname{polylog}\left(2, \frac{1-c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}+\frac{\operatorname{polylog}\left(2, \frac{1-c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2 c^{2 / 3}} \\
& -\frac{\operatorname{polylog}\left(2, \frac{1-c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}-\frac{\operatorname{polylog}\left(2, \frac{1}{2}+\frac{c^{1 / 3} \sqrt{x}}{2}\right)}{2 c^{2 / 3}}-\frac{\operatorname{polylog}\left(2, \frac{1+c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2 c^{2 / 3}}+\frac{\operatorname{polylog}\left(2, \frac{1+c^{1 / 3} \sqrt{x}}{\left.1+(-1)^{1 / 3}\right)}\right.}{2 c^{2 / 3}} \\
& +\frac{\operatorname{polylog}\left(2, \frac{1+c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}-\frac{\operatorname{polylog}\left(2, \frac{1+c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{2 c^{2 / 3}}
\end{aligned}
$$

Result(type 8, 10 leaves):

$$
\int \operatorname{arctanh}\left(c x^{3 / 2}\right)^{2} \mathrm{~d} x
$$

Problem 65: Unable to integrate problem.

$$
\int \frac{\operatorname{arctanh}\left(c x^{3 / 2}\right)^{2}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 4, 2777 leaves, 196 steps):

$$
-\frac{\ln \left(1-c x^{3 / 2}\right)^{2}}{8 x^{2}}-\frac{\ln \left(1+c x^{3 / 2}\right)^{2}}{8 x^{2}}-\frac{3 c^{4 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right)}{2}-\frac{c^{4 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right)^{2}}{8}+\frac{3 c^{4 / 3} \ln \left(1+c^{2 / 3} x-c^{1 / 3} \sqrt{x}\right)}{4}
$$

$-\frac{3 c^{4 / 3} \ln \left(1+c^{1 / 3} \sqrt{x}\right)}{2}-\frac{c^{4 / 3} \ln \left(1+c^{1 / 3} \sqrt{x}\right)^{2}}{8}+\frac{3 c^{4 / 3} \ln \left(1+c^{2 / 3} x+c^{1 / 3} \sqrt{x}\right)}{4}+\frac{c^{4 / 3} \operatorname{polylog}\left(2, \frac{1}{2}-\frac{c^{1 / 3} \sqrt{x}}{2}\right)}{4}$
$+\frac{c^{4 / 3} \operatorname{polylog}\left(2, \frac{1-c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{4}-\frac{c^{4 / 3} \operatorname{polylog}\left(2, \frac{1-c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{4}-\frac{c^{4 / 3} \operatorname{polylog}\left(2, \frac{1-c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{4}+\frac{c^{4 / 3} \operatorname{polylog}\left(2, \frac{1-c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{4}$
$+\frac{c^{4 / 3} \operatorname{polylog}\left(2, \frac{1}{2}+\frac{c^{1 / 3} \sqrt{x}}{2}\right)}{4}+\frac{c^{4 / 3} \operatorname{polylog}\left(2, \frac{1+c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{4}-\frac{c^{4 / 3} \operatorname{polylog}\left(2, \frac{1+c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{4}-\frac{c^{4 / 3} \operatorname{polylog}\left(2, \frac{1+c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{4}$
$+\frac{c^{4 / 3} \text { polylog }\left(2, \frac{1+c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{4}+\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(\frac{(-1)^{2 / 3}\left(1-c^{1 / 3} \sqrt{x}\right)}{1+(-1)^{2 / 3}}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$-\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(-\frac{(-1)^{2 / 3}\left(1+c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{2 / 3}}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$+\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(-(-1)^{2 / 3}+c^{1 / 3} \sqrt{x}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$+\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(\frac{1}{2}-\frac{(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{2}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$-\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(\frac{(-1)^{1 / 3}-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$+\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(\frac{(-1)^{1 / 3}-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right) \ln \left(\frac{1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{4}$
$+\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(-\frac{(-1)^{2 / 3}\left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{2 / 3}}\right)}{4}$
$-\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{(-1)^{1 / 3}+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{4}$
$+\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(\frac{1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right) \ln \left(\frac{(-1)^{1 / 3}+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{4}+\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$-\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{4}-\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(-\frac{(-1)^{1 / 3}\left(1-c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{1 / 3}}\right) \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$+\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(\frac{(-1)^{1 / 3}\left(1+c^{1 / 3} \sqrt{x}\right)}{1+(-1)^{1 / 3}}\right) \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$-\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left((-1)^{1 / 3}+c^{1 / 3} \sqrt{x}\right) \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{4}-\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{1}{2}+\frac{(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{4}$
$-\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{4}+\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$+\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(\frac{(-1)^{1 / 3}\left(1-c^{1 / 3} \sqrt{x}\right)}{1+(-1)^{1 / 3}}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$-\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left((-1)^{1 / 3}-c^{1 / 3} \sqrt{x}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$-\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(-\frac{(-1)^{1 / 3}\left(1+c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{1 / 3}}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$+\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(-\frac{(-1)^{1 / 3}\left((-1)^{1 / 3}+c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{2 / 3}}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$-\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(\frac{1}{2}-\frac{(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{2}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{4}+\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$-\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{4}-\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(-\frac{(-1)^{2 / 3}\left(1-c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{2 / 3}}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$+\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(-(-1)^{2 / 3}-c^{1 / 3} \sqrt{x}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$+\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(\frac{(-1)^{2 / 3}\left(1+c^{1 / 3} \sqrt{x}\right)}{1+(-1)^{2 / 3}}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$+\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{1}{2}+\frac{(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{4}-\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{4}$
$+\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{4}-\frac{c^{4 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{(-1)^{1 / 3}+c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{4}$
$-\frac{c^{4 / 3} \ln \left(1+c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{-(-1)^{2 / 3}-c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{4}+\frac{c^{4 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{(-1)^{2 / 3}+c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{4}$
$+\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)^{2}}{8}+\frac{(-1)^{1 / 3} c^{4 / 3} \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)^{2}}{8}-\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)^{2}}{8}$
$-\frac{(-1)^{2 / 3} c^{4 / 3} \ln \left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)^{2}}{8}-\frac{(-1)^{1 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1}{1+(-1)^{2 / 3}}-c^{1 / 3} \sqrt{x}\right)}{4}$
$+\frac{(-1)^{2 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1}{1-(-1)^{1 / 3}}+c^{1 / 3} \sqrt{x}\right)}{4}-\frac{(-1)^{1 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1}{1+(-1)^{2 / 3}}+c^{1 / 3} \sqrt{x}\right)}{4}$
$-\frac{(-1)^{1 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1}{2}-\frac{(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{4}-\frac{(-1)^{1 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{4}$
$+\frac{(-1)^{1 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{4}+\frac{(-1)^{1 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{4}$
$-\frac{(-1)^{1 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1}{2}+\frac{(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{4}-\frac{(-1)^{1 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{4}$
$+\frac{(-1)^{1 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{4}+\frac{(-1)^{1 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{4}$
$+\frac{(-1)^{2 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1}{2}-\frac{(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{4}-\frac{(-1)^{2 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{4}$
$+\frac{(-1)^{2 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{4}+\frac{(-1)^{2 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{(-1)^{1 / 3}-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{4}$
$+\frac{(-1)^{2 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1}{2}+\frac{(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{4}+\frac{(-1)^{2 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{4}$

$$
\begin{aligned}
& -\frac{(-1)^{2 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{4}+\frac{(-1)^{2 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{4} \\
& +\frac{(-1)^{2 / 3} c^{4 / 3} \operatorname{polylog}\left(2, \frac{(-1)^{1 / 3}+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{4}-\frac{3 c^{4 / 3} \arctan \left(\frac{\left(1-2 c^{1 / 3} \sqrt{x}\right) \sqrt{3}}{3}\right) \sqrt{3}}{2} \\
& -\frac{3 c^{4 / 3} \arctan \left(\frac{\left(1+2 c^{1 / 3} \sqrt{x}\right) \sqrt{3}}{3}\right) \sqrt{3}}{2}+\frac{3 c \ln \left(1-c x^{3 / 2}\right)}{2 \sqrt{x}}-\frac{3 c \ln \left(1+c x^{3 / 2}\right)}{2 \sqrt{x}}+\frac{\ln \left(1-c x^{3 / 2}\right) \ln \left(1+c x^{3 / 2}\right)}{4 x^{2}} \\
& +\frac{c^{4 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1-c^{1 / 3} \sqrt{x}\right)}{4}-\frac{c^{4 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1-c^{1 / 3} \sqrt{x}\right)}{4}+\frac{c^{4 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{-(-1)^{1 / 3}+c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{4} \\
& -\frac{c^{4 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{-(-1)^{2 / 3}+c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{4}+\frac{c^{4 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{1}{2}+\frac{c^{1 / 3} \sqrt{x}}{2}\right)}{4}-\frac{c^{4 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1+c^{1 / 3} \sqrt{x}\right)}{4} \\
& +\frac{c^{4 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1+c^{1 / 3} \sqrt{x}\right)}{4}+\frac{c^{4 / 3} \ln \left(\frac{1}{2}-\frac{c^{1 / 3} \sqrt{x}}{2}\right) \ln \left(1+c^{1 / 3} \sqrt{x}\right)}{4}-\frac{c^{4 / 3} \ln \left(\frac{(-1)^{1 / 3}-c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right) \ln \left(1+c^{1 / 3} \sqrt{x}\right)}{4} \\
& +\frac{c^{4 / 3} \ln \left(\frac{(-1)^{2 / 3}-c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right) \ln \left(1+c^{1 / 3} \sqrt{x}\right)}{4}+\frac{c^{4 / 3} \ln \left(1+c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{-(-1)^{1 / 3}-c^{1 / 3} \sqrt{x}}{\left.1-(-1)^{1 / 3}\right)}\right.}{4}
\end{aligned}
$$

Result(type 8, 14 leaves):

$$
\int \frac{\operatorname{arctanh}\left(c x^{3} / 2\right)^{2}}{x^{3}} \mathrm{~d} x
$$

Problem 66: Unable to integrate problem.

$$
\int \frac{\operatorname{arctanh}\left(c x^{3} / 2\right)^{2}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 4, 2632 leaves, 160 steps):

$$
\begin{aligned}
& \frac{c^{2 / 3} \operatorname{polylog}\left(2, \frac{1+c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{2}-\frac{\ln \left(1-c x^{3 / 2}\right)^{2}}{4 x}-\frac{\ln \left(1+c x^{3 / 2}\right)^{2}}{4 x}-\frac{c^{2 / 3} \ln \left(-1-c^{1 / 3} \sqrt{x}\right)^{2}}{4}-\frac{c^{2 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right)^{2}}{4} \\
& \quad+\frac{c^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{2}-\frac{c^{1 / 3} \sqrt{x}}{2}\right)}{2}+\frac{c^{2 / 3} \operatorname{polylog}\left(2, \frac{1-c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2}-\frac{c^{2 / 3} \operatorname{polylog}\left(2, \frac{1-c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2}-\frac{c^{2 / 3} \operatorname{polylog}\left(2, \frac{1-c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2}
\end{aligned}
$$

$+\frac{c^{2 / 3} \operatorname{polylog}\left(2, \frac{1-c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{2}+\frac{c^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{2}+\frac{c^{1 / 3} \sqrt{x}}{2}\right)}{2}+\frac{c^{2 / 3} \operatorname{polylog}\left(2, \frac{1+c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2}-\frac{c^{2 / 3} \operatorname{polylog}\left(2, \frac{1+c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2}$
$-\frac{c^{2 / 3} \operatorname{polylog}\left(2, \frac{1+c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2}+\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(-1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(-1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2}-\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(\frac{(-1)^{2 / 3}\left(1-c^{1 / 3} \sqrt{x}\right)}{1+(-1)^{2 / 3}}\right) \ln \left(-1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$+\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(-\frac{(-1)^{2 / 3}\left(1+c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{2 / 3}}\right) \ln \left(-1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(-(-1)^{2 / 3}+c^{1 / 3} \sqrt{x}\right) \ln \left(-1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(-1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{1}{2}-\frac{(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2}-\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$+\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2}+\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(-\frac{(-1)^{2 / 3}\left(1-c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{2 / 3}}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(-(-1)^{2 / 3}-c^{1 / 3} \sqrt{x}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(\frac{(-1)^{2 / 3}\left(1+c^{1 / 3} \sqrt{x}\right)}{1+(-1)^{2 / 3}}\right) \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$+\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(-1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{(-1)^{1 / 3}-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{1}{2}+\frac{(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(\frac{(-1)^{1 / 3}-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right) \ln \left(\frac{1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2}$
$-\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(-1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(-\frac{(-1)^{2 / 3}\left(1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{2 / 3}}\right)}{2}$
$+\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{(-1)^{1 / 3}+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(\frac{1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right) \ln \left(\frac{(-1)^{1 / 3}+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2}-\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(-1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$+\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(-1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2}+\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(-\frac{(-1)^{1 / 3}\left(1-c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{1 / 3}}\right) \ln \left(-1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$-\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(\frac{(-1)^{1 / 3}\left(1+c^{1 / 3} \sqrt{x}\right)}{1+(-1)^{1 / 3}}\right) \ln \left(-1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$+\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left((-1)^{1 / 3}+c^{1 / 3} \sqrt{x}\right) \ln \left(-1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$+\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(-1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{1}{2}+\frac{(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2}+\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$-\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2}-\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(\frac{(-1)^{1 / 3}\left(1-c^{1 / 3} \sqrt{x}\right)}{1+(-1)^{1 / 3}}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$+\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left((-1)^{1 / 3}-c^{1 / 3} \sqrt{x}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$+\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(-\frac{(-1)^{1 / 3}\left(1+c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{1 / 3}}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$-\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(-\frac{(-1)^{1 / 3}\left((-1)^{1 / 3}+c^{1 / 3} \sqrt{x}\right)}{1-(-1)^{2 / 3}}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2}$
$+\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(\frac{1}{2}-\frac{(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{2}\right) \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)}{2}-\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)^{2}}{4}$
$+\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(-1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)^{2}}{4}+\frac{(-1)^{1 / 3} c^{2 / 3} \ln \left(1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}\right)^{2}}{4}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{1-(-1)^{1 / 3}}-c^{1 / 3} \sqrt{x}\right)}{2}+\frac{(-1)^{2 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{1+(-1)^{2 / 3}}-c^{1 / 3} \sqrt{x}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{1-(-1)^{1 / 3}}+c^{1 / 3} \sqrt{x}\right)}{2}+\frac{(-1)^{2 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{1+(-1)^{2 / 3}}+c^{1 / 3} \sqrt{x}\right)}{2}$
$+\frac{(-1)^{2 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{2}-\frac{(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2}+\frac{(-1)^{2 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2}$
$-\frac{(-1)^{2 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2}-\frac{(-1)^{2 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2}$
$+\frac{(-1)^{2 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{2}+\frac{(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2}+\frac{(-1)^{2 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2}$
$-\frac{(-1)^{2 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2}-\frac{(-1)^{2 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{2}-\frac{(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2}+\frac{(-1)^{1 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{2}-\frac{(-1)^{1 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{(-1)^{1 / 3}-(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1}{2}+\frac{(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{2}\right)}{2}+\frac{(-1)^{1 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2}$
$-\frac{(-1)^{1 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{1+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{2}-\frac{(-1)^{1 / 3} c^{2 / 3} \operatorname{polylog}\left(2, \frac{(-1)^{1 / 3}+(-1)^{2 / 3} c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2}$
$+\frac{\ln \left(1-c x^{3 / 2}\right) \ln \left(1+c x^{3 / 2}\right)}{2 x}-\frac{c^{2 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(-1-c^{1 / 3} \sqrt{x}\right)}{2}+\frac{c^{2 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(-1-c^{1 / 3} \sqrt{x}\right)}{2}$
$+\frac{c^{2 / 3} \ln \left(-1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{1}{2}-\frac{c^{1 / 3} \sqrt{x}}{2}\right)}{2}+\frac{c^{2 / 3} \ln \left(1-c x^{3 / 2}\right) \ln \left(1-c^{1 / 3} \sqrt{x}\right)}{2}-\frac{c^{2 / 3} \ln \left(1+c x^{3 / 2}\right) \ln \left(1-c^{1 / 3} \sqrt{x}\right)}{2}$

$$
\begin{aligned}
& \left.+\frac{c^{2 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{-(-1)^{1 / 3}+c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2}-\frac{c^{2 / 3} \ln \left(-1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{(-1)^{1 / 3}-c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2}\right) \\
& \left.-\frac{c^{2 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{-(-1)^{2 / 3}+c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2}+\frac{c^{2 / 3} \ln \left(-1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{(-1)^{2 / 3}-c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{2}\right) \\
& \left.+\frac{c^{2 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{1}{2}+\frac{c^{1 / 3} \sqrt{x}}{2}\right)}{2}+\frac{c^{2 / 3} \ln \left(-1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{-(-1)^{1 / 3}-c^{1 / 3} \sqrt{x}}{1-(-1)^{1 / 3}}\right)}{2}\right) \\
& -\frac{c^{2 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{(-1)^{1 / 3}+c^{1 / 3} \sqrt{x}}{1+(-1)^{1 / 3}}\right)}{2}-\frac{c^{2 / 3} \ln \left(-1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{-(-1)^{2 / 3}-c^{1 / 3} \sqrt{x}}{1-(-1)^{2 / 3}}\right)}{2} \\
& +\frac{c^{2 / 3} \ln \left(1-c^{1 / 3} \sqrt{x}\right) \ln \left(\frac{(-1)^{2 / 3}+c^{1 / 3} \sqrt{x}}{1+(-1)^{2 / 3}}\right)}{2}-\frac{(-1)^{2 / 3} c^{2 / 3} \ln \left(-1+(-1)^{1 / 3} c^{1 / 3} \sqrt{x}\right)^{2}}{4}
\end{aligned}
$$

Result(type 8, 14 leaves):

$$
\int \frac{\operatorname{arctanh}\left(c x^{3 / 2}\right)^{2}}{x^{2}} \mathrm{~d} x
$$

Problem 67: Unable to integrate problem.

$$
\int x^{2}\left(a+b \operatorname{arctanh}\left(c x^{n}\right)\right) \mathrm{d} x
$$

Optimal(type 5, 60 leaves, 2 steps):

$$
\frac{x^{3}\left(a+b \operatorname{arctanh}\left(c x^{n}\right)\right)}{3}-\frac{b c n x^{3+n} \text { hypergeom }\left(\left[1, \frac{3+n}{2 n}\right],\left[\frac{3(1+n)}{2 n}\right], c^{2} x^{2 n}\right)}{3(3+n)}
$$

Result(type 8, 16 leaves):

$$
\int x^{2}\left(a+b \operatorname{arctanh}\left(c x^{n}\right)\right) \mathrm{d} x
$$

Problem 68: Unable to integrate problem.

$$
\int \frac{a+b \operatorname{arctanh}\left(c x^{n}\right)}{x^{2}} \mathrm{~d} x
$$

Optimal(type 5, 63 leaves, 2 steps):

$$
\frac{-a-b \operatorname{arctanh}\left(c x^{n}\right)}{x}-\frac{b c n x^{-1+n} \text { hypergeom }\left(\left[1, \frac{-1+n}{2 n}\right],\left[\frac{3}{2}-\frac{1}{2 n}\right], c^{2} x^{2 n}\right)}{1-n}
$$

Result(type 8, 16 leaves):

$$
\int \frac{a+b \operatorname{arctanh}\left(c x^{n}\right)}{x^{2}} \mathrm{~d} x
$$

Test results for the 17 problems in "7.3.3 (d+e $x)^{\wedge} m\left(a+b \operatorname{arctanh}\left(c x^{\wedge} n\right)\right)^{\wedge} p . t x t "$
Problem 1: Result more than twice size of optimal antiderivative.

$$
\int(e x+d)^{4}(a+b \operatorname{arctanh}(c x)) \mathrm{d} x
$$

Optimal(type 3, 137 leaves, 6 steps):
$\frac{b d e\left(2 c^{2} d^{2}+e^{2}\right) x}{c^{3}}+\frac{b e^{2}\left(10 c^{2} d^{2}+e^{2}\right) x^{2}}{10 c^{3}}+\frac{b d e^{3} x^{3}}{3 c}+\frac{b e^{4} x^{4}}{20 c}+\frac{(e x+d)^{5}(a+b \operatorname{arctanh}(c x))}{5 e}+\frac{b(d c+e)^{5} \ln (-c x+1)}{10 c^{5} e}$
$-\frac{b(d c-e)^{5} \ln (c x+1)}{10 c^{5} e}$
Result(type 3, 394 leaves):
$\frac{a d^{5}}{5 e}+\frac{b e^{4} x^{2}}{10 c^{3}}+\frac{b e^{4} \ln (c x+1)}{10 c^{5}}+\frac{b e^{4} \ln (c x-1)}{10 c^{5}}+\frac{b \ln (c x+1) d^{4}}{2 c}+\frac{b \ln (c x-1) d^{4}}{2 c}+\frac{b e^{4} \operatorname{arctanh}(c x) x^{5}}{5}+\frac{b \ln (c x-1) d^{5}}{10 e}+\frac{b \operatorname{arctanh}(c x) d^{5}}{5 e}$
$-\frac{b \ln (c x+1) d^{5}}{10 e}+b \operatorname{arctanh}(c x) x d^{4}+a e^{3} x^{4} d+2 a e^{2} x^{3} d^{2}+2 a e x^{2} d^{3}+\frac{a e^{4} x^{5}}{5}+a x d^{4}+\frac{b e^{2} x^{2} d^{2}}{c}-\frac{b e^{3} \ln (c x+1) d}{2 c^{4}}+\frac{b e \ln (c x-1) d^{3}}{c^{2}}$
$+\frac{b e^{2} \ln (c x-1) d^{2}}{c^{3}}+\frac{b e^{3} \ln (c x-1) d}{2 c^{4}}-\frac{b e \ln (c x+1) d^{3}}{c^{2}}+\frac{b e^{2} \ln (c x+1) d^{2}}{c^{3}}+b e^{3} \operatorname{arctanh}(c x) x^{4} d+2 b e^{2} \operatorname{arctanh}(c x) x^{3} d^{2}$
$+2 b e \operatorname{arctanh}(c x) x^{2} d^{3}+\frac{2 b e d^{3} x}{c}+\frac{b e^{3} d x}{c^{3}}+\frac{b d e^{3} x^{3}}{3 c}+\frac{b e^{4} x^{4}}{20 c}$

Problem 2: Result more than twice size of optimal antiderivative.

$$
\int(e x+d)^{2}(a+b \operatorname{arctanh}(c x)) \mathrm{d} x
$$

Optimal(type 3, 88 leaves, 6 steps):

$$
\frac{b d e x}{c}+\frac{b e^{2} x^{2}}{6 c}+\frac{(e x+d)^{3}(a+b \operatorname{arctanh}(c x))}{3 e}+\frac{b(d c+e)^{3} \ln (-c x+1)}{6 c^{3} e}-\frac{b(d c-e)^{3} \ln (c x+1)}{6 c^{3} e}
$$

Result(type 3, 217 leaves):

$$
\begin{aligned}
& \frac{a e^{2} x^{3}}{3}+a e x^{2} d+a x d^{2}+\frac{a d^{3}}{3 e}+\frac{b e^{2} \operatorname{arctanh}(c x) x^{3}}{3}+b e \operatorname{arctanh}(c x) x^{2} d+b \operatorname{arctanh}(c x) x d^{2}+\frac{b \operatorname{arctanh}(c x) d^{3}}{3 e}+\frac{b e^{2} x^{2}}{6 c}+\frac{b d e x}{c}+\frac{b \ln (c x-1) d^{3}}{6 e} \\
& \quad+\frac{b \ln (c x-1) d^{2}}{2 c}+\frac{b e \ln (c x-1) d}{2 c^{2}}+\frac{b e^{2} \ln (c x-1)}{6 c^{3}}-\frac{b \ln (c x+1) d^{3}}{6 e}+\frac{b \ln (c x+1) d^{2}}{2 c}-\frac{b e \ln (c x+1) d}{2 c^{2}}+\frac{b e^{2} \ln (c x+1)}{6 c^{3}}
\end{aligned}
$$

Problem 4: Result more than twice size of optimal antiderivative.

$$
\int(e x+d)^{2}(a+b \operatorname{arctanh}(c x))^{2} \mathrm{~d} x
$$

Optimal(type 4, 241 leaves, 15 steps):

$$
\begin{aligned}
& \frac{2 a b d e x}{c}+\frac{b^{2} e^{2} x}{3 c^{2}}-\frac{b^{2} e^{2} \operatorname{arctanh}(c x)}{3 c^{3}}+\frac{2 b^{2} d e x \operatorname{arctanh}(c x)}{c}+\frac{b e^{2} x^{2}(a+b \operatorname{arctanh}(c x))}{3 c}+\frac{\left(3 c^{2} d^{2}+e^{2}\right)(a+b \operatorname{arctanh}(c x))^{2}}{3 c^{3}} \\
& -\frac{d\left(d^{2}+\frac{3 e^{2}}{c^{2}}\right)(a+b \operatorname{arctanh}(c x))^{2}}{3 e}+\frac{(e x+d)^{3}(a+b \operatorname{arctanh}(c x))^{2}}{3 e}-\frac{2 b\left(3 c^{2} d^{2}+e^{2}\right)(a+b \operatorname{arctanh}(c x)) \ln \left(\frac{2}{-c x+1}\right)}{3 c^{3}} \\
& +\frac{b^{2} d e \ln \left(-c^{2} x^{2}+1\right)}{c^{2}}-\frac{b^{2}\left(3 c^{2} d^{2}+e^{2}\right) \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{3 c^{3}}
\end{aligned}
$$

Result(type 4, 1049 leaves):
$\frac{2 a b d e x}{c}+\frac{2 b^{2} d e x \operatorname{arctanh}(c x)}{c}-\frac{b^{2} e d \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{2 c^{2}}+\frac{b^{2} e \operatorname{arctanh}(c x) \ln (c x-1) d}{c^{2}}-\frac{b^{2} e \operatorname{arctanh}(c x) \ln (c x+1) d}{c^{2}}$
$+\frac{a b e \ln (c x-1) d}{c^{2}}-\frac{a b e \ln (c x+1) d}{c^{2}}-\frac{b^{2} e d \ln (c x-1) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{2 c^{2}}+\frac{b^{2} e d \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{2 c^{2}}+2 a b e \operatorname{arctanh}(c x) x^{2} d$
$+\frac{b^{2} \operatorname{arctanh}(c x) \ln (c x+1) d^{2}}{c}+\frac{a b \ln (c x-1) d^{2}}{c}+\frac{a b \ln (c x+1) d^{2}}{c}+\frac{a b e^{2} x^{2}}{3 c}-\frac{b^{2} d^{3} \ln (c x-1) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{6 e}$
$+\frac{b^{2} d^{3} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{6 e}-\frac{b^{2} d^{3} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{6 e}+\frac{b^{2} \operatorname{arctanh}(c x) \ln (c x-1) d^{3}}{3 e}-\frac{b^{2} \operatorname{arctanh}(c x) \ln (c x+1) d^{3}}{3 e}$
$+\frac{2 a b e^{2} \operatorname{arctanh}(c x) x^{3}}{3}+b^{2} e \operatorname{arctanh}(c x)^{2} x^{2} d-\frac{a b \ln (c x+1) d^{3}}{3 e}+\frac{2 a b \operatorname{arctanh}(c x) d^{3}}{3 e}+\frac{a b \ln (c x-1) d^{3}}{3 e}-\frac{b^{2} e^{2} \ln (c x-1) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{6 c^{3}}$
$-\frac{b^{2} e^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{6 c^{3}}+\frac{b^{2} e^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{6 c^{3}}+\frac{b^{2} e^{2} \operatorname{arctanh}(c x) \ln (c x-1)}{3 c^{3}}+\frac{b^{2} e^{2} \operatorname{arctanh}(c x) \ln (c x+1)}{3 c^{3}}$
$+\frac{a b e^{2} \ln (c x-1)}{3 c^{3}}+\frac{a b e^{2} \ln (c x+1)}{3 c^{3}}+\frac{b^{2} e \ln (c x+1) d}{c^{2}}+\frac{b^{2} e \ln (c x-1) d}{c^{2}}+\frac{b^{2} e d \ln (c x+1)^{2}}{4 c^{2}}+\frac{b^{2} e d \ln (c x-1)^{2}}{4 c^{2}}+\frac{b^{2} e^{2} \operatorname{arctanh}(c x) x^{2}}{3 c}$
$-\frac{b^{2} d^{2} \ln (c x-1) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{2 c}-\frac{b^{2} d^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{2 c}+\frac{b^{2} d^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{2 c}+\frac{b^{2} \operatorname{arctanh}(c x) \ln (c x-1) d^{2}}{c}$
$+2 a b \operatorname{arctanh}(c x) x d^{2}+a^{2} e x^{2} d+\frac{b^{2} e^{2} \operatorname{arctanh}(c x)^{2} x^{3}}{3}-\frac{b^{2} e^{2} \ln (c x+1)}{6 c^{3}}+\frac{b^{2} e^{2} \ln (c x-1)^{2}}{12 c^{3}}-\frac{b^{2} e^{2} \operatorname{dilog}\left(\frac{c x}{2}+\frac{1}{2}\right)}{3 c^{3}}-\frac{b^{2} e^{2} \ln (c x+1)^{2}}{12 c^{3}}$

$$
\begin{aligned}
& +\frac{b^{2} e^{2} \ln (c x-1)}{6 c^{3}}-\frac{b^{2} d^{2} \operatorname{dilog}\left(\frac{c x}{2}+\frac{1}{2}\right)}{c}-\frac{b^{2} d^{2} \ln (c x+1)^{2}}{4 c}+\frac{b^{2} d^{2} \ln (c x-1)^{2}}{4 c}+b^{2} \operatorname{arctanh}(c x)^{2} x d^{2}+\frac{b^{2} d^{3} \ln (c x+1)^{2}}{12 e} \\
& +\frac{b^{2} \operatorname{arctanh}(c x)^{2} d^{3}}{3 e}+\frac{b^{2} d^{3} \ln (c x-1)^{2}}{12 e}+\frac{a^{2} d^{3}}{3 e}+\frac{b^{2} e^{2} x}{3 c^{2}}+a^{2} x d^{2}+\frac{a^{2} e^{2} x^{3}}{3}
\end{aligned}
$$

Problem 5: Result more than twice size of optimal antiderivative.

$$
\int(e x+d)^{3}(a+b \operatorname{arctanh}(c x))^{3} \mathrm{~d} x
$$

Optimal(type 4, 580 leaves, 29 steps):
$\frac{3 a b^{2} d e^{2} x}{c^{2}}+\frac{b^{3} e^{3} x}{4 c^{3}}-\frac{b^{3} e^{3} \operatorname{arctanh}(c x)}{4 c^{4}}+\frac{3 b^{3} d e^{2} x \operatorname{arctanh}(c x)}{c^{2}}+\frac{b^{2} e^{3} x^{2}(a+b \operatorname{arctanh}(c x))}{4 c^{2}}-\frac{3 b d e^{2}(a+b \operatorname{arctanh}(c x))^{2}}{2 c^{3}}$
$+\frac{b e^{3}(a+b \operatorname{arctanh}(c x))^{2}}{4 c^{4}}+\frac{3 b e\left(6 c^{2} d^{2}+e^{2}\right)(a+b \operatorname{arctanh}(c x))^{2}}{4 c^{4}}+\frac{3 b e\left(6 c^{2} d^{2}+e^{2}\right) x(a+b \operatorname{arctanh}(c x))^{2}}{4 c^{3}}$
$+\frac{3 b d e^{2} x^{2}(a+b \operatorname{arctanh}(c x))^{2}}{2 c}+\frac{b e^{3} x^{3}(a+b \operatorname{arctanh}(c x))^{2}}{4 c}+\frac{d\left(c^{2} d^{2}+e^{2}\right)(a+b \operatorname{arctanh}(c x))^{3}}{c^{3}}$
$-\frac{\left(c^{4} d^{4}+6 c^{2} d^{2} e^{2}+e^{4}\right)(a+b \operatorname{arctanh}(c x))^{3}}{4 c^{4} e}+\frac{(e x+d)^{4}(a+b \operatorname{arctanh}(c x))^{3}}{4 e}-\frac{b^{2} e^{3}(a+b \operatorname{arctanh}(c x)) \ln \left(\frac{2}{-c x+1}\right)}{2 c^{4}}$
$-\frac{3 b^{2} e\left(6 c^{2} d^{2}+e^{2}\right)(a+b \operatorname{arctanh}(c x)) \ln \left(\frac{2}{-c x+1}\right)}{2 c^{4}}-\frac{3 b d\left(c^{2} d^{2}+e^{2}\right)(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{-c x+1}\right)}{c^{3}}+\frac{3 b^{3} d e^{2} \ln \left(-c^{2} x^{2}+1\right)}{2 c^{3}}$
$-\frac{b^{3} e^{3} \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{4 c^{4}}-\frac{3 b^{3} e\left(6 c^{2} d^{2}+e^{2}\right) \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{4 c^{4}}$
$-\frac{3 b^{2} d\left(c^{2} d^{2}+e^{2}\right)(a+b \operatorname{arctanh}(c x)) \operatorname{poly} \log \left(2,1-\frac{2}{-c x+1}\right)}{c^{3}}+\frac{3 b^{3} d\left(c^{2} d^{2}+e^{2}\right) \operatorname{poly} \log \left(3,1-\frac{2}{-c x+1}\right)}{2 c^{3}}$
Result(type ?, 6148 leaves): Display of huge result suppressed!
Problem 6: Result more than twice size of optimal antiderivative.

$$
\int(e x+d)^{2}(a+b \operatorname{arctanh}(c x))^{3} \mathrm{~d} x
$$

Optimal(type 4, 373 leaves, 20 steps):
$\frac{a b^{2} e^{2} x}{c^{2}}+\frac{b^{3} e^{2} x \operatorname{arctanh}(c x)}{c^{2}}+\frac{3 b d e(a+b \operatorname{arctanh}(c x))^{2}}{c^{2}}-\frac{b e^{2}(a+b \operatorname{arctanh}(c x))^{2}}{2 c^{3}}+\frac{3 b d e x(a+b \operatorname{arctanh}(c x))^{2}}{c}$

$$
\begin{aligned}
& +\frac{b e^{2} x^{2}(a+b \operatorname{arctanh}(c x))^{2}}{2 c}+\frac{\left(3 c^{2} d^{2}+e^{2}\right)(a+b \operatorname{arctanh}(c x))^{3}}{3 c^{3}}-\frac{d\left(d^{2}+\frac{3 e^{2}}{c^{2}}\right)(a+b \operatorname{arctanh}(c x))^{3}}{3 e}+\frac{(e x+d)^{3}(a+b \operatorname{arctanh}(c x))^{3}}{3 e} \\
& -\frac{6 b^{2} d e(a+b \operatorname{arctanh}(c x)) \ln \left(\frac{2}{-c x+1}\right)}{c^{2}}-\frac{b\left(3 c^{2} d^{2}+e^{2}\right)(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{-c x+1}\right)}{c^{3}}+\frac{b^{3} e^{2} \ln \left(-c^{2} x^{2}+1\right)}{2 c^{3}} \\
& -\frac{3 b^{3} d e \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{c^{2}}-\frac{b^{2}\left(3 c^{2} d^{2}+e^{2}\right)(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{c^{3}}+\frac{b^{3}\left(3 c^{2} d^{2}+e^{2}\right) \operatorname{polylog}\left(3,1-\frac{2}{-c x+1}\right)}{2 c^{3}}
\end{aligned}
$$

Result(type ?, 4635 leaves): Display of huge result suppressed!
Problem 7: Result more than twice size of optimal antiderivative

$$
\int(e x+d)(a+b \operatorname{arctanh}(c x))^{3} \mathrm{~d} x
$$

Optimal(type 4, 232 leaves, 14 steps):
$\frac{3 b e(a+b \operatorname{arctanh}(c x))^{2}}{2 c^{2}}+\frac{3 b e x(a+b \operatorname{arctanh}(c x))^{2}}{2 c}+\frac{d(a+b \operatorname{arctanh}(c x))^{3}}{c}-\frac{\left(d^{2}+\frac{e^{2}}{c^{2}}\right)(a+b \operatorname{arctanh}(c x))^{3}}{2 e}$

$$
\begin{aligned}
& +\frac{(e x+d)^{2}(a+b \operatorname{arctanh}(c x))^{3}}{2 e}-\frac{3 b^{2} e(a+b \operatorname{arctanh}(c x)) \ln \left(\frac{2}{-c x+1}\right)}{c^{2}}-\frac{3 b d(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{-c x+1}\right)}{c} \\
& -\frac{3 b^{3} e \text { polylog }\left(2,1-\frac{2}{-c x+1}\right)}{2 c^{2}}-\frac{3 b^{2} d(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{c}+\frac{3 b^{3} d \operatorname{poly} \log \left(3,1-\frac{2}{-c x+1}\right)}{2 c}
\end{aligned}
$$

Result(type ?, 12529 leaves): Display of huge result suppressed!
Problem 8: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))^{3}}{e x+d} \mathrm{~d} x
$$

Optimal(type 4, 260 leaves, 1 step):

$$
\begin{aligned}
& -\frac{(a+b \operatorname{arctanh}(c x))^{3} \ln \left(\frac{2}{c x+1}\right)}{e}+\frac{(a+b \operatorname{arctanh}(c x))^{3} \ln \left(\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{e}+\frac{3 b(a+b \operatorname{arctanh}(c x))^{2} \operatorname{polylog}\left(2,1-\frac{2}{c x+1}\right)}{2 e} \\
& -\frac{3 b(a+b \operatorname{arctanh}(c x))^{2} \operatorname{poly} \log \left(2,1-\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{2 e}+\frac{3 b^{2}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(3,1-\frac{2}{c x+1}\right)}{2 e} \\
& -\frac{3 b^{2}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(3,1-\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{2 e}+\frac{3 b^{3} \operatorname{polylog}\left(4,1-\frac{2}{c x+1}\right)}{4 e}-\frac{3 b^{3} \operatorname{polylog}\left(4,1-\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{4 e}
\end{aligned}
$$

Result(type ?, 2366 leaves): Display of huge result suppressed!

Problem 9: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))^{3}}{(e x+d)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 501 leaves, 9 steps):

$$
\begin{aligned}
-\frac{(a+b \operatorname{arctanh}(c x))^{3}}{e(e x+d)}+\frac{3 b c(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{-c x+1}\right)}{2 e(d c+e)}-\frac{3 b c(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{c x+1}\right)}{2(d c-e) e}+\frac{3 b c(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{c x+1}\right)}{c^{2} d^{2}-e^{2}} \\
-\frac{3 b c(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{c^{2} d^{2}-e^{2}}+\frac{3 b^{2} c(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{2 e(d c+e)}
\end{aligned}
$$

$$
+\frac{3 b^{2} c(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{c x+1}\right)}{2(d c-e) e}-\frac{3 b^{2} c(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{c x+1}\right)}{c^{2} d^{2}-e^{2}}
$$

$$
+\frac{3 b^{2} c(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{c^{2} d^{2}-e^{2}}-\frac{3 b^{3} c \operatorname{polylog}\left(3,1-\frac{2}{-c x+1}\right)}{4 e(d c+e)}+\frac{3 b^{3} c \operatorname{polylog}\left(3,1-\frac{2}{c x+1}\right)}{4(d c-e) e}
$$

$$
-\frac{3 b^{3} c \text { polylog }\left(3,1-\frac{2}{c x+1}\right)}{2\left(c^{2} d^{2}-e^{2}\right)}+\frac{3 b^{3} c \text { polylog }\left(3,1-\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{2\left(c^{2} d^{2}-e^{2}\right)}
$$

Result(type ?, 3719 leaves): Display of huge result suppressed!
Problem 11: Unable to integrate problem.

$$
\int(e x+d)\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 4, 900 leaves, 77 steps):


$$
\begin{aligned}
& +\frac{b^{2} d \arctan (x \sqrt{c}) \ln \left(\frac{(1+\mathrm{I})(1-x \sqrt{c})}{1-\mathrm{I} x \sqrt{c}}\right)}{\sqrt{c}}+\frac{b^{2} d \operatorname{arctanh}(x \sqrt{c}) \ln \left(-\frac{2(1-x \sqrt{-c}) \sqrt{c}}{(\sqrt{-c}-\sqrt{c})(1+x \sqrt{c})}\right)}{\sqrt{c}} \\
& +\frac{b^{2} d \operatorname{arctanh}(x \sqrt{c}) \ln \left(\frac{2(1+x \sqrt{-c}) \sqrt{c}}{(\sqrt{-c}+\sqrt{c})(1+x \sqrt{c})}\right)}{\sqrt{c}}+\frac{b^{2} d \arctan (x \sqrt{c}) \ln \left(\frac{(1-\mathrm{I})(1+x \sqrt{c})}{1-\mathrm{I} x \sqrt{c}}\right)}{\sqrt{c}}+\frac{\mathrm{I} b^{2} d \operatorname{polylog}\left(2,1-\frac{2}{1-\mathrm{I} x \sqrt{c}}\right)}{\sqrt{c}} \\
& +\frac{\mathrm{I} b^{2} d \operatorname{polylog}\left(2,1-\frac{2}{1+\mathrm{I} x \sqrt{c}}\right)}{\sqrt{c}}-\frac{b e\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right) \ln \left(\frac{2}{-c x^{2}+1}\right)}{c}-a b d x \ln \left(-c x^{2}+1\right)+a b d x \ln \left(c x^{2}+1\right)+\frac{\operatorname{Ib^{2}d\operatorname {arctan}(x\sqrt {c})^{2}}}{\sqrt{c}} \\
& +\frac{b^{2} d x \ln \left(-c x^{2}+1\right)^{2}}{4}+\frac{b^{2} d x \ln \left(c x^{2}+1\right)^{2}}{4}-\frac{b^{2} d \operatorname{polylog}\left(2,1+\frac{2(1-x \sqrt{-c}) \sqrt{c}}{(\sqrt{-c}-\sqrt{c})(1+x \sqrt{c})}\right)}{2 \sqrt{c}} \\
& -\frac{b^{2} d \operatorname{polylog}\left(2,1-\frac{2(1+x \sqrt{-c}) \sqrt{c}}{(\sqrt{-c}+\sqrt{c})(1+x \sqrt{c})}\right)}{2 \sqrt{c}}-\frac{b^{2} d \operatorname{arctanh}(x \sqrt{c})^{2}}{\sqrt{c}}+\frac{b^{2} d \operatorname{polylog}\left(2,1-\frac{2}{1-x \sqrt{c}}\right)}{\sqrt{c}}+\frac{b^{2} d \operatorname{polylog}\left(2,1-\frac{2}{1+x \sqrt{c}}\right)}{\sqrt{c}} \\
& 2 c
\end{aligned}
$$

Result(type 8, 20 leaves):

$$
\int(e x+d)\left(a+b \operatorname{arctanh}\left(c x^{2}\right)\right)^{2} \mathrm{~d} x
$$

Problem 13: Result more than twice size of optimal antiderivative.

$$
\int \frac{a+b \operatorname{arctanh}(c \sqrt{x})}{-c^{2} x+1} \mathrm{~d} x
$$

Optimal(type 4, 70 leaves, 5 steps):

$$
-\frac{(a+b \operatorname{arctanh}(c \sqrt{x}))^{2}}{b c^{2}}+\frac{2(a+b \operatorname{arctanh}(c \sqrt{x})) \ln \left(\frac{2}{1-c \sqrt{x}}\right)}{c^{2}}+\frac{b \operatorname{polylog}\left(2,1-\frac{2}{1-c \sqrt{x}}\right)}{c^{2}}
$$

Result(type 4, 185 leaves):
$-\frac{a \ln (c \sqrt{x}-1)}{c^{2}}-\frac{a \ln (1+c \sqrt{x})}{c^{2}}-\frac{b \operatorname{arctanh}(c \sqrt{x}) \ln (c \sqrt{x}-1)}{c^{2}}-\frac{b \operatorname{arctanh}(c \sqrt{x}) \ln (1+c \sqrt{x})}{c^{2}}-\frac{b \ln (c \sqrt{x}-1)^{2}}{4 c^{2}}+\frac{b \operatorname{dilog}\left(\frac{c \sqrt{x}}{2}+\frac{1}{2}\right)}{c^{2}}$

$$
+\frac{b \ln (c \sqrt{x}-1) \ln \left(\frac{c \sqrt{x}}{2}+\frac{1}{2}\right)}{2 c^{2}}+\frac{b \ln (1+c \sqrt{x})^{2}}{4 c^{2}}+\frac{b \ln \left(-\frac{c \sqrt{x}}{2}+\frac{1}{2}\right) \ln \left(\frac{c \sqrt{x}}{2}+\frac{1}{2}\right)}{2 c^{2}}-\frac{b \ln \left(-\frac{c \sqrt{x}}{2}+\frac{1}{2}\right) \ln (1+c \sqrt{x})}{2 c^{2}}
$$

Problem 14: Result more than twice size of optimal antiderivative.

$$
\int \frac{a+b \operatorname{arctanh}(c \sqrt{x})}{x^{2}\left(-c^{2} x+1\right)} \mathrm{d} x
$$

Optimal(type 4, 105 leaves, 9 steps):
$b c^{2} \operatorname{arctanh}(c \sqrt{x})+\frac{-a-b \operatorname{arctanh}(c \sqrt{x})}{x}+\frac{c^{2}(a+b \operatorname{arctanh}(c \sqrt{x}))^{2}}{b}+2 c^{2}(a+b \operatorname{arctanh}(c \sqrt{x})) \ln \left(2-\frac{2}{1+c \sqrt{x}}\right)-b c^{2} \operatorname{polylog}(2,-1$

$$
\left.+\frac{2}{1+c \sqrt{x}}\right)-\frac{b c}{\sqrt{x}}
$$

Result(type 4, 314 leaves):

$$
\begin{aligned}
-\frac{a}{x} & +2 c^{2} a \ln (c \sqrt{x})-c^{2} a \ln (1+c \sqrt{x})-c^{2} a \ln (c \sqrt{x}-1)-\frac{b \operatorname{arctanh}(c \sqrt{x})}{x}+2 c^{2} b \ln (c \sqrt{x}) \operatorname{arctanh}(c \sqrt{x})-c^{2} b \operatorname{arctanh}(c \sqrt{x}) \ln (1+c \sqrt{x}) \\
& -c^{2} b \operatorname{arctanh}(c \sqrt{x}) \ln (c \sqrt{x}-1)+\frac{c^{2} b \ln (1+c \sqrt{x})}{2}-\frac{c^{2} b \ln (c \sqrt{x}-1)}{2}-\frac{b c}{\sqrt{x}}-c^{2} b \operatorname{dilog}(1+c \sqrt{x})-c^{2} b \ln (c \sqrt{x}) \ln (1+c \sqrt{x}) \\
& -c^{2} b \operatorname{dilog}(c \sqrt{x})-\frac{c^{2} b \ln (c \sqrt{x}-1)^{2}}{4}+c^{2} b \operatorname{dilog}\left(\frac{c \sqrt{x}}{2}+\frac{1}{2}\right)+\frac{c^{2} b \ln (c \sqrt{x}-1) \ln \left(\frac{c \sqrt{x}}{2}+\frac{1}{2}\right)}{2}+\frac{c^{2} b \ln (1+c \sqrt{x})^{2}}{4} \\
& +\frac{c^{2} b \ln \left(-\frac{c \sqrt{x}}{2}+\frac{1}{2}\right) \ln \left(\frac{c \sqrt{x}}{2}+\frac{1}{2}\right)}{2}-\frac{c^{2} b \ln \left(-\frac{c \sqrt{x}}{2}+\frac{1}{2}\right) \ln (1+c \sqrt{x})}{2}
\end{aligned}
$$

Test results for the 143 problems in "7.3.4 u (a+b arctanh(c x))^p.txt"
Problem 10: Result more than twice size of optimal antiderivative.

$$
\int(d x c+d)^{3}(a+b \operatorname{arctanh}(c x)) \mathrm{d} x
$$

Optimal(type 3, 78 leaves, 4 steps):

$$
b d^{3} x+\frac{b d^{3}(c x+1)^{2}}{4 c}+\frac{b d^{3}(c x+1)^{3}}{12 c}+\frac{d^{3}(c x+1)^{4}(a+b \operatorname{arctanh}(c x))}{4 c}+\frac{2 b d^{3} \ln (-c x+1)}{c}
$$

Result(type 3, 161 leaves):
$\frac{c^{3} x^{4} a d^{3}}{4}+c^{2} x^{3} a d^{3}+\frac{3 c x^{2} a d^{3}}{2}+a x d^{3}+\frac{d^{3} a}{4 c}+\frac{c^{3} b d^{3} \operatorname{arctanh}(c x) x^{4}}{4}+c^{2} b d^{3} x^{3} \operatorname{arctanh}(c x)+\frac{3 c b d^{3} \operatorname{arctanh}(c x) x^{2}}{2}+b d^{3} x \operatorname{arctanh}(c x)$

$$
+\frac{b d^{3} \operatorname{arctanh}(c x)}{4 c}+\frac{c^{2} b d^{3} x^{3}}{12}+\frac{c b d^{3} x^{2}}{2}+\frac{7 b d^{3} x}{4}+\frac{2 b d^{3} \ln (c x-1)}{c}
$$

Problem 16: Result more than twice size of optimal antiderivative.

$$
\int \frac{a+b \operatorname{arctanh}(c x)}{x(d x c+d)} \mathrm{d} x
$$

Optimal(type 4, 44 leaves, 2 steps):

$$
\frac{(a+b \operatorname{arctanh}(c x)) \ln \left(2-\frac{2}{c x+1}\right)}{d}-\frac{b \operatorname{polylog}\left(2,-1+\frac{2}{c x+1}\right)}{2 d}
$$

Result(type 4, 155 leaves):
$\frac{a \ln (c x)}{d}-\frac{a \ln (c x+1)}{d}+\frac{b \operatorname{arctanh}(c x) \ln (c x)}{d}-\frac{b \operatorname{arctanh}(c x) \ln (c x+1)}{d}+\frac{b \ln (c x+1)^{2}}{4 d}+\frac{b \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{2 d}$

$$
-\frac{b \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{2 d}+\frac{b \operatorname{dilog}\left(\frac{c x}{2}+\frac{1}{2}\right)}{2 d}-\frac{b \operatorname{dilog}(c x+1)}{2 d}-\frac{b \ln (c x) \ln (c x+1)}{2 d}-\frac{b \operatorname{dilog}(c x)}{2 d}
$$

Problem 17: Result more than twice size of optimal antiderivative.

$$
\int \frac{a+b \operatorname{arctanh}(c x)}{x^{3}(d x c+d)} \mathrm{d} x
$$

Optimal(type 4, 139 leaves, 12 steps):

$$
\begin{aligned}
& -\frac{b c}{2 d x}+\frac{b c^{2} \operatorname{arctanh}(c x)}{2 d}+\frac{-a-b \operatorname{arctanh}(c x)}{2 x^{2} d}+\frac{c(a+b \operatorname{arctanh}(c x))}{d x}-\frac{b c^{2} \ln (x)}{d}+\frac{b c^{2} \ln \left(-c^{2} x^{2}+1\right)}{2 d}+\frac{c^{2}(a+b \operatorname{arctanh}(c x)) \ln \left(2-\frac{2}{c x+1}\right)}{d} \\
& \quad-\frac{b c^{2} \operatorname{polylog}\left(2,-1+\frac{2}{c x+1}\right)}{2 d}
\end{aligned}
$$

Result(type 4, 285 leaves):

$$
\begin{aligned}
& -\frac{a}{2 d x^{2}}+\frac{c^{2} a \ln (c x)}{d}+\frac{c a}{d x}-\frac{c^{2} a \ln (c x+1)}{d}-\frac{b \operatorname{arctanh}(c x)}{2 d x^{2}}+\frac{c^{2} b \operatorname{arctanh}(c x) \ln (c x)}{d}+\frac{c b \operatorname{arctanh}(c x)}{d x}-\frac{c^{2} b \operatorname{arctanh}(c x) \ln (c x+1)}{d} \\
& \quad+\frac{3 c^{2} b \ln (c x+1)}{4 d}+\frac{c^{2} b \ln (c x-1)}{4 d}-\frac{b c}{2 d x}-\frac{c^{2} b \ln (c x)}{d}-\frac{c^{2} b \operatorname{dilog}(c x+1)}{2 d}-\frac{c^{2} b \ln (c x) \ln (c x+1)}{2 d}-\frac{c^{2} b \operatorname{dilog}(c x)}{2 d}+\frac{c^{2} b \ln (c x+1)^{2}}{4 d} \\
& \quad+\frac{c^{2} b \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{2 d}-\frac{c^{2} b \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{2 d}+\frac{c^{2} b \operatorname{dilog}\left(\frac{c x}{2}+\frac{1}{2}\right)}{2 d}
\end{aligned}
$$

Problem 24: Result more than twice size of optimal antiderivative

$$
\int \frac{(d x c+d)^{2}(a+b \operatorname{arctanh}(c x))^{2}}{x} \mathrm{~d} x
$$

Optimal(type 4, 268 leaves, 19 steps):
$a b c d^{2} x+b^{2} c d^{2} x \operatorname{arctanh}(c x)+\frac{3 d^{2}(a+b \operatorname{arctanh}(c x))^{2}}{2}+2 c d^{2} x(a+b \operatorname{arctanh}(c x))^{2}+\frac{c^{2} d^{2} x^{2}(a+b \operatorname{arctanh}(c x))^{2}}{2}-2 d^{2}(a$

$$
\begin{aligned}
& +b \operatorname{arctanh}(c x))^{2} \operatorname{arctanh}\left(-1+\frac{2}{-c x+1}\right)-4 b d^{2}(a+b \operatorname{arctanh}(c x)) \ln \left(\frac{2}{-c x+1}\right)+\frac{b^{2} d^{2} \ln \left(-c^{2} x^{2}+1\right)}{2}-2 b^{2} d^{2} \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right) \\
& \left.-b d^{2}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)+b d^{2}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,-1+\frac{2}{-c x+1}\right)+\frac{2}{-c x+1}\right) \\
& -\frac{b^{2} d^{2} \operatorname{polylog}\left(3,-1+\frac{2}{-c x+1}\right)}{2}
\end{aligned}
$$

Result(type 4, 1081 leaves):

$$
\begin{aligned}
& \frac{d^{2} a^{2} c^{2} x^{2}}{2}+d^{2} b^{2} \operatorname{arctanh}(c x)^{2} \ln (c x)-d^{2} b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)+d^{2} b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(1+\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right) \\
& \quad+2 d^{2} b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2,-\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)+d^{2} b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(1-\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)+2 d^{2} b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2, \frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right) \\
& \quad-d^{2} b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)-4 d^{2} b^{2} \operatorname{arctanh}(c x) \ln \left(1+\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right)-4 d^{2} b^{2} \operatorname{arctanh}(c x) \ln \left(1-\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right)-d^{2} a b \operatorname{dilog}(c x
\end{aligned}
$$

$$
+1)-d^{2} a b \operatorname{dilog}(c x)+\frac{5 a b \ln (c x-1) d^{2}}{2}+\frac{3 a b \ln (c x+1) d^{2}}{2}+2 a^{2} c x d^{2}
$$

$$
-\frac{\mathrm{I} d^{2} b^{2} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{2} \operatorname{arctanh}(c x)^{2}}{2}
$$

$$
-\frac{\mathrm{I} d^{2} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{2} \operatorname{arctanh}(c x)^{2}}{2}+2 d^{2} a b \operatorname{arctanh}(c x) \ln (c x)
$$

$$
+\frac{\mathrm{I} d^{2} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{3} \operatorname{arctanh}(c x)^{2}}{2}+d^{2} a b \operatorname{arctanh}(c x) c^{2} x^{2}+4 a b \operatorname{arctanh}(c x) c x d^{2}-d^{2} a b \ln (c x) \ln (c x+1)
$$

$+\frac{d^{2} b^{2} \operatorname{arctanh}(c x)^{2} c^{2} x^{2}}{2}+2 b^{2} \operatorname{arctanh}(c x)^{2} c x d^{2}+a b c d^{2} x+b^{2} c d^{2} x \operatorname{arctanh}(c x)$
$+\frac{\mathrm{I} d^{2} b^{2} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{arctanh}(c x)^{2}}{2}+d^{2} a^{2} \ln (c x)-2 d^{2} b^{2} \operatorname{polylog}(3$,
$\left.-\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)-2 d^{2} b^{2}$ polylog $\left(3, \frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)+\frac{d^{2} b^{2} \operatorname{polylog}\left(3,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{2}+\frac{3 d^{2} b^{2} \operatorname{arctanh}(c x)^{2}}{2}-4 d^{2} b^{2} \operatorname{dilog}\left(1+\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right)$
$-4 d^{2} b^{2} \operatorname{dilog}\left(1-\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right)+d^{2} b^{2} \operatorname{arctanh}(c x)-d^{2} b^{2} \ln \left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)$

Problem 25: Result more than twice size of optimal antiderivative.

$$
\int \frac{(c d x+d)^{2}(a+b \operatorname{arctanh}(c x))^{2}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 4, 283 leaves, 17 steps):
$2 c d^{2}(a+b \operatorname{arctanh}(c x))^{2}-\frac{d^{2}(a+b \operatorname{arctanh}(c x))^{2}}{x}+c^{2} d^{2} x(a+b \operatorname{arctanh}(c x))^{2}-4 c d^{2}(a+b \operatorname{arctanh}(c x))^{2} \operatorname{arctanh}\left(-1+\frac{2}{-c x+1}\right)-2 b c d^{2}(a$
$+b \operatorname{arctanh}(c x)) \ln \left(\frac{2}{-c x+1}\right)+2 b c d^{2}(a+b \operatorname{arctanh}(c x)) \ln \left(2-\frac{2}{c x+1}\right)-b^{2} c d^{2} \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)-2 b c d^{2}(a$
$+b \operatorname{arctanh}(c x))$ polylog$\left(2,1-\frac{2}{-c x+1}\right)+2 b c d^{2}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,-1+\frac{2}{-c x+1}\right)-b^{2} c d^{2} \operatorname{polylog}\left(2,-1+\frac{2}{c x+1}\right)$
$+b^{2} c d^{2}$ polylog $\left(3,1-\frac{2}{-c x+1}\right)-b^{2} c d^{2} \operatorname{poly} \log \left(3,-1+\frac{2}{-c x+1}\right)$
Result(type ?, 6038 leaves): Display of huge result suppressed!
Problem 26: Result more than twice size of optimal antiderivative.
$\int \frac{(c d x+d)^{3}(a+b \operatorname{arctanh}(c x))^{2}}{x^{4}} \mathrm{~d} x$
Optimal(type 4, 374 leaves, 28 steps):

$$
\begin{aligned}
& -\frac{b^{2} c^{2} d^{3}}{3 x}+\frac{b^{2} c^{3} d^{3} \operatorname{arctanh}(c x)}{3}-\frac{b c d^{3}(a+b \operatorname{arctanh}(c x))}{3 x^{2}}-\frac{3 b c^{2} d^{3}(a+b \operatorname{arctanh}(c x))}{x}+\frac{29 c^{3} d^{3}(a+b \operatorname{arctanh}(c x))^{2}}{6} \\
& -\frac{d^{3}(a+b \operatorname{arctanh}(c x))^{2}}{3 x^{3}}-\frac{3 c d^{3}(a+b \operatorname{arctanh}(c x))^{2}}{2 x^{2}}-\frac{3 c^{2} d^{3}(a+b \operatorname{arctanh}(c x))^{2}}{x}-2 c^{3} d^{3}(a+b \operatorname{arctanh}(c x))^{2} \operatorname{arctanh}\left(-1+\frac{2}{-c x+1}\right) \\
& +3 b^{2} c^{3} d^{3} \ln (x)-\frac{3 b^{2} c^{3} d^{3} \ln \left(-c^{2} x^{2}+1\right)}{2}+\frac{20 b c^{3} d^{3}(a+b \operatorname{arctanh}(c x)) \ln \left(2-\frac{2}{c x+1}\right)}{3}-b c^{3} d^{3}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right) \\
& +b c^{3} d^{3}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,-1+\frac{2}{-c x+1}\right)-\frac{10 b^{2} c^{3} d^{3} \operatorname{polylog}\left(2,-1+\frac{2}{c x+1}\right)}{3}+\frac{b^{2} c^{3} d^{3} \operatorname{polylog}\left(3,1-\frac{2}{-c x+1}\right)}{2} \\
& -\frac{b^{2} c^{3} d^{3} \operatorname{polylog}\left(3,-1+\frac{2}{-c x+1}\right)}{2}
\end{aligned}
$$

Result(type 4, 1336 leaves):

$$
-\frac{\mathrm{I} c^{3} d^{3} b^{2} \operatorname{\pi csgn}\left(\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{2} \operatorname{arctanh}(c x)^{2}}{2}
$$

$$
-\frac{\mathrm{I} c^{3} d^{3} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{2} \operatorname{arctanh}(c x)^{2}}{2}-\frac{3 c^{2} d^{3} a b}{x}-\frac{c d^{3} a b}{3 x^{2}}-\frac{3 c d^{3} b^{2} \operatorname{arctanh}(c x)^{2}}{2 x^{2}}
$$

$$
-\frac{c d^{3} b^{2} \operatorname{arctanh}(c x)}{3 x^{2}}-\frac{3 c^{2} d^{3} b^{2} \operatorname{arctanh}(c x)}{x}-\frac{3 c^{2} d^{3} b^{2} \operatorname{arctanh}(c x)^{2}}{x}+2 c^{3} d^{3} b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2,-\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)
$$

$$
+c^{3} d^{3} b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(1-\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)-c^{3} d^{3} a b \operatorname{dilog}(c x)+\frac{20 c^{3} d^{3} b^{2} \operatorname{arctanh}(c x) \ln \left(1+\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{3}-\frac{c^{3} d^{3} b^{2} \sqrt{-c^{2} x^{2}+1}}{3\left(-\sqrt{-c^{2} x^{2}+1}+c x+1\right)}
$$

$$
+\frac{c^{3} d^{3} b^{2} \sqrt{-c^{2} x^{2}+1}}{3\left(\sqrt{-c^{2} x^{2}+1}+c x+1\right)}+c^{3} d^{3} b^{2} \ln (c x) \operatorname{arctanh}(c x)^{2}+c^{3} d^{3} b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(1+\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)-c^{3} d^{3} b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}(2,
$$

$$
\left.-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)-c^{3} d^{3} b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)+2 c^{3} d^{3} b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2, \frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)+\frac{20 c^{3} d^{3} a b \ln (c x)}{3}
$$

$$
-\frac{11 c^{3} d^{3} a b \ln (c x+1)}{6}-\frac{29 c^{3} d^{3} a b \ln (c x-1)}{6}-c^{3} d^{3} a b \operatorname{dilog}(c x+1)-\frac{2 d^{3} a b \operatorname{arctanh}(c x)}{3 x^{3}}
$$

$$
\begin{aligned}
& +\frac{\mathrm{I} c^{3} d^{3} b^{2} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{arctanh}(c x)^{2}}{2} \\
& +\frac{\mathrm{I} c^{3} d^{3} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{3} \operatorname{arctanh}(c x)^{2}}{2}-\frac{3 c d^{3} a^{2}}{2 x^{2}}-\frac{3 c^{2} d^{3} a^{2}}{x}-\frac{d^{3} b^{2} \operatorname{arctanh}(c x)^{2}}{3 x^{3}}+3 c^{3} d^{3} b^{2} \ln \left(1+\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right) \\
& -2 c^{3} d^{3} b^{2} \operatorname{polylog}\left(3,-\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)+\frac{c^{3} d^{3} b^{2} \operatorname{polylog}\left(3,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{2}-2 c^{3} d^{3} b^{2} \operatorname{polylog}\left(3, \frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)-\frac{20 c^{3} d^{3} b^{2} \operatorname{dilog}\left(\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{3} \\
& +\frac{20 c^{3} d^{3} b^{2} \operatorname{dilog}\left(1+\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{3}+3 c^{3} d^{3} b^{2} \ln \left(\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}-1\right)-\frac{11 c^{3} d^{3} b^{2} \operatorname{arctanh}(c x)^{2}}{6}+c^{3} d^{3} a^{2} \ln (c x)-\frac{d^{3} a^{2}}{3 x^{3}} \\
& +2 c^{3} d^{3} a b \ln (c x) \operatorname{arctanh}(c x)-c^{3} d^{3} a b \ln (c x) \ln (c x+1)-\frac{3 c d^{3} a b \operatorname{arctanh}(c x)}{x^{2}}-\frac{6 c^{2} d^{3} a b \operatorname{arctanh}(c x)}{x}-\frac{8 b^{2} c^{3} d^{3} \operatorname{arctanh}(c x)}{3}
\end{aligned}
$$

Problem 27: Result more than twice size of optimal antiderivative.

$$
\int \frac{(c d x+d)^{3}(a+b \operatorname{arctanh}(c x))^{2}}{x^{5}} \mathrm{~d} x
$$

Optimal(type 4, 259 leaves, 18 steps):

$$
\begin{aligned}
& -\frac{b^{2} c^{2} d^{3}}{12 x^{2}}-\frac{b^{2} c^{3} d^{3}}{x}+b^{2} c^{4} d^{3} \operatorname{arctanh}(c x)-\frac{b c d^{3}(a+b \operatorname{arctanh}(c x))}{6 x^{3}}-\frac{b c^{2} d^{3}(a+b \operatorname{arctanh}(c x))}{x^{2}}-\frac{7 b c^{3} d^{3}(a+b \operatorname{arctanh}(c x))}{2 x} \\
& \quad-\frac{d^{3}(c x+1)^{4}(a+b \operatorname{arctanh}(c x))^{2}}{4 x^{4}}+4 a b c^{4} d^{3} \ln (x)+\frac{11 b^{2} c^{4} d^{3} \ln (x)}{3}+4 b c^{4} d^{3}(a+b \operatorname{arctanh}(c x)) \ln \left(\frac{2}{-c x+1}\right)-\frac{11 b^{2} c^{4} d^{3} \ln \left(-c^{2} x^{2}+1\right)}{6} \\
& \quad-2 b^{2} c^{4} d^{3} \operatorname{polylog}(2,-c x)+2 b^{2} c^{4} d^{3} \operatorname{poly} \log (2, c x)+2 b^{2} c^{4} d^{3} \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)
\end{aligned}
$$

Result(type 4, 645 leaves):

$$
\begin{array}{r}
-\frac{c d^{3} a b}{6 x^{3}}-\frac{7 c^{3} d^{3} a b}{2 x}-\frac{c^{2} d^{3} a b}{x^{2}}-\frac{c d^{3} b^{2} \operatorname{arctanh}(c x)}{6 x^{3}}-\frac{3 c^{2} d^{3} b^{2} \operatorname{arctanh}(c x)^{2}}{2 x^{2}}-\frac{c^{2} d^{3} b^{2} \operatorname{arctanh}(c x)}{x^{2}}-\frac{7 c^{3} d^{3} b^{2} \operatorname{arctanh}(c x)}{2 x}-\frac{c^{3} d^{3} b^{2} \operatorname{arctanh}(c x)^{2}}{x} \\
-\frac{c d^{3} b^{2} \operatorname{arctanh}(c x)^{2}}{x^{3}}+4 c^{4} d^{3} b^{2} \ln (c x) \operatorname{arctanh}(c x)-\frac{c^{4} d^{3} b^{2} \operatorname{arctanh}(c x) \ln (c x+1)}{4}-\frac{15 c^{4} d^{3} b^{2} \operatorname{arctanh}(c x) \ln (c x-1)}{4}-2 c^{4} d^{3} b^{2} \ln (c x) \ln (c x
\end{array}
$$

$$
\begin{aligned}
& +1)+\frac{15 c^{4} d^{3} b^{2} \ln (c x-1) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{8}-\frac{c^{4} d^{3} b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{8}+\frac{c^{4} d^{3} b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{8}+4 c^{4} d^{3} a b \ln (c x) \\
& -\frac{c^{4} d^{3} a b \ln (c x+1)}{4}-\frac{15 c^{4} d^{3} a b \ln (c x-1)}{4}-\frac{d^{3} a b \operatorname{arctanh}(c x)}{2 x^{4}}-\frac{d^{3} a^{2}}{4 x^{4}}-\frac{7 c^{4} d^{3} b^{2} \ln (c x-1)}{3}-2 c^{4} d^{3} b^{2} \operatorname{dilog}(c x+1)-2 c^{4} d^{3} b^{2} \operatorname{dilog}(c x) \\
& -\frac{15 c^{4} d^{3} b^{2} \ln (c x-1)^{2}}{16}+2 c^{4} d^{3} b^{2} \operatorname{dilog}\left(\frac{c x}{2}+\frac{1}{2}\right)+\frac{c^{4} d^{3} b^{2} \ln (c x+1)^{2}}{16}-\frac{c d^{3} a^{2}}{x^{3}}-\frac{3 c^{2} d^{3} a^{2}}{2 x^{2}}-\frac{c^{3} d^{3} a^{2}}{x}-\frac{d^{3} b^{2} \operatorname{arctanh}(c x)^{2}}{4 x^{4}} \\
& +\frac{11 c^{4} d^{3} b^{2} \ln (c x)}{3}-\frac{4 c^{4} d^{3} b^{2} \ln (c x+1)}{3}-\frac{2 c d^{3} a b \operatorname{arctanh}(c x)}{x^{3}}-\frac{3 c^{2} d^{3} a b \operatorname{arctanh}(c x)}{x^{2}}-\frac{2 c^{3} d^{3} a b \operatorname{arctanh}(c x)}{x}-\frac{b^{2} c^{2} d^{3}}{12 x^{2}}-\frac{b^{2} c^{3} d^{3}}{x}
\end{aligned}
$$

Problem 28: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))^{2}}{x^{2}(c d x+d)} \mathrm{d} x
$$

Optimal(type 4, 160 leaves, 8 steps):
$\frac{c(a+b \operatorname{arctanh}(c x))^{2}}{d}-\frac{(a+b \operatorname{arctanh}(c x))^{2}}{d x}+\frac{2 b c(a+b \operatorname{arctanh}(c x)) \ln \left(2-\frac{2}{c x+1}\right)}{d}-\frac{c(a+b \operatorname{arctanh}(c x))^{2} \ln \left(2-\frac{2}{c x+1}\right)}{d}$

$$
-\frac{b^{2} c \text { polylog }\left(2,-1+\frac{2}{c x+1}\right)}{d}+\frac{b c(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,-1+\frac{2}{c x+1}\right)}{d}+\frac{b^{2} c \operatorname{polylog}\left(3,-1+\frac{2}{c x+1}\right)}{2 d}
$$

Result(type ?, 7285 leaves): Display of huge result suppressed!
Problem 29: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))^{2}}{x^{3}(c d x+d)} \mathrm{d} x
$$

Optimal(type 4, 242 leaves, 17 steps):
$-\frac{b c(a+b \operatorname{arctanh}(c x))}{d x}-\frac{c^{2}(a+b \operatorname{arctanh}(c x))^{2}}{2 d}-\frac{(a+b \operatorname{arctanh}(c x))^{2}}{2 x^{2} d}+\frac{c(a+b \operatorname{arctanh}(c x))^{2}}{d x}+\frac{b^{2} c^{2} \ln (x)}{d}-\frac{b^{2} c^{2} \ln \left(-c^{2} x^{2}+1\right)}{2 d}$
$-\frac{2 b c^{2}(a+b \operatorname{arctanh}(c x)) \ln \left(2-\frac{2}{c x+1}\right)}{d}+\frac{c^{2}(a+b \operatorname{arctanh}(c x))^{2} \ln \left(2-\frac{2}{c x+1}\right)}{d}+\frac{b^{2} c^{2} \operatorname{polylog}\left(2,-1+\frac{2}{c x+1}\right)}{d}$

$$
-\frac{b c^{2}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,-1+\frac{2}{c x+1}\right)}{d}-\frac{b^{2} c^{2} \operatorname{polylog}\left(3,-1+\frac{2}{c x+1}\right)}{2 d}
$$

Result(type 4, 1849 leaves):

$$
-\frac{\mathrm{I} c^{2} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right)^{3} \operatorname{arctanh}(c x)^{2}}{2 d}-\frac{\mathrm{I} c^{2} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right)^{3} \operatorname{arctanh}(c x)^{2}}{2 d}
$$

$$
+\frac{\mathrm{I} c^{2} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{3} \operatorname{arctanh}(c x)^{2}}{2 d}+\frac{\mathrm{I} c^{2} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right)^{2} \operatorname{arctanh}(c x)^{2}}{d}-\frac{a^{2}}{2 d x^{2}}-\frac{c a b}{d x}
$$

$$
+\frac{\mathrm{I} c^{2} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{arctanh}(c x)^{2}}{2 d}
$$

$$
-\frac{\mathrm{I} c^{2} b^{2} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{2} \operatorname{arctanh}(c x)^{2}}{2 d}
$$

$$
+\frac{\mathrm{I} c^{2} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right)^{2} \operatorname{arctanh}(c x)^{2}}{2 d}
$$

$$
-\frac{\mathrm{I} c^{2} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right) \operatorname{arctanh}(c x)^{2}}{2 d}-\frac{\mathrm{I} c^{2} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{2} \operatorname{arctanh}(c x)^{2}}{2 d}
$$

$$
+\frac{c a^{2}}{d x}-\frac{b^{2} \operatorname{arctanh}(c x)^{2}}{2 d x^{2}}-\frac{2 c^{2} b^{2} \text { polylog }\left(3,-\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{d}-\frac{2 c^{2} b^{2} \operatorname{polylog}\left(3, \frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{d}+\frac{2 c^{2} b^{2} \operatorname{dilog}\left(\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{d}
$$

$$
-\frac{2 c^{2} b^{2} \operatorname{dilog}\left(1+\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{d}+\frac{3 c^{2} b^{2} \operatorname{arctanh}(c x)^{2}}{2 d}-\frac{2 c^{2} b^{2} \operatorname{arctanh}(c x)^{3}}{3 d}+\frac{c^{2} a^{2} \ln (c x)}{d}-\frac{c^{2} a^{2} \ln (c x+1)}{d}+\frac{c^{2} b^{2} \ln \left(\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}-1\right)}{d}
$$

$$
+\frac{c^{2} b^{2} \ln \left(1+\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{d}-\frac{c^{2} b^{2} \operatorname{arctanh}(c x)}{d}-\frac{c b^{2} \operatorname{arctanh}(c x)}{d x}+\frac{c b^{2} \operatorname{arctanh}(c x)^{2}}{d x}-\frac{c^{2} a b \operatorname{dilog}(c x+1)}{d}-\frac{c^{2} a b \operatorname{dilog}(c x)}{d}
$$

$$
\begin{aligned}
& +\frac{c^{2} a b \ln (c x+1)^{2}}{2 d}+\frac{c^{2} a b \operatorname{dilog}\left(\frac{c x}{2}+\frac{1}{2}\right)}{d}+\frac{c^{2} b^{2} \ln (c x) \operatorname{arctanh}(c x)^{2}}{d}+\frac{c^{2} b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(1+\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{d} \\
& -\frac{c^{2} b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{d}+\frac{2 c^{2} b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2, \frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{d}+\frac{2 c^{2} b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2,-\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{d} \\
& +\frac{c^{2} b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(1-\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{d}-\frac{2 c^{2} b^{2} \operatorname{arctanh}(c x) \ln \left(1+\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{d}-\frac{c^{2} b^{2} \operatorname{arctanh}(c x)^{2} \ln (c x+1)}{d} \\
& +\frac{2 c^{2} b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{d}+\frac{c^{2} b^{2} \ln (2) \operatorname{arctanh}(c x)^{2}}{d}-\frac{2 c^{2} a b \ln (c x)}{d}+\frac{3 c^{2} a b \ln (c x+1)}{2 d}+\frac{c^{2} a b \ln (c x-1)}{2 d}-\frac{a b \operatorname{arctanh}(c x)}{d x^{2}} \\
& \left.+\frac{\mathrm{I} c^{2} b^{2} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right)}{2 d}\right) \operatorname{arctanh}(c x)^{2}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \\
& -\frac{\mathrm{I} c^{2} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{arctanh}(c x)^{2}}{2 d}-\frac{2 c^{2} a b \operatorname{arctanh}(c x) \ln (c x+1)}{d} \\
& -\frac{c^{2} a b \ln (c x) \ln (c x+1)}{d}-\frac{c^{2} a b \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{d}+\frac{c^{2} a b \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{d}+\frac{2 c^{2} a b \ln (c x) \operatorname{arctanh}(c x)}{d} \\
& +\frac{2 c a b \operatorname{arctanh}(c x)}{d x}
\end{aligned}
$$

Problem 30: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))^{2}}{x^{4}(c d x+d)} \mathrm{d} x
$$

Optimal(type 4, 314 leaves, 26 steps):

$$
\begin{aligned}
& -\frac{b^{2} c^{2}}{3 d x}+\frac{b^{2} c^{3} \operatorname{arctanh}(c x)}{3 d}-\frac{b c(a+b \operatorname{arctanh}(c x))}{3 x^{2} d}+\frac{b c^{2}(a+b \operatorname{arctanh}(c x))}{d x}+\frac{5 c^{3}(a+b \operatorname{arctanh}(c x))^{2}}{6 d}-\frac{(a+b \operatorname{arctanh}(c x))^{2}}{3 x^{3} d} \\
& \quad+\frac{c(a+b \operatorname{arctanh}(c x))^{2}}{2 x^{2} d}-\frac{c^{2}(a+b \operatorname{arctanh}(c x))^{2}}{d x}-\frac{b^{2} c^{3} \ln (x)}{d}+\frac{b^{2} c^{3} \ln \left(-c^{2} x^{2}+1\right)}{2 d}+\frac{8 b c^{3}(a+b \operatorname{arctanh}(c x)) \ln \left(2-\frac{2}{c x+1}\right)}{3 d}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{c^{3}(a+b \operatorname{arctanh}(c x))^{2} \ln \left(2-\frac{2}{c x+1}\right)}{d}-\frac{4 b^{2} c^{3} \operatorname{polylog}\left(2,-1+\frac{2}{c x+1}\right)}{3 d}+\frac{b c^{3}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,-1+\frac{2}{c x+1}\right)}{d} \\
& +\frac{b^{2} c^{3} \operatorname{poly} \log \left(3,-1+\frac{2}{c x+1}\right)}{2 d}
\end{aligned}
$$

Result (type ?, 2018 leaves): Display of huge result suppressed!
Problem 31: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2}(a+b \operatorname{arctanh}(c x))^{2}}{(c d x+d)^{3}} \mathrm{~d} x
$$

Optimal(type 4, 249 leaves, 26 steps):

$$
\begin{aligned}
& -\frac{b^{2}}{16 c^{3} d^{3}(c x+1)^{2}}+\frac{13 b^{2}}{16 c^{3} d^{3}(c x+1)}-\frac{13 b^{2} \operatorname{arctanh}(c x)}{16 d^{3} c^{3}}-\frac{b(a+b \operatorname{arctanh}(c x))}{4 c^{3} d^{3}(c x+1)^{2}}+\frac{7 b(a+b \operatorname{arctanh}(c x))}{4 c^{3} d^{3}(c x+1)}-\frac{7(a+b \operatorname{arctanh}(c x))^{2}}{8 d^{3} c^{3}} \\
& -\frac{(a+b \operatorname{arctanh}(c x))^{2}}{2 c^{3} d^{3}(c x+1)^{2}}+\frac{2(a+b \operatorname{arctanh}(c x))^{2}}{c^{3} d^{3}(c x+1)}-\frac{(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{c x+1}\right)}{d^{3} c^{3}}+\frac{2}{d^{3} c^{3}}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{c x+1}\right) \\
& +\frac{b^{2} \operatorname{polylog}\left(3,1-\frac{2}{c x+1}\right)}{2 d^{3} c^{3}}
\end{aligned}
$$

Result(type 4, 1249 leaves):

$$
\begin{aligned}
& -\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right)^{2} \operatorname{arctanh}(c x)^{2}}{c^{3} d^{3}}-\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right.}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{arctanh}(c x)^{2}}{2 c^{3} d^{3}} \\
& -\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right)^{2} \operatorname{arctanh}(c x)^{2}}{2 c^{3} d^{3}} \\
& -\frac{a^{2}}{2 c^{3} d^{3}(c x+1)^{2}}+\frac{2 a^{2}}{c^{3} d^{3}(c x+1)}+\frac{a^{2} \ln (c x+1)}{c^{3} d^{3}} \\
& +\frac{b^{2} \text { polylog }\left(3,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{2 c^{3} d^{3}}-\frac{7 b^{2} \operatorname{arctanh}(c x)^{2}}{8 c^{3} d^{3}}+\frac{2 b^{2} \operatorname{arctanh}(c x)^{3}}{3 c^{3} d^{3}} \\
& +\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{arctanh}(c x)^{2}}{2 c^{3} d^{3}}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right)^{3} \operatorname{arctanh}(c x)^{2}}{2 c^{3} d^{3}}+\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right)^{3} \operatorname{arctanh}(c x)^{2}}{2 c^{3} d^{3}}-\frac{b^{2} \operatorname{arctanh}(c x) x^{2}}{16 c d^{3}(c x+1)^{2}}+\frac{b^{2} \operatorname{arctanh}(c x) x}{8 c^{2} d^{3}(c x+1)^{2}} \\
& -\frac{3 b^{2} \operatorname{arctanh}(c x) x}{4 c^{2} d^{3}(c x+1)}-\frac{a b \operatorname{arctanh}(c x)}{c^{3} d^{3}(c x+1)^{2}}+\frac{4 a b \operatorname{arctanh}(c x)}{c^{3} d^{3}(c x+1)}+\frac{a b \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{c^{3} d^{3}}-\frac{a b \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{c^{3} d^{3}} \\
& +\frac{2 a b \operatorname{arctanh}(c x) \ln (c x+1)}{c^{3} d^{3}}+\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right) \operatorname{arctanh}(c x)^{2}}{2 c^{3} d^{3}}-\frac{b^{2}}{64 c^{3} d^{3}(c x+1)^{2}}+\frac{3 b^{2}}{8 c^{3} d^{3}(c x+1)} \\
& +\frac{7 a b}{4 c^{3} d^{3}(c x+1)}-\frac{3 b^{2} x}{8 c^{2} d^{3}(c x+1)}-\frac{b^{2} x^{2}}{64 c d^{3}(c x+1)^{2}}+\frac{b^{2} x}{32 c^{2} d^{3}(c x+1)^{2}}+\frac{b^{2} \operatorname{arctanh}(c x)^{2} \ln (c x+1)}{c^{3} d^{3}} \\
& -\frac{b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{c^{3} d^{3}}-\frac{2 b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{c^{3} d^{3}}-\frac{b^{2} \operatorname{arctanh}(c x)^{2}}{2 c^{3} d^{3}(c x+1)^{2}}+\frac{2 b^{2} \operatorname{arctanh}(c x)^{2}}{c^{3} d^{3}(c x+1)}-\frac{b^{2} \operatorname{arctanh}(c x)}{16 c^{3} d^{3}(c x+1)^{2}} \\
& +\frac{3 b^{2} \operatorname{arctanh}(c x)}{4 c^{3} d^{3}(c x+1)}+\frac{7 a b \ln (c x-1)}{8 c^{3} d^{3}}-\frac{7 a b \ln (c x+1)}{8 c^{3} d^{3}}-\frac{a b \ln (c x+1)^{2}}{2 c^{3} d^{3}}-\frac{a b \operatorname{dilog}\left(\frac{c x}{2}+\frac{1}{2}\right)}{c^{3} d^{3}}-\frac{b^{2} \ln (2) \operatorname{arctanh}(c x)^{2}}{c^{3} d^{3}}-\frac{a b}{4 c^{3} d^{3}(c x+1)^{2}}
\end{aligned}
$$

Problem 32: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))^{2}}{x^{2}(c d x+d)^{3}} d x
$$

Optimal(type 4, 428 leaves, 36 steps):

$$
\begin{aligned}
& -\frac{b^{2} c}{16 d^{3}(c x+1)^{2}}-\frac{19 b^{2} c}{16 d^{3}(c x+1)}+\frac{19 b^{2} c \operatorname{arctanh}(c x)}{16 d^{3}}-\frac{b c(a+b \operatorname{arctanh}(c x))}{4 d^{3}(c x+1)^{2}}-\frac{9 b c(a+b \operatorname{arctanh}(c x))}{4 d^{3}(c x+1)}+\frac{17 c(a+b \operatorname{arctanh}(c x))^{2}}{8 d^{3}} \\
& -\frac{(a+b \operatorname{arctanh}(c x))^{2}}{x d^{3}}-\frac{c(a+b \operatorname{arctanh}(c x))^{2}}{2 d^{3}(c x+1)^{2}}-\frac{2 c(a+b \operatorname{arctanh}(c x))^{2}}{d^{3}(c x+1)}+\frac{6 c(a+b \operatorname{arctanh}(c x))^{2} \operatorname{arctanh}\left(-1+\frac{2}{-c x+1}\right)}{d^{3}} \\
& -\frac{3 c(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{c x+1}\right)}{d^{3}}+\frac{2 b c(a+b \operatorname{arctanh}(c x)) \ln \left(2-\frac{2}{c x+1}\right)}{d^{3}}+\frac{3 b c(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{d^{3}} \\
& -\frac{3 b c(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,-1+\frac{2}{-c x+1}\right)}{d^{3}}+\frac{3 b c(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{c x+1}\right)}{d^{3}}-\frac{b^{2} c \operatorname{polylog}\left(2,-1+\frac{2}{c x+1}\right)}{d^{3}}
\end{aligned}
$$

$$
-\frac{3 b^{2} c \text { polylog }\left(3,1-\frac{2}{-c x+1}\right)}{2 d^{3}}+\frac{3 b^{2} c \text { polylog }\left(3,-1+\frac{2}{-c x+1}\right)}{2 d^{3}}+\frac{3 b^{2} c \operatorname{poly} \log \left(3,1-\frac{2}{c x+1}\right)}{2 d^{3}}
$$

Result(type ?, 7646 leaves): Display of huge result suppressed!
Problem 33: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))^{2}}{(c x+1)^{4}} \mathrm{~d} x
$$

Optimal(type 3, 158 leaves, 18 steps):
$-\frac{b^{2}}{54 c(c x+1)^{3}}-\frac{5 b^{2}}{144 c(c x+1)^{2}}-\frac{11 b^{2}}{144 c(c x+1)}+\frac{11 b^{2} \operatorname{arctanh}(c x)}{144 c}-\frac{b(a+b \operatorname{arctanh}(c x))}{9 c(c x+1)^{3}}-\frac{b(a+b \operatorname{arctanh}(c x))}{12 c(c x+1)^{2}}-\frac{b(a+b \operatorname{arctanh}(c x))}{12 c(c x+1)}$

$$
+\frac{(a+b \operatorname{arctanh}(c x))^{2}}{24 c}-\frac{(a+b \operatorname{arctanh}(c x))^{2}}{3 c(c x+1)^{3}}
$$

Result(type 3, 385 leaves):
$-\frac{a^{2}}{3 c(c x+1)^{3}}-\frac{b^{2} \operatorname{arctanh}(c x)^{2}}{3 c(c x+1)^{3}}-\frac{b^{2} \operatorname{arctanh}(c x)}{9 c(c x+1)^{3}}-\frac{b^{2} \operatorname{arctanh}(c x)}{12 c(c x+1)^{2}}-\frac{b^{2} \operatorname{arctanh}(c x)}{12 c(c x+1)}+\frac{b^{2} \operatorname{arctanh}(c x) \ln (c x+1)}{24 c}-\frac{b^{2} \operatorname{arctanh}(c x) \ln (c x-1)}{24 c}$

$$
\begin{aligned}
& -\frac{b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{48 c}+\frac{b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{48 c}+\frac{b^{2} \ln (c x-1) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{48 c}-\frac{b^{2} \ln (c x+1)^{2}}{96 c}-\frac{b^{2} \ln (c x-1)^{2}}{96 c} \\
& +\frac{11 b^{2} \ln (c x+1)}{288 c}-\frac{11 b^{2} \ln (c x-1)}{288 c}-\frac{11 b^{2}}{144 c(c x+1)}-\frac{5 b^{2}}{144 c(c x+1)^{2}}-\frac{b^{2}}{54 c(c x+1)^{3}}-\frac{2 a b \operatorname{arctanh}(c x)}{3 c(c x+1)^{3}}-\frac{a b}{9 c(c x+1)^{3}} \\
& -\frac{a b}{12 c(c x+1)^{2}}-\frac{a b}{12 c(c x+1)}+\frac{a b \ln (c x+1)}{24 c}-\frac{a b \ln (c x-1)}{24 c}
\end{aligned}
$$

Problem 34: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)^{2}}{-a c x^{2}+c x} \mathrm{~d} x
$$

Optimal(type 4, 65 leaves, 4 steps):

$$
\frac{\operatorname{arctanh}(a x)^{2} \ln \left(2-\frac{2}{-a x+1}\right)}{c}+\frac{\operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-1+\frac{2}{-a x+1}\right)}{c}-\frac{\operatorname{polylog}\left(3,-1+\frac{2}{-a x+1}\right)}{2 c}
$$

Result(type 4, 716 leaves):
$\frac{\operatorname{arctanh}(a x)^{2} \ln (a x)}{c}-\frac{\operatorname{arctanh}(a x)^{2} \ln (a x-1)}{c}-\frac{\operatorname{arctanh}(a x)^{2} \ln \left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{c}+\frac{\operatorname{arctanh}(a x)^{2} \ln \left(1+\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}$

$$
\begin{aligned}
& +\frac{2 \operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}-\frac{2 \operatorname{polylog}\left(3,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{\operatorname{arctanh}(a x)^{2} \ln \left(1-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c} \\
& +\frac{2 \operatorname{arctanh}(a x) \operatorname{polylog}\left(2, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}-\frac{2 \operatorname{polylog}\left(3, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{\mathrm{I} \pi \operatorname{arctanh}(a x)^{2}}{c} \\
& -\frac{\mathrm{I} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \pi \operatorname{arctanh}(a x)^{2}}{2 c}-\frac{\mathrm{I} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \pi \operatorname{arctanh}(a x)^{2}}{c} \\
& +\frac{\mathrm{I} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{3} \pi \operatorname{arctanh}(a x)^{2}}{c}+\frac{\mathrm{I} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \pi \operatorname{arctanh}(a x)^{2}}{2 c} \\
& +\frac{\operatorname{Icsgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{3} \pi \operatorname{arctanh}(a x)^{2}}{2 c}-\frac{I \operatorname{csgn}\left(I\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \pi \operatorname{arctanh}(a x)^{2}}{2 c} \\
& +\frac{\ln (2) \operatorname{arctanh}(a x)^{2}}{c}
\end{aligned}
$$

Problem 35: Result more than twice size of optimal antiderivative.

$$
\int(c x+1)^{3}(a+b \operatorname{arctanh}(c x))^{3} \mathrm{~d} x
$$

Optimal(type 4, 288 leaves, 26 steps):
$3 a b^{2} x+\frac{b^{3} x}{4}-\frac{b^{3} \operatorname{arctanh}(c x)}{4 c}+3 b^{3} x \operatorname{arctanh}(c x)+\frac{b^{2} c x^{2}(a+b \operatorname{arctanh}(c x))}{4}+\frac{4 b(a+b \operatorname{arctanh}(c x))^{2}}{c}+\frac{21 b x(a+b \operatorname{arctanh}(c x))^{2}}{4}$
$+\frac{3 b c x^{2}(a+b \operatorname{arctanh}(c x))^{2}}{2}+\frac{b c^{2} x^{3}(a+b \operatorname{arctanh}(c x))^{2}}{4}+\frac{(c x+1)^{4}(a+b \operatorname{arctanh}(c x))^{3}}{4 c}-\frac{11 b^{2}(a+b \operatorname{arctanh}(c x)) \ln \left(\frac{2}{-c x+1}\right)}{c}$
$-\frac{6 b(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{-c x+1}\right)}{c}+\frac{3 b^{3} \ln \left(-c^{2} x^{2}+1\right)}{2 c}-\frac{11 b^{3} \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{2 c}$

$$
-\frac{6 b^{2}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{c}+\frac{3 b^{3} \operatorname{polylog}\left(3,1-\frac{2}{-c x+1}\right)}{c}
$$

Result(type 4, 962 leaves):

$$
\begin{aligned}
& -\frac{6 a b^{2} \ln (c x-1) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{c}+\frac{c^{2} a b^{2} x^{3} \operatorname{arctanh}(c x)}{2}+\frac{9 c a^{2} b \operatorname{arctanh}(c x) x^{2}}{2}+\frac{9 c a b^{2} \operatorname{arctanh}(c x)^{2} x^{2}}{2}+3 c a b^{2} \operatorname{arctanh}(c x) x^{2} \\
& +3 c^{2} x^{3} a b^{2} \operatorname{arctanh}(c x)^{2}+3 c^{2} x^{3} a^{2} b \operatorname{arctanh}(c x)+\frac{3 c^{3} a^{2} b x^{4} \operatorname{arctanh}(c x)}{4}+\frac{3 c^{3} a b^{2} x^{4} \operatorname{arctanh}(c x)^{2}}{4}-\frac{6 \mathrm{I} b^{3} \pi \operatorname{arctanh}(c x)^{2}}{c} \\
& +\frac{12 a b^{2} \operatorname{arctanh}(c x) \ln (c x-1)}{c}+c^{2} x^{3} a^{3}+\frac{3 c x^{2} a^{3}}{2}+\frac{c^{3} x^{4} a^{3}}{4}+\frac{4 b^{3} \operatorname{arctanh}(c x)^{2}}{c}-\frac{3 b^{3} \ln \left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{c}+\frac{b^{3} \operatorname{arctanh}(c x)^{3}}{4 c} \\
& +\frac{3 b^{3} \text { polylog }\left(3,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{c}-\frac{11 b^{3} \operatorname{dilog}\left(1-\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right)}{c}-\frac{11 b^{3} \operatorname{dilog}\left(1+\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right)}{c}+\frac{21 b^{3} x \operatorname{arctanh}(c x)^{2}}{4}+b^{3} \operatorname{arctanh}(c x)^{3} x \\
& +\frac{21 a^{2} b x}{4}-\frac{13 a b^{2}}{4 c}-\frac{b^{3}}{4 c}+\frac{3 c a^{2} b x^{2}}{2}+\frac{c^{2} a^{2} b x^{3}}{4}+\frac{c a b^{2} x^{2}}{4}+\frac{21 a b^{2} \operatorname{arctanh}(c x) x}{2}+3 a^{2} b \operatorname{arctanh}(c x) x+3 \operatorname{arctanh}(c x)^{2} x a b^{2} \\
& +\frac{6 b^{3} \operatorname{arctanh}(c x)^{2} \ln (c x-1)}{c}-\frac{6 b^{3} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{c}-\frac{11 b^{3} \operatorname{arctanh}(c x) \ln \left(1-\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right)}{c} \\
& -\frac{11 b^{3} \operatorname{arctanh}(c x) \ln \left(1+\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right)}{c}+\frac{3 a b^{2} \operatorname{arctanh}(c x)^{2}}{4 c}+\frac{3 a^{2} b \operatorname{arctanh}(c x)}{4 c}-\frac{6 a b^{2} \operatorname{dilog}\left(\frac{c x}{2}+\frac{1}{2}\right)}{c}-\frac{6 b^{3} \ln (2) \operatorname{arctanh}(c x)^{2}}{c} \\
& +\frac{6 a^{2} b \ln (c x-1)}{c}+\frac{3 a b^{2} \ln (c x-1)^{2}}{c}+\frac{4 a b^{2} \ln (c x+1)}{c}+\frac{7 a b^{2} \ln (c x-1)}{c}+c^{2} x^{3} b^{3} \operatorname{arctanh}(c x)^{3}+\frac{c^{2} b^{3} x^{3} \operatorname{arctanh}(c x)^{2}}{4} \\
& +\frac{c^{3} b^{3} x^{4} \operatorname{arctanh}(c x)^{3}}{4}+\frac{c b^{3} \operatorname{arctanh}(c x) x^{2}}{4}+\frac{3 c b^{3} \operatorname{arctanh}(c x)^{2} x^{2}}{2}+\frac{3 c b^{3} \operatorname{arctanh}(c x)^{3} x^{2}}{2}+\frac{a^{3}}{4 c}+a^{3} x \\
& -\frac{6 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{3} \operatorname{arctanh}(c x)^{2}}{c}+\frac{6 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{2} \operatorname{arctanh}(c x)^{2}}{c}+3 a b^{2} x+\frac{11 b^{3} \operatorname{arctanh}(c x)}{4 c}+3 b^{3} x \operatorname{arctanh}(c x) \\
& +\frac{b^{3} x}{4}
\end{aligned}
$$

Problem 36: Result more than twice size of optimal antiderivative.

$$
\int(c x+1)(a+b \operatorname{arctanh}(c x))^{3} \mathrm{~d} x
$$

Optimal(type 4, 181 leaves, 11 steps):
$\frac{3 b(a+b \operatorname{arctanh}(c x))^{2}}{2 c}+\frac{3 b x(a+b \operatorname{arctanh}(c x))^{2}}{2}+\frac{(c x+1)^{2}(a+b \operatorname{arctanh}(c x))^{3}}{2 c}-\frac{3 b^{2}(a+b \operatorname{arctanh}(c x)) \ln \left(\frac{2}{-c x+1}\right)}{c}$

$$
\begin{aligned}
& -\frac{3 b(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{-c x+1}\right)}{c}-\frac{3 b^{3} \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{2 c}-\frac{3 b^{2}(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{c} \\
& +\frac{3 b^{3} \operatorname{poly} \log \left(3,1-\frac{2}{-c x+1}\right)}{2 c}
\end{aligned}
$$

Result(type ?, 6502 leaves): Display of huge result suppressed!
Problem 37: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))^{3}}{(c x+1)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 127 leaves, 11 steps):

$$
\begin{aligned}
& -\frac{3 b^{3}}{4 c(c x+1)}+\frac{3 b^{3} \operatorname{arctanh}(c x)}{4 c}-\frac{3 b^{2}(a+b \operatorname{arctanh}(c x))}{2 c(c x+1)}+\frac{3 b(a+b \operatorname{arctanh}(c x))^{2}}{4 c}-\frac{3 b(a+b \operatorname{arctanh}(c x))^{2}}{2 c(c x+1)}+\frac{(a+b \operatorname{arctanh}(c x))^{3}}{2 c} \\
& \quad-\frac{(a+b \operatorname{arctanh}(c x))^{3}}{c(c x+1)}
\end{aligned}
$$

Result(type 3, 1912 leaves):

$$
\begin{aligned}
& -\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{arctanh}(c x)^{2} x}{8(c x+1)}+\frac{3 a b^{2} \ln (c x-1) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{4 c} \\
& -\frac{3 a b^{2} \operatorname{arctanh}(c x) \ln (c x-1)}{2 c}-\frac{3 b^{3} \operatorname{arctanh}(c x)^{2} \ln (c x-1)}{4 c}-\frac{3 a^{2} b \ln (c x-1)}{4 c}-\frac{3 a b^{2} \ln (c x-1)^{2}}{8 c}+\frac{3 a b^{2} \ln (c x+1)}{4 c}-\frac{3 a b^{2} \ln (c x-1)}{4 c} \\
& -\frac{3 b^{3} \operatorname{arctanh}(c x)^{2} \ln \left(\frac{c x+1}{\sqrt{-c^{2} x^{2}+1}}\right)}{2 c}+\frac{3 b^{3} \operatorname{arctanh}(c x)^{2} \ln (c x+1)}{4 c}+\frac{3 a^{2} b \ln (c x+1)}{4 c}-\frac{3 a b^{2} \ln (c x+1)^{2}}{8 c} \\
& +\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right.}\right)}{8 c(c x+1)} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{arctanh}(c x)^{2} \\
& +\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{arctanh}(c x)^{2} x}{8(c x+1)}-\frac{3 a b^{2}}{2 c(c x+1)}-\frac{3 a^{2} b}{2 c(c x+1)}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{3 b^{3} \operatorname{arctanh}(c x)}{4 c(c x+1)}-\frac{3 b^{3} \operatorname{arctanh}(c x)^{2}}{4 c(c x+1)}-\frac{b^{3} \operatorname{arctanh}(c x)^{3}}{2 c(c x+1)}+\frac{b^{3} \operatorname{arctanh}(c x)^{3} x}{2(c x+1)}+\frac{3 b^{3} \operatorname{arctanh}(c x)^{2} x}{4(c x+1)}+\frac{3 b^{3} \operatorname{arctanh}(c x) x}{4(c x+1)} \\
& +\frac{3 a b^{2} \operatorname{arctanh}(c x) \ln (c x+1)}{2 c}-\frac{3 a b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln \left(\frac{c x}{2}+\frac{1}{2}\right)}{4 c}+\frac{3 a b^{2} \ln \left(-\frac{c x}{2}+\frac{1}{2}\right) \ln (c x+1)}{4 c} \\
& -\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right)^{2} \operatorname{arctanh}(c x)^{2} x}{4(c x+1)} \\
& \frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right)^{2} \operatorname{arctanh}(c x)^{2} x}{8(c x+1)} \\
& +\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right) \operatorname{arctanh}(c x)^{2} x}{8(c x+1)}-\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right)^{2} \operatorname{arctanh}(c x)^{2}}{4 c(c x+1)} \\
& -\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right)^{2} \operatorname{arctanh}(c x)^{2}}{8 c(c x+1))} \\
& 8 c(c x+1) \\
& +\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)}{\sqrt{-c^{2} x^{2}+1}}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right) \operatorname{arctanh}(c x)^{2}}{8 c(c x+1)} \\
& -\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{arctanh}(c x)^{2}}{8 c(c x+1)}-\frac{3 b^{3}}{8 c(c x+1)}-\frac{3 a b^{2} \operatorname{arctanh}(c x)}{c(c x+1)}-\frac{3 a^{2} b \operatorname{arctanh}(c x)}{c(c x+1)} \\
& -\frac{3 a b^{2} \operatorname{arctanh}(c x)^{2}}{c(c x+1)}+\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{3} \operatorname{arctanh}(c x)^{2} x}{4(c x+1)}+\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right)^{3} \operatorname{arctanh}(c x)^{2} x}{8(c x+1)} \\
& +\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right)^{3} \operatorname{arctanh}(c x)^{2} x}{8(c x+1)}-\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{2} \operatorname{arctanh}(c x)^{2} x}{4(c x+1)}
\end{aligned}
$$

$$
\begin{aligned}
& 3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}\right)^{3} \operatorname{arctanh}(c x)^{2} \\
+ & \frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{2} \operatorname{arctanh}(c x)^{2}}{4 c(c x+1)}-\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{3} \operatorname{arctanh}(c x)^{2}}{4 c(c x+1)}+\frac{3 \mathrm{I} b^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(c x+1)^{2}}{-c^{2} x^{2}+1}\right)^{3} \operatorname{arctanh}(c x)^{2}}{8 c(c x+1)}+\frac{3 \mathrm{I} b^{3} \pi \operatorname{arctanh}(c x)^{2} x}{4(c x+1)}+\frac{3 \mathrm{I} b^{3} \pi \operatorname{arctanh}(c x)^{2}}{4 c(c x+1)} \\
+ & \frac{a^{3}}{c(c x+1)}+\frac{3 b^{3} x}{8(c x+1)}
\end{aligned}
$$

Problem 38: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))^{3}}{(c x+1)^{4}} d x
$$

Optimal(type 3, 249 leaves, 42 steps):

```
\[
-\frac{b^{3}}{108 c(c x+1)^{3}}-\frac{19 b^{3}}{576 c(c x+1)^{2}}-\frac{85 b^{3}}{576 c(c x+1)}+\frac{85 b^{3} \operatorname{arctanh}(c x)}{576 c}-\frac{b^{2}(a+b \operatorname{arctanh}(c x))}{18 c(c x+1)^{3}}-\frac{5 b^{2}(a+b \operatorname{arctanh}(c x))}{48 c(c x+1)^{2}}
\]
\[
-\frac{11 b^{2}(a+b \operatorname{arctanh}(c x))}{48 c(c x+1)}+\frac{11 b(a+b \operatorname{arctanh}(c x))^{2}}{96 c}-\frac{b(a+b \operatorname{arctanh}(c x))^{2}}{6 c(c x+1)^{3}}-\frac{b(a+b \operatorname{arctanh}(c x))^{2}}{8 c(c x+1)^{2}}-\frac{b(a+b \operatorname{arctanh}(c x))^{2}}{8 c(c x+1)}
\]
\[
+\frac{(a+b \operatorname{arctanh}(c x))^{3}}{24 c}-\frac{(a+b \operatorname{arctanh}(c x))^{3}}{3 c(c x+1)^{3}}
\]
Result(type ?, 3672 leaves): Display of huge result suppressed!
Problem 39: Result more than twice size of optimal antiderivative.
```

$$
\int \frac{\operatorname{arctanh}(a x)^{3}}{a c x^{2}+c x} \mathrm{~d} x
$$

Optimal(type 4, 87 leaves, 5 steps):
$\frac{\operatorname{arctanh}(a x)^{3} \ln \left(2-\frac{2}{a x+1}\right)}{c}-\frac{3 \operatorname{arctanh}(a x)^{2} \operatorname{polylog}\left(2,-1+\frac{2}{a x+1}\right)}{2 c}-\frac{3 \operatorname{arctanh}(a x) \operatorname{polylog}\left(3,-1+\frac{2}{a x+1}\right)}{2 c}-\frac{3 \operatorname{polylog}\left(4,-1+\frac{2}{a x+1}\right)}{4 c}$
Result(type 4, 1225 leaves):
$-\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{3}}{2 c}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)^{3}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{\operatorname{arctanh}(a x)^{3}}}{2 c}-\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{3}}{2 c}$
$+\frac{\operatorname{arctanh}(a x)^{3} \ln (a x)}{c}+\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{2}}{c}$

$$
-\frac{I \pi \operatorname{csgn}\left(I\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \operatorname{arctanh}(a x)^{3}}{2 c}
$$

$$
+\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2}}{2 c}
$$

$$
+\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)}{2 c}-\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{2 c}
$$

$$
-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \operatorname{arctanh}(a x)^{3}}{2 c}
$$

$$
-\frac{\operatorname{arctanh}(a x)^{3} \ln (a x+1)}{c}+\frac{\operatorname{arctanh}(a x)^{3} \ln \left(1+\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}
$$

$$
+\frac{3 \operatorname{arctanh}(a x)^{2} \operatorname{polylog}\left(2,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}-\frac{6 \operatorname{arctanh}(a x) \operatorname{polylog}\left(3,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{\operatorname{arctanh}(a x)^{3} \ln \left(1-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}
$$

$$
+\frac{3 \operatorname{arctanh}(a x)^{2} \operatorname{polylog}\left(2, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}-\frac{6 \operatorname{arctanh}(a x) \operatorname{polylog}\left(3, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}-\frac{\operatorname{arctanh}(a x)^{3} \ln \left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{c}
$$

$$
+\frac{2 \operatorname{arctanh}(a x)^{3} \ln \left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{\operatorname{arctanh}(a x)^{3} \ln (2)}{c}-\frac{\operatorname{arctanh}(a x)^{4}}{2 c}+\frac{6 \operatorname{polylog}\left(4, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{6 \operatorname{polylog}\left(4,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}
$$

$$
\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)}{2 c}
$$

$$
+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{3}}{2 c}
$$

Problem 40: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)^{3}}{x^{3}(a c x+c)} \mathrm{d} x
$$

Optimal(type 4, 287 leaves, 18 steps):
$\frac{3 a^{2} \operatorname{arctanh}(a x)^{2}}{2 c}-\frac{3 a \operatorname{arctanh}(a x)^{2}}{2 c x}-\frac{a^{2} \operatorname{arctanh}(a x)^{3}}{2 c}-\frac{\operatorname{arctanh}(a x)^{3}}{2 c x^{2}}+\frac{a \operatorname{arctanh}(a x)^{3}}{c x}+\frac{3 a^{2} \operatorname{arctanh}(a x) \ln \left(2-\frac{2}{a x+1}\right)}{c}$

$$
\begin{aligned}
& -\frac{3 a^{2} \operatorname{arctanh}(a x)^{2} \ln \left(2-\frac{2}{a x+1}\right)}{c}+\frac{a^{2} \operatorname{arctanh}(a x)^{3} \ln \left(2-\frac{2}{a x+1}\right)}{c}-\frac{3 a^{2} \operatorname{polylog}\left(2,-1+\frac{2}{a x+1}\right)}{2 c} \\
& +\frac{3 a^{2} \operatorname{arctanh}(a x) \operatorname{poly} \log \left(2,-1+\frac{2}{a x+1}\right)}{c}-\frac{3 a^{2} \operatorname{arctanh}(a x)^{2} \operatorname{polylog}\left(2,-1+\frac{2}{a x+1}\right)}{2 c}+\frac{3 a^{2} \operatorname{poly} \log \left(3,-1+\frac{2}{a x+1}\right)}{2 c} \\
& -\frac{3 a^{2} \operatorname{arctanh}(a x) \operatorname{polylog}\left(3,-1+\frac{2}{a x+1}\right)}{2 c}-\frac{3 a^{2} \operatorname{polylog}\left(4,-1+\frac{2}{a x+1}\right)}{4 c}
\end{aligned}
$$

Result(type 4, 663 leaves):

$$
-\frac{3 a^{2} \operatorname{arctanh}(a x)^{2} \ln \left(1+\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}-\frac{6 a^{2} \operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}-\frac{3 a^{2} \operatorname{arctanh}(a x)^{2} \ln \left(1-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}
$$

$$
-\frac{6 a^{2} \operatorname{arctanh}(a x) \operatorname{polylog}\left(2, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{6 a^{2} \operatorname{polylog}\left(3,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{6 a^{2} \operatorname{polylog}\left(3, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}
$$

$$
+\frac{a^{2} \operatorname{arctanh}(a x)^{3} \ln \left(1+\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{3 a^{2} \operatorname{arctanh}(a x)^{2} \operatorname{polylog}\left(2,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}-\frac{6 a^{2} \operatorname{arctanh}(a x) \operatorname{polylog}\left(3,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}
$$

$$
+\frac{a^{2} \operatorname{arctanh}(a x)^{3} \ln \left(1-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{3 a^{2} \operatorname{arctanh}(a x)^{2} \operatorname{polylog}\left(2, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}-\frac{6 a^{2} \operatorname{arctanh}(a x) \operatorname{polylog}\left(3, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}
$$

$$
\begin{aligned}
& -\frac{a^{2} \operatorname{arctanh}(a x)^{4}}{2 c}+\frac{6 a^{2} \text { polylog }\left(4, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{6 a^{2} \operatorname{polylog}\left(4,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{a \operatorname{arctanh}(a x)^{3}}{c x}-\frac{3 a \operatorname{arctanh}(a x)^{2}}{2 c x}-\frac{\operatorname{arctanh}(a x)^{3}}{2 c x^{2}} \\
& -\frac{3 a^{2} \operatorname{arctanh}(a x)^{2}}{2 c}+\frac{3 a^{2} \operatorname{polylog}\left(2, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{3 a^{2} \operatorname{polylog}\left(2,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{3 a^{2} \operatorname{arctanh}(a x)^{3}}{2 c} \\
& +\frac{3 a^{2} \operatorname{arctanh}(a x) \ln \left(1-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}+\frac{3 a^{2} \operatorname{arctanh}(a x) \ln \left(1+\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{c}
\end{aligned}
$$

Problem 45: Result more than twice size of optimal antiderivative.

$$
\int \frac{x(a+b \operatorname{arctanh}(c x))^{2}}{e x+d} \mathrm{~d} x
$$

Optimal(type 4, 275 leaves, 8 steps):
$\frac{(a+b \operatorname{arctanh}(c x))^{2}}{e c}+\frac{x(a+b \operatorname{arctanh}(c x))^{2}}{e}-\frac{2 b(a+b \operatorname{arctanh}(c x)) \ln \left(\frac{2}{-c x+1}\right)}{e c}+\frac{d(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{c x+1}\right)}{e^{2}}$

$$
\begin{aligned}
& -\frac{d(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{e^{2}}-\frac{b^{2} \operatorname{polylog}\left(2,1-\frac{2}{-c x+1}\right)}{e c}-\frac{b d(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{c x+1}\right)}{e^{2}} \\
& +\frac{b d(a+b \operatorname{arctanh}(c x)) \operatorname{poly} \log \left(2,1-\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{e^{2}}-\frac{b^{2} d \operatorname{poly} \log \left(3,1-\frac{2}{c x+1}\right)}{2 e^{2}}+\frac{b^{2} d \operatorname{poly} \log \left(3,1-\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{2 e^{2}}
\end{aligned}
$$

Result(type ?, 13911 leaves): Display of huge result suppressed!
Problem 46: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))^{2}}{e x+d} \mathrm{~d} x
$$

Optimal(type 4, 184 leaves, 1 step):
$-\frac{(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2}{c x+1}\right)}{e}+\frac{(a+b \operatorname{arctanh}(c x))^{2} \ln \left(\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{e}+\frac{b(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2}{c x+1}\right)}{e}$
$-\frac{b(a+b \operatorname{arctanh}(c x)) \operatorname{polylog}\left(2,1-\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{e}+\frac{b^{2} \operatorname{polylog}\left(3,1-\frac{2}{c x+1}\right)}{2 e}-\frac{b^{2} \operatorname{polylog}\left(3,1-\frac{2 c(e x+d)}{(d c+e)(c x+1)}\right)}{2 e}$
Result(type 4, 1169 leaves):
$\frac{a^{2} \ln (c e x+d c)}{e}+\frac{b^{2} \ln (c e x+d c) \operatorname{arctanh}(c x)^{2}}{e}-\frac{b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right) e+d c\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)\right)}{e}$

$$
\begin{aligned}
& -\frac{b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{e}+\frac{b^{2} \operatorname{polylog}\left(3,-\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)}{2 e} \\
& +\frac{\mathrm{I} b^{2} \operatorname{arctanh}(c x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right) e+d c\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{3}}{2 e} \\
& -\frac{\mathrm{I} b^{2} \operatorname{arctanh}(c x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right) e+d c\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{csgn}\left(\mathrm{I}\left(\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right) e+d c\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)\right)\right)}{2 e} \\
& +\frac{1}{2 e}\left(\mathrm { I } b ^ { 2 } \operatorname { c s g n } ( \frac { \mathrm { I } } { 1 + \frac { ( c x + 1 ) ^ { 2 } } { - c ^ { 2 } x ^ { 2 } + 1 } } ) \operatorname { a r c t a n h } ( c x ) ^ { 2 } \pi \operatorname { c s g n } ( \frac { \mathrm { I } ( ( \frac { ( c x + 1 ) ^ { 2 } } { - c ^ { 2 } x ^ { 2 } + 1 } - 1 ) e + d c ( 1 + \frac { ( c x + 1 ) ^ { 2 } } { - c ^ { 2 } x ^ { 2 } + 1 } ) ) } { 1 + \frac { ( c x + 1 ) ^ { 2 } } { - c ^ { 2 } x ^ { 2 } + 1 } } ) \operatorname { c s g n } \left(\mathrm { I } \left(\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right) e+d c(1\right.\right.\right. \\
& \left.\left.+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)\right)-\frac{\mathrm{I} b^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right) \operatorname{arctanh}(c x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\left(\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}-1\right) e+d c\left(1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}\right)\right)}{1+\frac{(c x+1)^{2}}{-c^{2} x^{2}+1}}\right)^{2}}{2 e} \\
& +\frac{b^{2} \operatorname{arctanh}(c x)^{2} \ln \left(1-\frac{(d c+e)(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)(-d c+e)}\right)}{d c+e}+\frac{b^{2} \operatorname{arctanh}(c x) \operatorname{polylog}\left(2, \frac{(d c+e)(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)(-d c+e)}\right)}{d c+e} \\
& -\frac{b^{2} \text { polylog }\left(3, \frac{(d c+e)(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)(-d c+e)}\right)}{2(d c+e)}+\frac{c b^{2} d \operatorname{arctanh}(c x)^{2} \ln \left(1-\frac{(d c+e)(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)(-d c+e)}\right)}{e(d c+e)} \\
& +\frac{c b^{2} d \operatorname{arctanh}(c x) \operatorname{polylog}\left(2, \frac{(d c+e)(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)(-d c+e)}\right)}{e(d c+e)}-\frac{c b^{2} d \operatorname{polylog}\left(3, \frac{(d c+e)(c x+1)^{2}}{\left(-c^{2} x^{2}+1\right)(-d c+e)}\right)}{2 e(d c+e)}+\frac{2 a b \ln (c e x+d c) \operatorname{arctanh}(c x)}{e}
\end{aligned}
$$

$+\frac{a b \ln (c e x+d c) \ln \left(\frac{c e x-e}{-d c-e}\right)}{e}+\frac{a b \operatorname{dilog}\left(\frac{c e x-e}{-d c-e}\right)}{e}-\frac{a b \ln (c e x+d c) \ln \left(\frac{c e x+e}{-d c+e}\right)}{e}-\frac{a b \operatorname{dilog}\left(\frac{c e x+e}{-d c+e}\right)}{e}$

Problem 55: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(-x^{2} a^{2}+1\right)^{2} \operatorname{arctanh}(a x)^{2}}{x} \mathrm{~d} x
$$

Optimal(type 4, 170 leaves, 23 steps):

$$
\frac{x^{2} a^{2}}{12}-\frac{3 a x \operatorname{arctanh}(a x)}{2}+\frac{a^{3} x^{3} \operatorname{arctanh}(a x)}{6}+\frac{3 \operatorname{arctanh}(a x)^{2}}{4}-a^{2} x^{2} \operatorname{arctanh}(a x)^{2}+\frac{a^{4} x^{4} \operatorname{arctanh}(a x)^{2}}{4}-2 \operatorname{arctanh}(a x)^{2} \operatorname{arctanh}\left(-1+\frac{2}{-a x+1}\right)
$$

$$
-\frac{2 \ln \left(-x^{2} a^{2}+1\right)}{3}-\operatorname{arctanh}(a x) \operatorname{polylog}\left(2,1-\frac{2}{-a x+1}\right)+\operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-1+\frac{2}{-a x+1}\right)+\frac{\operatorname{poly} \log \left(3,1-\frac{2}{-a x+1}\right)}{2}
$$

$$
-\frac{\operatorname{polylog}\left(3,-1+\frac{2}{-a x+1}\right)}{2}
$$

Result(type 4, 727 leaves):
$\frac{(a x-3)(a x+1) \operatorname{arctanh}(a x)}{2}+\frac{\left(x^{2} a^{2}-4 a x+7\right)(a x+1) \operatorname{arctanh}(a x)}{6}$

$$
-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \operatorname{arctanh}(a x)^{2}}{2}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{3} \operatorname{arctanh}(a x)^{2}}{2}-2 \operatorname{polylog}(3,
$$

$\left.-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)-2$ polylog $\left(3, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)-\operatorname{arctanh}(a x)^{2} \ln \left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)+\operatorname{arctanh}(a x)^{2} \ln \left(1+\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)$
$+2 \operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+\operatorname{arctanh}(a x)^{2} \ln \left(1-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+2 \operatorname{arctanh}(a x) \operatorname{polylog}\left(2, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+\frac{x^{2} a^{2}}{12}-(a x$
$+1) \operatorname{arctanh}(a x)+\ln (a x) \operatorname{arctanh}(a x)^{2}-\operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)-\frac{1}{12}$
$-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \operatorname{arctanh}(a x)^{2}}{2}+\frac{3 \operatorname{arctanh}(a x)^{2}}{4}+\frac{4 \ln \left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{3}+\frac{\operatorname{polylog}\left(3,-\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{2}$
$-a^{2} x^{2} \operatorname{arctanh}(a x)^{2}+\frac{a^{4} x^{4} \operatorname{arctanh}(a x)^{2}}{4}+\frac{I \pi \operatorname{csgn}\left(I\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{\left.1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{arctanh}(a x)^{2}}\right.}{2}$

Problem 56: Result more than twice size of optimal antiderivative.

$$
\int\left(-x^{2} a^{2}+1\right)^{2} \operatorname{arctanh}(a x)^{3} \mathrm{~d} x
$$

Optimal(type 4, 223 leaves, 12 steps):

$$
\begin{aligned}
& \frac{x^{2} a^{2}-1}{20 a}-x \operatorname{arctanh}(a x)-\frac{x\left(-x^{2} a^{2}+1\right) \operatorname{arctanh}(a x)}{10}+\frac{2\left(-x^{2} a^{2}+1\right) \operatorname{arctanh}(a x)^{2}}{5 a}+\frac{3\left(-x^{2} a^{2}+1\right)^{2} \operatorname{arctanh}(a x)^{2}}{20 a}+\frac{8 \operatorname{arctanh}(a x)^{3}}{15 a} \\
& +\frac{8 x \operatorname{arctanh}(a x)^{3}}{15}+\frac{4 x\left(-x^{2} a^{2}+1\right) \operatorname{arctanh}(a x)^{3}}{15}+\frac{x\left(-x^{2} a^{2}+1\right)^{2} \operatorname{arctanh}(a x)^{3}}{5}-\frac{8 \operatorname{arctanh}(a x)^{2} \ln \left(\frac{2}{-a x+1}\right)}{5 a}-\frac{\ln \left(-x^{2} a^{2}+1\right)}{2 a} \\
& -\frac{8 \operatorname{arctanh}(a x) \operatorname{polylog}\left(2,1-\frac{2}{-a x+1}\right)}{5 a}+\frac{4 \operatorname{polylog}\left(3,1-\frac{2}{-a x+1}\right)}{5 a}
\end{aligned}
$$

Result(type 4, 891 leaves):

$$
\begin{aligned}
& -\frac{1}{20 a}+\frac{2 \operatorname{Iarctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{3}}{5 a}+\frac{4 \mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \operatorname{arctanh}(a x)^{2}}{5 a}-\frac{7 a x^{2} \operatorname{arctanh}(a x)^{2}}{10} \\
& -\frac{2 a^{2} x^{3} \operatorname{arctanh}(a x)^{3}}{3}+\frac{3 a^{3} x^{4} \operatorname{arctanh}(a x)^{2}}{20}+\frac{a^{4} \operatorname{arctanh}(a x)^{3} x^{5}}{5}+\frac{a^{2} x^{3} \operatorname{arctanh}(a x)}{10}+\frac{\ln \left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{a}+\frac{4 \operatorname{polylog}\left(3,-\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{5 a} \\
& -\frac{4 \mathrm{I} \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)}{5 a}+\frac{2 \mathrm{I} \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)^{2}}{5 a} \\
& -\frac{2 \operatorname{Icsgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2}}{5 a} \\
& -\frac{2 \mathrm{I} \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2}}{5 a}+\frac{2 \operatorname{Iarctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{3}}{5 a}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{4 \mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{3} \operatorname{arctanh}(a x)^{2}}{5 a} \\
& +\frac{2 \mathrm{I} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)}{5 a}-\frac{\operatorname{arctanh}(a x)}{a}-\frac{4 \mathrm{I} \pi \operatorname{arctanh}(a x)^{2}}{5 a} \\
& - \\
& -\frac{8 \ln (2) \operatorname{arctanh}(a x)^{2}}{5 a}-\frac{8 \operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{5 a}+\frac{4 \operatorname{arctanh}(a x)^{2} \ln (a x+1)}{5 a}+\frac{4 \operatorname{arctanh}(a x)^{2} \ln (a x-1)}{5 a} \\
& -\frac{8 \operatorname{arctanh}(a x)^{2} \ln \left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{5 a}+\frac{a x^{2}}{20}-\frac{11 x \operatorname{arctanh}(a x)}{10}+\frac{11 \operatorname{arctanh}(a x)^{2}}{20 a}+\frac{8 \operatorname{arctanh}(a x)^{3}}{15 a}+x \operatorname{arctanh}(a x)^{3}
\end{aligned}
$$

Problem 60: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2} \operatorname{arctanh}(a x)}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 3, 38 leaves, 4 steps):

$$
-\frac{x \operatorname{arctanh}(a x)}{a^{2}}+\frac{\operatorname{arctanh}(a x)^{2}}{2 a^{3}}-\frac{\ln \left(-x^{2} a^{2}+1\right)}{2 a^{3}}
$$

Result(type 3, 144 leaves):
$-\frac{x \operatorname{arctanh}(a x)}{a^{2}}-\frac{\operatorname{arctanh}(a x) \ln (a x-1)}{2 a^{3}}+\frac{\operatorname{arctanh}(a x) \ln (a x+1)}{2 a^{3}}-\frac{\ln (a x-1)^{2}}{8 a^{3}}+\frac{\ln (a x-1) \ln \left(\frac{a x}{2}+\frac{1}{2}\right)}{4 a^{3}}-\frac{\ln (a x-1)}{2 a^{3}}-\frac{\ln (a x+1)}{2 a^{3}}$

$$
-\frac{\ln (a x+1)^{2}}{8 a^{3}}-\frac{\ln \left(-\frac{a x}{2}+\frac{1}{2}\right) \ln \left(\frac{a x}{2}+\frac{1}{2}\right)}{4 a^{3}}+\frac{\ln \left(-\frac{a x}{2}+\frac{1}{2}\right) \ln (a x+1)}{4 a^{3}}
$$

Problem 62: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)}{x\left(-x^{2} a^{2}+1\right)} d x
$$

Optimal(type 4, 41 leaves, 3 steps):

$$
\frac{\operatorname{arctanh}(a x)^{2}}{2}+\operatorname{arctanh}(a x) \ln \left(2-\frac{2}{a x+1}\right)-\frac{\operatorname{polylog}\left(2,-1+\frac{2}{a x+1}\right)}{2}
$$

Result(type 4, 129 leaves):
$\ln (a x) \operatorname{arctanh}(a x)-\frac{\operatorname{arctanh}(a x) \ln (a x+1)}{2}-\frac{\operatorname{arctanh}(a x) \ln (a x-1)}{2}-\frac{\ln (a x-1)^{2}}{8}+\frac{\operatorname{dilog}\left(\frac{a x}{2}+\frac{1}{2}\right)}{2}+\frac{\ln (a x-1) \ln \left(\frac{a x}{2}+\frac{1}{2}\right)}{4}$

$$
+\frac{\ln (a x+1)^{2}}{8}-\frac{\left(\ln (a x+1)-\ln \left(\frac{a x}{2}+\frac{1}{2}\right)\right) \ln \left(-\frac{a x}{2}+\frac{1}{2}\right)}{4}-\frac{\operatorname{dilog}(a x+1)}{2}-\frac{\ln (a x) \ln (a x+1)}{2}-\frac{\operatorname{dilog}(a x)}{2}
$$

Problem 63: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)}{x^{3}\left(-x^{2} a^{2}+1\right)} \mathrm{d} x
$$

Optimal(type 4, 74 leaves, 7 steps):

$$
-\frac{a}{2 x}+\frac{a^{2} \operatorname{arctanh}(a x)}{2}-\frac{\operatorname{arctanh}(a x)}{2 x^{2}}+\frac{a^{2} \operatorname{arctanh}(a x)^{2}}{2}+a^{2} \operatorname{arctanh}(a x) \ln \left(2-\frac{2}{a x+1}\right)-\frac{a^{2} \operatorname{polylog}\left(2,-1+\frac{2}{a x+1}\right)}{2}
$$

Result(type 4, 208 leaves):

$$
\begin{aligned}
& -\frac{\operatorname{arctanh}(a x)}{2 x^{2}}+a^{2} \ln (a x) \operatorname{arctanh}(a x)-\frac{a^{2} \operatorname{arctanh}(a x) \ln (a x+1)}{2}-\frac{a^{2} \operatorname{arctanh}(a x) \ln (a x-1)}{2}+\frac{a^{2} \ln (a x+1)}{4}-\frac{a^{2} \ln (a x-1)}{4}-\frac{a}{2 x} \\
& -\frac{a^{2} \ln (a x-1)^{2}}{8}+\frac{a^{2} \operatorname{dilog}\left(\frac{a x}{2}+\frac{1}{2}\right)}{2}+\frac{a^{2} \ln (a x-1) \ln \left(\frac{a x}{2}+\frac{1}{2}\right)}{4}+\frac{a^{2} \ln (a x+1)^{2}}{8}+\frac{a^{2} \ln \left(-\frac{a x}{2}+\frac{1}{2}\right) \ln \left(\frac{a x}{2}+\frac{1}{2}\right)}{4} \\
& -\frac{a^{2} \ln \left(-\frac{a x}{2}+\frac{1}{2}\right) \ln (a x+1)}{4}-\frac{a^{2} \operatorname{dilog}(a x+1)}{2}-\frac{a^{2} \ln (a x) \ln (a x+1)}{2}-\frac{a^{2} \operatorname{dilog}(a x)}{2}
\end{aligned}
$$

Problem 64: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)^{2}}{x\left(-x^{2} a^{2}+1\right)} d x
$$

Optimal(type 4, 62 leaves, 4 steps):

$$
\frac{\operatorname{arctanh}(a x)^{3}}{3}+\operatorname{arctanh}(a x)^{2} \ln \left(2-\frac{2}{a x+1}\right)-\operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-1+\frac{2}{a x+1}\right)-\frac{\operatorname{polylog}\left(3,-1+\frac{2}{a x+1}\right)}{2}
$$

Result(type 4, 1196 leaves):

$$
-\frac{I \pi \operatorname{csgn}\left(I\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \operatorname{arctanh}(a x)^{2}}{2}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)^{2}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{\operatorname{arctanh}(a x)^{2}}}{2}
$$

$-2 \operatorname{polylog}\left(3,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)-2 \operatorname{polylog}\left(3, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+\operatorname{arctanh}(a x)^{2} \ln \left(1+\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+2 \operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)$ $+\operatorname{arctanh}(a x)^{2} \ln \left(1-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+2 \operatorname{arctanh}(a x) \operatorname{polylog}\left(2, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+\ln (a x) \operatorname{arctanh}(a x)^{2}-\operatorname{arctanh}(a x)^{2} \ln \left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)$ $+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{3} \operatorname{arctanh}(a x)^{2}}{2}+\ln (2) \operatorname{arctanh}(a x)^{2}-\frac{\operatorname{arctanh}(a x)^{2} \ln (a x+1)}{2}-\frac{\operatorname{arctanh}(a x)^{2} \ln (a x-1)}{2}$ $+\operatorname{arctanh}(a x)^{2} \ln \left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+\frac{\mathrm{I} \pi \operatorname{arctanh}(a x)^{2}}{2}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2} \operatorname{arctanh}(a x)^{2}}{4}$ $+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2}}{4}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{arctanh}(a x)^{2}}{4}$ $+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{2} \operatorname{arctanh}(a x)^{2}}{2}$
$+\frac{I \pi \operatorname{csgn}\left(I\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{I}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{I\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2}}{2}-\frac{I \pi \operatorname{csgn}\left(\frac{I(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{3} \operatorname{arctanh}(a x)^{2}}{4}$ $-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{3} \operatorname{arctanh}(a x)^{2}}{4}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \operatorname{arctanh}(a x)^{2}}{2}$
$+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{3} \operatorname{arctanh}(a x)^{2}}{2}-\frac{\operatorname{arctanh}(a x)^{3}}{3}$

$$
-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2}}{4}
$$

Problem 65: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)^{2}}{x^{2}\left(-x^{2} a^{2}+1\right)} d x
$$

Optimal(type 4, 64 leaves, 6 steps):

$$
a \operatorname{arctanh}(a x)^{2}-\frac{\operatorname{arctanh}(a x)^{2}}{x}+\frac{a \operatorname{arctanh}(a x)^{3}}{3}+2 a \operatorname{arctanh}(a x) \ln \left(2-\frac{2}{a x+1}\right)-a \operatorname{polylog}\left(2,-1+\frac{2}{a x+1}\right)
$$

Result(type ?, 4502 leaves): Display of huge result suppressed!
Problem 66: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2} \operatorname{arctanh}(a x)^{3}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 4, 99 leaves, 7 steps):
$-\frac{\operatorname{arctanh}(a x)^{3}}{a^{3}}-\frac{x \operatorname{arctanh}(a x)^{3}}{a^{2}}+\frac{\operatorname{arctanh}(a x)^{4}}{4 a^{3}}+\frac{3 \operatorname{arctanh}(a x)^{2} \ln \left(\frac{2}{-a x+1}\right)}{a^{3}}+\frac{3 \operatorname{arctanh}(a x) \operatorname{polylog}\left(2,1-\frac{2}{-a x+1}\right)}{a^{3}}$

$$
-\frac{3 \text { polylog }\left(3,1-\frac{2}{-a x+1}\right)}{2 a^{3}}
$$

Result(type 4, 796 leaves):

$$
\begin{aligned}
& -\frac{x \operatorname{arctanh}(a x)^{3}}{a^{2}}-\frac{\operatorname{arctanh}(a x)^{3} \ln (a x-1)}{2 a^{3}}+\frac{\operatorname{arctanh}(a x)^{3} \ln (a x+1)}{2 a^{3}}-\frac{\operatorname{arctanh}(a x)^{3} \ln \left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{a^{3}}+\frac{\operatorname{arctanh}(a x)^{4}}{4 a^{3}}-\frac{\operatorname{arctanh}(a x)^{3}}{a^{3}} \\
& -\frac{I \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{2}}{2 a^{3}}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{3} \operatorname{arctanh}(a x)^{3}}{2 a^{3}} \\
& \left.-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2 a^{3}} \operatorname{arctanh}(a x)^{3}}{2}+\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{4 a^{3}}\right)
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{3}}{4 a^{3}}+\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{3}}{4 a^{3}} \\
& -\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)}{4 a^{3}} \\
& +\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)}{4 a^{3}} \\
& -\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{cosgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2}}{4 a^{3}}+\frac{\mathrm{I} \pi \operatorname{arctanh}(a x)^{3}}{2 a^{3}}+\frac{3 \operatorname{arctanh}(a x)^{2} \ln \left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{a^{3}} \\
& +\frac{3 \operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{a^{3}}-\frac{3 \operatorname{polylog}\left(3,-\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{2 a^{3}}
\end{aligned}
$$

Problem 67: Result more than twice size of optimal antiderivative.

$$
\int \frac{x \operatorname{arctanh}(a x)^{3}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 4, 100 leaves, 5 steps):
$-\frac{\operatorname{arctanh}(a x)^{4}}{4 a^{2}}+\frac{\operatorname{arctanh}(a x)^{3} \ln \left(\frac{2}{-a x+1}\right)}{a^{2}}+\frac{3 \operatorname{arctanh}(a x)^{2} \operatorname{polylog}\left(2,1-\frac{2}{-a x+1}\right)}{2 a^{2}}-\frac{3 \operatorname{arctanh}(a x) \operatorname{polylog}\left(3,1-\frac{2}{-a x+1}\right)}{2 a^{2}}$

$$
+\frac{3 \text { polylog }\left(4,1-\frac{2}{-a x+1}\right)}{4 a^{2}}
$$

Result(type 4, 784 leaves):
$-\frac{\operatorname{arctanh}(a x)^{3} \ln (a x-1)}{2 a^{2}}-\frac{\operatorname{arctanh}(a x)^{3} \ln (a x+1)}{2 a^{2}}+\frac{\operatorname{arctanh}(a x)^{3} \ln \left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{a^{2}}-\frac{\operatorname{arctanh}(a x)^{4}}{4 a^{2}}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\left.1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{2} \operatorname{arctanh}(a x)^{3}}\right.}{2 a^{2}}$

$$
\begin{aligned}
& +\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{3} \operatorname{arctanh}(a x)^{3}}{2 a^{2}}-\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{3}}{4 a^{2}} \\
& -\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)}{4 a^{2}}+\frac{\mathrm{I} \pi \operatorname{arctanh}(a x)^{3}}{2 a^{2}} \\
& -\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{4 a^{2}}-\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{3}}{4 a^{2}} \\
& +\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2}}{4 a^{2}} \\
& +\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)}{4 a^{2}}+\frac{\mathrm{I} \operatorname{arctanh}(a x)^{3} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{2}}{2 a^{2}} \\
& +\frac{\operatorname{arctanh}(a x)^{3} \ln (2)}{a^{2}}+\frac{3 \operatorname{arctanh}(a x)^{2} \operatorname{polylog}\left(2,-\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{2 a^{2}}-\frac{3 \operatorname{arctanh}(a x) \operatorname{polylog}\left(3,-\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{2 a^{2}}+\frac{3 \operatorname{polylog}\left(4,-\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}{4 a^{2}}
\end{aligned}
$$

Problem 68: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)^{3}}{x^{3}\left(-x^{2} a^{2}+1\right)} d x
$$

Optimal(type 4, 182 leaves, 13 steps):
$\frac{3 a^{2} \operatorname{arctanh}(a x)^{2}}{2}-\frac{3 a \operatorname{arctanh}(a x)^{2}}{2 x}+\frac{a^{2} \operatorname{arctanh}(a x)^{3}}{2}-\frac{\operatorname{arctanh}(a x)^{3}}{2 x^{2}}+\frac{a^{2} \operatorname{arctanh}(a x)^{4}}{4}+3 a^{2} \operatorname{arctanh}(a x) \ln \left(2-\frac{2}{a x+1}\right)+a^{2} \operatorname{arctanh}(a x)^{3} \ln (2$
$\left.-\frac{2}{a x+1}\right)-\frac{3 a^{2} \operatorname{polylog}\left(2,-1+\frac{2}{a x+1}\right)}{2}-\frac{3 a^{2} \operatorname{arctanh}(a x)^{2} \operatorname{polylog}\left(2,-1+\frac{2}{a x+1}\right)}{2}-\frac{3 a^{2} \operatorname{arctanh}(a x) \operatorname{polylog}\left(3,-1+\frac{2}{a x+1}\right)}{2}$

$$
-\frac{3 a^{2} \operatorname{poly} \log \left(4,-1+\frac{2}{a x+1}\right)}{4}
$$

Result(type 4, 405 leaves):

```
\(-\frac{a^{2} \operatorname{arctanh}(a x)^{4}}{4}+\frac{a^{2} \operatorname{arctanh}(a x)^{3}}{2}-\frac{3 a^{2} \operatorname{arctanh}(a x)^{2}}{2}-\frac{3 a \operatorname{arctanh}(a x)^{2}}{2 x}-\frac{\operatorname{arctanh}(a x)^{3}}{2 x^{2}}+a^{2} \operatorname{arctanh}(a x)^{3} \ln \left(1+\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)\)
\(+3 a^{2} \operatorname{arctanh}(a x)^{2} \operatorname{polylog}\left(2,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)-6 a^{2} \operatorname{arctanh}(a x) \operatorname{polylog}\left(3,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+6 a^{2} \operatorname{polylog}\left(4,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)\)
\(+a^{2} \operatorname{arctanh}(a x)^{3} \ln \left(1-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+3 a^{2} \operatorname{arctanh}(a x)^{2} \operatorname{polylog}\left(2, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)-6 a^{2} \operatorname{arctanh}(a x) \operatorname{polylog}\left(3, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)\)
\(+6 a^{2} \operatorname{polylog}\left(4, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+3 a^{2} \operatorname{arctanh}(a x) \ln \left(1-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+3 a^{2} \operatorname{polylog}\left(2, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+3 a^{2} \operatorname{arctanh}(a x) \ln \left(1+\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)\)
\(+3 a^{2} \operatorname{polylog}\left(2,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)\)
```

Problem 71: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3} \operatorname{arctanh}(a x)}{\left(-x^{2} a^{2}+1\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 99 leaves, 8 steps):

$$
-\frac{x}{4 a^{3}\left(-x^{2} a^{2}+1\right)}-\frac{\operatorname{arctanh}(a x)}{4 a^{4}}+\frac{\operatorname{arctanh}(a x)}{2 a^{4}\left(-x^{2} a^{2}+1\right)}+\frac{\operatorname{arctanh}(a x)^{2}}{2 a^{4}}-\frac{\operatorname{arctanh}(a x) \ln \left(\frac{2}{-a x+1}\right)}{a^{4}}-\frac{\operatorname{poly} \log \left(2,1-\frac{2}{-a x+1}\right)}{2 a^{4}}
$$

Result(type 4, 202 leaves):
$\frac{\operatorname{arctanh}(a x)}{4 a^{4}(a x+1)}+\frac{\operatorname{arctanh}(a x) \ln (a x+1)}{2 a^{4}}-\frac{\operatorname{arctanh}(a x)}{4 a^{4}(a x-1)}+\frac{\operatorname{arctanh}(a x) \ln (a x-1)}{2 a^{4}}+\frac{1}{8 a^{4}(a x+1)}-\frac{\ln (a x+1)}{8 a^{4}}+\frac{1}{8 a^{4}(a x-1)}+\frac{\ln (a x-1)}{8 a^{4}}$

$$
+\frac{\ln (a x-1)^{2}}{8 a^{4}}-\frac{\operatorname{dilog}\left(\frac{a x}{2}+\frac{1}{2}\right)}{2 a^{4}}-\frac{\ln (a x-1) \ln \left(\frac{a x}{2}+\frac{1}{2}\right)}{4 a^{4}}-\frac{\ln \left(-\frac{a x}{2}+\frac{1}{2}\right) \ln \left(\frac{a x}{2}+\frac{1}{2}\right)}{4 a^{4}}+\frac{\ln \left(-\frac{a x}{2}+\frac{1}{2}\right) \ln (a x+1)}{4 a^{4}}-\frac{\ln (a x+1)^{2}}{8 a^{4}}
$$

Problem 72: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)}{x^{2}\left(-x^{2} a^{2}+1\right)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 74 leaves, 10 steps):

$$
-\frac{a}{4\left(-x^{2} a^{2}+1\right)}-\frac{\operatorname{arctanh}(a x)}{x}+\frac{a^{2} x \operatorname{arctanh}(a x)}{2\left(-x^{2} a^{2}+1\right)}+\frac{3 a \operatorname{arctanh}(a x)^{2}}{4}+a \ln (x)-\frac{a \ln \left(-x^{2} a^{2}+1\right)}{2}
$$

Result(type 3, 179 leaves):
$-\frac{\operatorname{arctanh}(a x)}{x}-\frac{a \operatorname{arctanh}(a x)}{4(a x+1)}+\frac{3 a \operatorname{arctanh}(a x) \ln (a x+1)}{4}-\frac{a \operatorname{arctanh}(a x)}{4(a x-1)}-\frac{3 a \operatorname{arctanh}(a x) \ln (a x-1)}{4}+a \ln (a x)-\frac{a}{8(a x+1)}-\frac{a \ln (a x+1)}{2}$

$$
\begin{aligned}
& +\frac{a}{8(a x-1)}-\frac{a \ln (a x-1)}{2}-\frac{3 a \ln (a x-1)^{2}}{16}+\frac{3 a \ln (a x-1) \ln \left(\frac{a x}{2}+\frac{1}{2}\right)}{8}-\frac{3 a \ln \left(-\frac{a x}{2}+\frac{1}{2}\right) \ln \left(\frac{a x}{2}+\frac{1}{2}\right)}{8} \\
& +\frac{3 a \ln \left(-\frac{a x}{2}+\frac{1}{2}\right) \ln (a x+1)}{8}-\frac{3 a \ln (a x+1)^{2}}{16}
\end{aligned}
$$

Problem 73: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2} \operatorname{arctanh}(a x)^{2}}{\left(-x^{2} a^{2}+1\right)^{2}} d x
$$

Optimal(type 3, 84 leaves, 4 steps):

$$
\frac{x}{4 a^{2}\left(-x^{2} a^{2}+1\right)}+\frac{\operatorname{arctanh}(a x)}{4 a^{3}}-\frac{\operatorname{arctanh}(a x)}{2 a^{3}\left(-x^{2} a^{2}+1\right)}+\frac{x \operatorname{arctanh}(a x)^{2}}{2 a^{2}\left(-x^{2} a^{2}+1\right)}-\frac{\operatorname{arctanh}(a x)^{3}}{6 a^{3}}
$$

Result(type 3, 1739 leaves):

$$
+\frac{\mathrm{I} \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right) x^{2}}{4 a(a x-1)(a x+1)}+\frac{\mathrm{I} \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left.\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)\right)^{2} x^{2}}\right.}{8 a(a x-1)(a x+1)}
$$

$$
-\underline{\operatorname{Iarctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)^{2} x^{2} . . . ~}
$$

$$
8 a(a x-1)(a x+1)
$$

$$
\begin{aligned}
& -\frac{\mathrm{I} \operatorname{arctanh}(a x)^{2} \pi x^{2}}{4 a(a x-1)(a x+1)}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{3} \operatorname{arctanh}(a x)^{2}}{4 a^{3}(a x-1)(a x+1)}+\frac{\mathrm{I} \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{3}}{8 a^{3}(a x-1)(a x+1)} \\
& +\frac{\mathrm{I} \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{3}}{8 a^{3}(a x-1)(a x+1)}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \operatorname{arctanh}(a x)^{2}}{4 a^{3}(a x-1)(a x+1)}-\frac{\operatorname{arctanh}(a x)^{2} \ln (a x+1)}{4 a^{3}} \\
& +\frac{\operatorname{arctanh}(a x)^{2} \ln (a x-1)}{4 a^{3}}+\frac{\operatorname{arctanh}(a x)^{2} \ln \left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)}{2 a^{3}}-\frac{\operatorname{arctanh}(a x)^{2}}{4 a^{3}(a x+1)}-\frac{\operatorname{arctanh}(a x)^{2}}{4 a^{3}(a x-1)} \\
& +\frac{\mathrm{I} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2} x^{2}}{} \\
& +\frac{8 a(a x-1)(a x+1)}{8}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{\mathrm{I} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)}{8 a^{3}(a x-1)(a x+1)}+\frac{\mathrm{I} \pi \operatorname{arctanh}(a x)^{2}}{4 a^{3}(a x-1)(a x+1)} \\
& -\frac{\operatorname{arctanh}(a x)^{3} x^{2}}{6 a(a x-1)(a x+1)}-\frac{\mathrm{I} \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{3} x^{2}}{8 a(a x-1)(a x+1)}+\frac{\mathrm{I} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \operatorname{arctanh}(a x)^{2} \pi x^{2}}{4 a(a x-1)(a x+1)}
\end{aligned}
$$

$$
\begin{aligned}
& 8 a^{3}(a x-1)(a x+1) \\
& \mathrm{I} \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2} \\
& 8 a^{3}(a x-1)(a x+1) \\
& 4 a^{3}(a x-1)(a x+1)
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{\operatorname{arctanh}(a x)}{4 a^{3}(a x-1)(a x+1)}+\frac{\operatorname{arctanh}(a x) x^{2}}{4 a(a x-1)(a x+1)} \\
& -\frac{\mathrm{I} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right) x^{2}}{8 a(a x-1)(a x+1)} \\
& 8 a(a x-1)(a x+1)
\end{aligned}
$$

Problem 74: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)^{3}}{x^{2}\left(-x^{2} a^{2}+1\right)^{2}} d x
$$

Optimal(type 4, 177 leaves, 12 steps):

$$
\begin{aligned}
& -\frac{3 a}{8\left(-x^{2} a^{2}+1\right)}+\frac{3 a^{2} x \operatorname{arctanh}(a x)}{4\left(-x^{2} a^{2}+1\right)}+\frac{3 a \operatorname{arctanh}(a x)^{2}}{8}-\frac{3 a \operatorname{arctanh}(a x)^{2}}{4\left(-x^{2} a^{2}+1\right)}+a \operatorname{arctanh}(a x)^{3}-\frac{\operatorname{arctanh}(a x)^{3}}{x}+\frac{a^{2} x \operatorname{arctanh}(a x)^{3}}{2\left(-x^{2} a^{2}+1\right)}+\frac{3 a \operatorname{arctanh}(a x)^{4}}{8} \\
& \quad+3 a \operatorname{arctanh}(a x)^{2} \ln \left(2-\frac{2}{a x+1}\right)-3 a \operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-1+\frac{2}{a x+1}\right)-\frac{3 a \operatorname{polylog}\left(3,-1+\frac{2}{a x+1}\right)}{2}
\end{aligned}
$$

Result(type 4, 441 leaves):
$-\frac{3 a}{32(a x+1)}+\frac{3 a}{32(a x-1)}-\frac{\operatorname{arctanh}(a x)^{3} a^{2} x}{8(a x-1)}+\frac{3 a^{2} x \operatorname{arctanh}(a x)^{2}}{16(a x-1)}-\frac{3 \operatorname{arctanh}(a x) a^{2} x}{16(a x-1)}+\frac{\operatorname{arctanh}(a x)^{3} a^{2} x}{8(a x+1)}+\frac{3 a^{2} x \operatorname{arctanh}(a x)^{2}}{16(a x+1)}$

$$
\begin{aligned}
& +\frac{3 \operatorname{arctanh}(a x) a^{2} x}{16(a x+1)}-6 a \operatorname{polylog}\left(3, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)-6 a \operatorname{polylog}\left(3,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)-\frac{3 a \operatorname{arctanh}(a x)}{16(a x+1)}-\frac{3 a \operatorname{arctanh}(a x)}{16(a x-1)}+\frac{3 a^{2} x}{32(a x-1)} \\
& +\frac{3 a^{2} x}{32(a x+1)}+3 a \operatorname{arctanh}(a x)^{2} \ln \left(1-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+3 a \operatorname{arctanh}(a x)^{2} \ln \left(1+\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+6 a \operatorname{arctanh}(a x) \operatorname{polylog}\left(2, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)
\end{aligned}
$$

$$
+6 a \operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)-\frac{a \operatorname{arctanh}(a x)^{3}}{8(a x-1)}-\frac{a \operatorname{arctanh}(a x)^{3}}{8(a x+1)}-\frac{3 a \operatorname{arctanh}(a x)^{2}}{16(a x+1)}+\frac{3 a \operatorname{arctanh}(a x)^{2}}{16(a x-1)}-a \operatorname{arctanh}(a x)^{3}
$$

$$
+\frac{3 a \operatorname{arctanh}(a x)^{4}}{8}-\frac{\operatorname{arctanh}(a x)^{3}}{x}
$$

Problem 82: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2} \operatorname{arctanh}(a x)^{2}}{\left(-x^{2} a^{2}+1\right)^{3}} \mathrm{~d} x
$$

Optimal(type 3, 147 leaves, 13 steps):
$\frac{x}{32 a^{2}\left(-x^{2} a^{2}+1\right)^{2}}-\frac{x}{64 a^{2}\left(-x^{2} a^{2}+1\right)}-\frac{\operatorname{arctanh}(a x)}{64 a^{3}}-\frac{\operatorname{arctanh}(a x)}{8 a^{3}\left(-x^{2} a^{2}+1\right)^{2}}+\frac{\operatorname{arctanh}(a x)}{8 a^{3}\left(-x^{2} a^{2}+1\right)}+\frac{x \operatorname{arctanh}(a x)^{2}}{4 a^{2}\left(-x^{2} a^{2}+1\right)^{2}}-\frac{x \operatorname{arctanh}(a x)^{2}}{8 a^{2}\left(-x^{2} a^{2}+1\right)}$

$$
-\frac{\operatorname{arctanh}(a x)^{3}}{24 a^{3}}
$$

Result (type ?, 2597 leaves): Display of huge result suppressed!
Problem 83: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)^{2}}{x\left(-x^{2} a^{2}+1\right)^{3}} \mathrm{~d} x
$$

Optimal(type 4, 178 leaves, 13 steps):
$\frac{1}{32\left(-x^{2} a^{2}+1\right)^{2}}+\frac{11}{32\left(-x^{2} a^{2}+1\right)}-\frac{a x \operatorname{arctanh}(a x)}{8\left(-x^{2} a^{2}+1\right)^{2}}-\frac{11 a x \operatorname{arctanh}(a x)}{16\left(-x^{2} a^{2}+1\right)}-\frac{11 \operatorname{arctanh}(a x)^{2}}{32}+\frac{\operatorname{arctanh}(a x)^{2}}{4\left(-x^{2} a^{2}+1\right)^{2}}+\frac{\operatorname{arctanh}(a x)^{2}}{2\left(-x^{2} a^{2}+1\right)}+\frac{\operatorname{arctanh}(a x)^{3}}{3}$

$$
+\operatorname{arctanh}(a x)^{2} \ln \left(2-\frac{2}{a x+1}\right)-\operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-1+\frac{2}{a x+1}\right)-\frac{\operatorname{polylog}\left(3,-1+\frac{2}{a x+1}\right)}{2}
$$

Result(type 4, 1400 leaves):

$$
\begin{aligned}
& -\frac{I \pi \operatorname{csgn}\left(I\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \operatorname{arctanh}(a x)^{2}}{2}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{2} \operatorname{arctanh}(a x)^{2}}{2} \\
& -2 \text { polylog }\left(3,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)-2 \text { polylog }\left(3, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+\operatorname{arctanh}(a x)^{2} \ln \left(1+\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+2 \operatorname{arctanh}(a x) \operatorname{polylog}\left(2,-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right) \\
& +\operatorname{arctanh}(a x)^{2} \ln \left(1-\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+2 \operatorname{arctanh}(a x) \operatorname{polylog}\left(2, \frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+\ln (a x) \operatorname{arctanh}(a x)^{2}-\operatorname{arctanh}(a x)^{2} \ln \left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right) \\
& +\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{3} \operatorname{arctanh}(a x)^{2}}{2} \\
& +\frac{\operatorname{arctanh}(a x)^{2}}{16(a x-1)^{2}}+\ln (2) \operatorname{arctanh}(a x)^{2}-\frac{\operatorname{arctanh}(a x)^{2} \ln (a x+1)}{2}-\frac{\operatorname{arctanh}(a x)^{2} \ln (a x-1)}{2} \\
& +\operatorname{arctanh}(a x)^{2} \ln \left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)+\frac{\mathrm{I} \pi \operatorname{arctanh}(a x)^{2}}{2}-\frac{5 \operatorname{arctanh}(a x)^{2}}{16(a x-1)} \\
& +\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2} \operatorname{arctanh}(a x)^{2}}{4} \\
& +\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2}}{4}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{arctanh}(a x)^{2}}{4} \\
& +\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{2} \operatorname{arctanh}(a x)^{2}}{2}+\frac{\operatorname{arctanh}(a x)^{2}}{16(a x+1)^{2}} \\
& +\frac{I \pi \operatorname{csgn}\left(I\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}-1\right)}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2}}{2}+\frac{5 \operatorname{arctanh}(a x)^{2}}{16(a x+1)}
\end{aligned}
$$

$$
-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{3} \operatorname{arctanh}(a x)^{2}}{4}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right)^{3} \operatorname{arctanh}(a x)^{2}}{4}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\left.1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)^{2} \operatorname{arctanh}(a x)^{2}}\right.}{2}
$$

$$
+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right)^{3} \operatorname{arctanh}(a x)^{2}}{2}+\frac{(a x+1)^{2}}{512(a x-1)^{2}}-\frac{3(a x+1)}{32(a x-1)}-\frac{3(a x-1)}{32(a x+1)}+\frac{(a x-1)^{2}}{512(a x+1)^{2}}-\frac{3 \operatorname{arctanh}(a x)(a x-1)}{16(a x+1)}
$$

$$
+\frac{\operatorname{arctanh}(a x)(a x-1)^{2}}{128(a x+1)^{2}}+\frac{3(a x+1) \operatorname{arctanh}(a x)}{16(a x-1)}-\frac{\operatorname{arctanh}(a x)(a x+1)^{2}}{128(a x-1)^{2}}-\frac{11 \operatorname{arctanh}(a x)^{2}}{32}-\frac{\operatorname{arctanh}(a x)^{3}}{3}
$$

$$
-\underline{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{-x^{2} a^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}\right)}\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(a x+1)^{2}}{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2}}
$$

Problem 84: Result more than twice size of optimal antiderivative.

$$
\int \frac{x \operatorname{arctanh}(a x)^{3}}{\left(-x^{2} a^{2}+1\right)^{3}} \mathrm{~d} x
$$

Optimal(type 3, 170 leaves, 9 steps):

$$
\begin{aligned}
& -\frac{3 x}{128 a\left(-x^{2} a^{2}+1\right)^{2}}-\frac{45 x}{256 a\left(-x^{2} a^{2}+1\right)}-\frac{45 \operatorname{arctanh}(a x)}{256 a^{2}}+\frac{3 \operatorname{arctanh}(a x)}{32 a^{2}\left(-x^{2} a^{2}+1\right)^{2}}+\frac{9 \operatorname{arctanh}(a x)}{32 a^{2}\left(-x^{2} a^{2}+1\right)}-\frac{3 x \operatorname{arctanh}(a x)^{2}}{16 a\left(-x^{2} a^{2}+1\right)^{2}}-\frac{9 x \operatorname{arctanh}(a x)^{2}}{32 a\left(-x^{2} a^{2}+1\right)} \\
& -\frac{3 \operatorname{arctanh}(a x)^{3}}{32 a^{2}}+\frac{\operatorname{arctanh}(a x)^{3}}{4 a^{2}\left(-x^{2} a^{2}+1\right)^{2}}
\end{aligned}
$$

Result(type ?, 2608 leaves): Display of huge result suppressed!
Problem 93: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)}{\left(-x^{2} a^{2}+1\right)^{4}} d x
$$

Optimal(type 3, 120 leaves, 4 steps):

$$
-\frac{1}{36 a\left(-x^{2} a^{2}+1\right)^{3}}-\frac{5}{96 a\left(-x^{2} a^{2}+1\right)^{2}}-\frac{5}{32 a\left(-x^{2} a^{2}+1\right)}+\frac{x \operatorname{arctanh}(a x)}{6\left(-x^{2} a^{2}+1\right)^{3}}+\frac{5 x \operatorname{arctanh}(a x)}{24\left(-x^{2} a^{2}+1\right)^{2}}+\frac{5 x \operatorname{arctanh}(a x)}{16\left(-x^{2} a^{2}+1\right)}+\frac{5 \operatorname{arctanh}(a x)^{2}}{32 a}
$$

Result(type 3, 280 leaves):
$-\frac{\operatorname{arctanh}(a x)}{48 a(a x+1)^{3}}-\frac{\operatorname{arctanh}(a x)}{16 a(a x+1)^{2}}-\frac{5 \operatorname{arctanh}(a x)}{32 a(a x+1)}+\frac{5 \operatorname{arctanh}(a x) \ln (a x+1)}{32 a}-\frac{\operatorname{arctanh}(a x)}{48 a(a x-1)^{3}}+\frac{\operatorname{arctanh}(a x)}{16 a(a x-1)^{2}}-\frac{5 \operatorname{arctanh}(a x)}{32 a(a x-1)}$

$$
\begin{aligned}
& -\frac{5 \operatorname{arctanh}(a x) \ln (a x-1)}{32 a}-\frac{5 \ln \left(-\frac{a x}{2}+\frac{1}{2}\right) \ln \left(\frac{a x}{2}+\frac{1}{2}\right)}{64 a}+\frac{5 \ln \left(-\frac{a x}{2}+\frac{1}{2}\right) \ln (a x+1)}{64 a}+\frac{5 \ln (a x-1) \ln \left(\frac{a x}{2}+\frac{1}{2}\right)}{64 a}-\frac{5 \ln (a x+1)^{2}}{128 a} \\
& -\frac{5 \ln (a x-1)^{2}}{128 a}-\frac{37}{384 a(a x+1)}+\frac{37}{384 a(a x-1)}+\frac{1}{288 a(a x-1)^{3}}-\frac{7}{384 a(a x+1)^{2}}-\frac{1}{288 a(a x+1)^{3}}-\frac{7}{384 a(a x-1)^{2}}
\end{aligned}
$$

Problem 94: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)^{3}}{\left(-x^{2} a^{2}+1\right)^{4}} d x
$$

Optimal(type 3, 263 leaves, 13 steps):

$$
\begin{array}{r}
-\frac{1}{216 a\left(-x^{2} a^{2}+1\right)^{3}}-\frac{65}{2304 a\left(-x^{2} a^{2}+1\right)^{2}}-\frac{245}{768 a\left(-x^{2} a^{2}+1\right)}+\frac{x \operatorname{arctanh}(a x)}{36\left(-x^{2} a^{2}+1\right)^{3}}+\frac{65 x \operatorname{arctanh}(a x)}{576\left(-x^{2} a^{2}+1\right)^{2}}+\frac{245 x \operatorname{arctanh}(a x)}{384\left(-x^{2} a^{2}+1\right)}+\frac{245 \operatorname{arctanh}(a x)^{2}}{768 a} \\
-\frac{\operatorname{arctanh}(a x)^{2}}{12 a\left(-x^{2} a^{2}+1\right)^{3}}-\frac{5 \operatorname{arctanh}(a x)^{2}}{32 a\left(-x^{2} a^{2}+1\right)^{2}}-\frac{15 \operatorname{arctanh}(a x)^{2}}{32 a\left(-x^{2} a^{2}+1\right)}+\frac{x \operatorname{arctanh}(a x)^{3}}{6\left(-x^{2} a^{2}+1\right)^{3}}+\frac{5 x \operatorname{arctanh}(a x)^{3}}{24\left(-x^{2} a^{2}+1\right)^{2}}+\frac{5 x \operatorname{arctanh}(a x)^{3}}{16\left(-x^{2} a^{2}+1\right)}+\frac{5 \operatorname{arctanh}(a x)^{4}}{64 a}
\end{array}
$$

Result(type ?, 3585 leaves): Display of huge result suppressed!
Problem 95: Unable to integrate problem.

$$
\int \frac{\sqrt{\operatorname{arctanh}(a x)}}{\left(-x^{2} a^{2}+1\right)^{4}} d x
$$

Optimal(type 4, 188 leaves, 21 steps):
$\frac{5 \operatorname{arctanh}(a x)^{3 / 2}}{24 a}+\frac{\operatorname{erf}(\sqrt{6} \sqrt{\operatorname{arctanh}(a x)}) \sqrt{6} \sqrt{\pi}}{4608 a}-\frac{\operatorname{erfi}(\sqrt{6} \sqrt{\operatorname{arctanh}(a x)}) \sqrt{6} \sqrt{\pi}}{4608 a}+\frac{15 \operatorname{erf}(\sqrt{2} \sqrt{\operatorname{arctanh}(a x)}) \sqrt{2} \sqrt{\pi}}{512 a}$
$-\frac{15 \operatorname{erfi}(\sqrt{2} \sqrt{\operatorname{arctanh}(a x)}) \sqrt{2} \sqrt{\pi}}{512 a}+\frac{3 \operatorname{erf}(2 \sqrt{\operatorname{arctanh}(a x)}) \sqrt{\pi}}{512 a}-\frac{3 \operatorname{erfi}(2 \sqrt{\operatorname{arctanh}(a x)}) \sqrt{\pi}}{512 a}+\frac{15 \sinh (2 \operatorname{arctanh}(a x)) \sqrt{\operatorname{arctanh}(a x)}}{64 a}$
$+\frac{3 \sinh (4 \operatorname{arctanh}(a x)) \sqrt{\operatorname{arctanh}(a x)}}{64 a}+\frac{\sinh (6 \operatorname{arctanh}(a x)) \sqrt{\operatorname{arctanh}(a x)}}{192 a}$
Result(type 8, 21 leaves):

$$
\int \frac{\sqrt{\operatorname{arctanh}(a x)}}{\left(-x^{2} a^{2}+1\right)^{4}} d x
$$

Problem 100: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(a x)}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Optimal(type 4, 79 leaves, 1 step):
$-\frac{2 \arctan \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \operatorname{arctanh}(a x)}{a}-\frac{\mathrm{I} \text { polylog }\left(2, \frac{-\mathrm{I} \sqrt{-a x+1}}{\sqrt{a x+1}}\right)}{a}+\frac{\mathrm{Ipolylog}\left(2, \frac{\mathrm{I} \sqrt{-a x+1}}{\sqrt{a x+1}}\right)}{a}$

Result(type 4, 365 leaves):
$\frac{\mathrm{I} \operatorname{arctanh}(a x) \ln \left(-\frac{\mathrm{I}}{\sqrt{-x^{2} a^{2}+1}}-\frac{\mathrm{I} a x}{\sqrt{-x^{2} a^{2}+1}}\right)}{2 a}-\frac{\mathrm{I} \ln \left((1-\mathrm{I}) \cosh \left(\frac{\operatorname{arctanh}(a x)}{2}\right)+(1+\mathrm{I}) \sinh \left(\frac{\operatorname{arctanh}(a x)}{2}\right)\right) \operatorname{arctanh}(a x)}{a}$


Problem 102: Unable to integrate problem.

$$
\int \frac{x^{3} \operatorname{arctanh}(a x)^{3}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Optimal(type 4, 253 leaves, 21 steps):
$\frac{\arcsin (a x)}{a^{4}}+\frac{5 \arctan \left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2}}{a^{4}}-\frac{5 \mathrm{I} \operatorname{arctanh}(a x) \operatorname{polylog}\left(2, \frac{-\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)}{a^{4}}+\frac{5 \mathrm{I} \operatorname{arctanh}(a x) \operatorname{polylog}\left(2, \frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)}{a^{4}}$

$$
+\frac{5 \mathrm{I} \text { polylog }\left(3, \frac{-\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)}{a^{4}}-\frac{5 \mathrm{I} \text { polylog }\left(3, \frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)}{a^{4}}-\frac{\operatorname{arctanh}(a x) \sqrt{-x^{2} a^{2}+1}}{a^{4}}-\frac{x \operatorname{arctanh}(a x)^{2} \sqrt{-x^{2} a^{2}+1}}{2 a^{3}}
$$

$$
-\frac{2 \operatorname{arctanh}(a x)^{3} \sqrt{-x^{2} a^{2}+1}}{3 a^{4}}-\frac{x^{2} \operatorname{arctanh}(a x)^{3} \sqrt{-x^{2} a^{2}+1}}{3 a^{2}}
$$

Result(type 8, 24 leaves):

$$
\int \frac{x^{3} \operatorname{arctanh}(a x)^{3}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Problem 103: Unable to integrate problem.

$$
\int \frac{x \operatorname{arctanh}(a x)^{3}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Optimal(type 4, 174 leaves, 9 steps):


$$
+\frac{6 \mathrm{I} \text { polylog }\left(3, \frac{-\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)}{a^{2}}-\frac{6 \mathrm{I} \text { polylog }\left(3, \frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)}{a^{2}}-\frac{\operatorname{arctanh}(a x)^{3} \sqrt{-x^{2} a^{2}+1}}{a^{2}}
$$

Result(type 8, 22 leaves):

$$
\int \frac{x \operatorname{arctanh}(a x)^{3}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Problem 104: Unable to integrate problem.

$$
\int \frac{\operatorname{arctanh}(a x)^{3}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Optimal(type 4, 219 leaves, 10 steps):


Result(type 8, 21 leaves):

$$
\int \frac{\operatorname{arctanh}(a x)^{3}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Problem 105: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3} \operatorname{arctanh}(a x)}{\left(-x^{2} a^{2}+1\right)^{3 / 2}} d x
$$

Optimal (type 3, 68 leaves, 5 steps):

$$
-\frac{\arcsin (a x)}{a^{4}}-\frac{x}{a^{3} \sqrt{-x^{2} a^{2}+1}}+\frac{\operatorname{arctanh}(a x)}{a^{4} \sqrt{-x^{2} a^{2}+1}}+\frac{\operatorname{arctanh}(a x) \sqrt{-x^{2} a^{2}+1}}{a^{4}}
$$

Result(type 3, 143 leaves):

$$
\begin{aligned}
& -\frac{(\operatorname{arctanh}(a x)-1) \sqrt{-(a x-1)(a x+1)}}{2 a^{4}(a x-1)}+\frac{(\operatorname{arctanh}(a x)+1) \sqrt{-(a x-1)(a x+1)}}{2 a^{4}(a x+1)}+\frac{\operatorname{arctanh}(a x) \sqrt{-(a x-1)(a x+1)}}{a^{4}} \\
& \quad+\frac{\mathrm{I} \ln \left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}-\mathrm{I}\right)}{a^{4}}-\frac{\mathrm{I} \ln \left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}+\mathrm{I}\right)}{a^{4}}
\end{aligned}
$$

Problem 120: Unable to integrate problem.

$$
\int \frac{\operatorname{arctanh}(a x)^{2} \sqrt{-x^{2} a^{2}+1}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 4, 231 leaves, 11 steps):
$-2 a \arctan \left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2}-4 a \operatorname{arctanh}(a x) \operatorname{arctanh}\left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)+2 \mathrm{I} a \operatorname{arctanh}(a x) \operatorname{polylog}\left(2, \frac{-\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)$

$$
-2 \mathrm{I} a \operatorname{arctanh}(a x) \operatorname{polylog}\left(2, \frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)+2 a \operatorname{polylog}\left(2,-\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)-2 a \operatorname{polylog}\left(2, \frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)-2 \mathrm{I} a \operatorname{polylog}\left(3, \frac{-\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)
$$

$$
+2 \mathrm{I} a \text { polylog}\left(3, \frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)-\frac{\operatorname{arctanh}(a x)^{2} \sqrt{-x^{2} a^{2}+1}}{x}
$$

Result(type 8, 24 leaves):

$$
\int \frac{\operatorname{arctanh}(a x)^{2} \sqrt{-x^{2} a^{2}+1}}{x^{2}} \mathrm{~d} x
$$

Problem 129: Unable to integrate problem.

$$
\int \sqrt{-x^{2} a^{2}+1} \operatorname{arctanh}(a x)^{2} \mathrm{~d} x
$$

Optimal(type 4, 200 leaves, 10 steps):
$-\frac{\arcsin (a x)}{a}+\frac{\arctan \left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right) \operatorname{arctanh}(a x)^{2}}{a}-\frac{\mathrm{I} \operatorname{arctanh}(a x) \operatorname{polylog}\left(2, \frac{-\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)}{a}+\frac{\mathrm{I} \operatorname{arctanh}(a x) \operatorname{polylog}\left(2, \frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)}{a}$

$$
+\frac{\mathrm{I} \text { polylog }\left(3, \frac{-\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)}{a}-\frac{\mathrm{I} \text { polylog }\left(3, \frac{\mathrm{I}(a x+1)}{\sqrt{-x^{2} a^{2}+1}}\right)}{a}+\frac{\operatorname{arctanh}(a x) \sqrt{-x^{2} a^{2}+1}}{a}+\frac{x \operatorname{arctanh}(a x)^{2} \sqrt{-x^{2} a^{2}+1}}{2}
$$

Result(type 8, 21 leaves):

$$
\int \sqrt{-x^{2} a^{2}+1} \operatorname{arctanh}(a x)^{2} \mathrm{~d} x
$$

Problem 136: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\left(-x^{2} a^{2}+1\right)^{5 / 2} \operatorname{arctanh}(a x)^{3}} \mathrm{~d} x
$$

Optimal(type 4, 67 leaves, 12 steps):

$$
-\frac{1}{2 a\left(-x^{2} a^{2}+1\right)^{3 / 2} \operatorname{arctanh}(a x)^{2}}-\frac{3 x}{2\left(-x^{2} a^{2}+1\right)^{3 / 2} \operatorname{arctanh}(a x)}+\frac{3 \operatorname{Chi}(\operatorname{arctanh}(a x))}{8 a}+\frac{9 \operatorname{Chi}(3 \operatorname{arctanh}(a x))}{8 a}
$$

Result(type 4, 179 leaves):
$\frac{1}{8 a\left(x^{2} a^{2}-1\right) \operatorname{arctanh}(a x)^{2}}\left(3 \operatorname{arctanh}(a x)^{2} \operatorname{Chi}(\operatorname{arctanh}(a x)) x^{2} a^{2}+9 \operatorname{arctanh}(a x)^{2} \operatorname{Chi}(3 \operatorname{arctanh}(a x)) x^{2} a^{2}-3 \operatorname{arctanh}(a x) \sinh (3 \operatorname{arctanh}(a x)) x^{2} a^{2}\right.$
$-\cosh (3 \operatorname{arctanh}(a x)) x^{2} a^{2}+3 \sqrt{-x^{2} a^{2}+1} \operatorname{arctanh}(a x) a x-3 \operatorname{Chi}(\operatorname{arctanh}(a x)) \operatorname{arctanh}(a x)^{2}-9 \operatorname{Chi}(3 \operatorname{arctanh}(a x)) \operatorname{arctanh}(a x)^{2}$
$\left.+3 \sinh (3 \operatorname{arctanh}(a x)) \operatorname{arctanh}(a x)+3 \sqrt{-x^{2} a^{2}+1}+\cosh (3 \operatorname{arctanh}(a x))\right)$

Problem 138: Result is not expressed in closed-form.

$$
\int \frac{\operatorname{arctanh}(x)}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 4, 293 leaves, 17 steps):

$$
\begin{aligned}
-\frac{\ln (1-x) \ln \left(\frac{\sqrt{-a}-x \sqrt{b}}{\sqrt{-a}-\sqrt{b}}\right)}{4 \sqrt{-a} \sqrt{b}}+\frac{\ln (1+x) \ln \left(\frac{\sqrt{-a}-x \sqrt{b}}{\sqrt{-a}+\sqrt{b}}\right)}{4 \sqrt{-a} \sqrt{b}}-\frac{\ln (1+x) \ln \left(\frac{\sqrt{-a}+x \sqrt{b}}{\sqrt{-a}-\sqrt{b}}\right)}{4 \sqrt{-a} \sqrt{b}}+\frac{\ln (1-x) \ln \left(\frac{\sqrt{-a}+x \sqrt{b}}{\sqrt{-a}+\sqrt{b}}\right)}{4 \sqrt{-a} \sqrt{b}} \\
-\frac{\operatorname{poly} \log \left(2,-\frac{(1-x) \sqrt{b}}{\sqrt{-a}-\sqrt{b}}\right)}{4 \sqrt{-a} \sqrt{b}}-\frac{\operatorname{poly} \log \left(2,-\frac{(1+x) \sqrt{b}}{\sqrt{-a}-\sqrt{b}}\right)}{4 \sqrt{-a} \sqrt{b}}+\frac{\operatorname{polylog}\left(2, \frac{(1-x) \sqrt{b}}{\sqrt{-a}+\sqrt{b}}\right)}{4 \sqrt{-a} \sqrt{b}}+\frac{\operatorname{polylog}\left(2, \frac{(1+x) \sqrt{b}}{\sqrt{-a}+\sqrt{b}}\right)}{4 \sqrt{-a} \sqrt{b}}
\end{aligned}
$$

Result(type 7, 89 leaves):

Problem 139: Unable to integrate problem.

$$
\int \frac{\operatorname{arctanh}(a x)}{\left(d x^{2}+c\right)^{9 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 247 leaves, 8 steps):

$$
\begin{aligned}
& \frac{a}{35 c\left(a^{2} c+d\right)\left(d x^{2}+c\right)^{5 / 2}}+\frac{a\left(11 a^{2} c+6 d\right)}{105 c^{2}\left(a^{2} c+d\right)^{2}\left(d x^{2}+c\right)^{3 / 2}}+\frac{x \operatorname{arctanh}(a x)}{7 c\left(d x^{2}+c\right)^{7 / 2}}+\frac{6 x \operatorname{arctanh}(a x)}{35 c^{2}\left(d x^{2}+c\right)^{5 / 2}}+\frac{8 x \operatorname{arctanh}(a x)}{35 c^{3}\left(d x^{2}+c\right)^{3 / 2}} \\
& -\frac{\left(35 a^{6} c^{3}+70 a^{4} c^{2} d+56 a^{2} c d^{2}+16 d^{3}\right) \operatorname{arctanh}\left(\frac{a \sqrt{d x^{2}+c}}{\sqrt{a^{2} c+d}}\right)}{35 c^{4}\left(a^{2} c+d\right)^{7 / 2}}+\frac{a\left(19 a^{4} c^{2}+22 a^{2} c d+8 d^{2}\right)}{35 c^{3}\left(a^{2} c+d\right)^{3} \sqrt{d x^{2}+c}}+\frac{16 x \operatorname{arctanh}(a x)}{35 c^{4} \sqrt{d x^{2}+c}}
\end{aligned}
$$

Result(type 8, 16 leaves):

$$
\int \frac{\operatorname{arctanh}(a x)}{\left(d x^{2}+c\right)^{9 / 2}} \mathrm{~d} x
$$

Problem 140: Result more than twice size of optimal antiderivative.

$$
\int x^{3}(a+b \operatorname{arctanh}(c x))\left(d+e \ln \left(-c^{2} x^{2}+1\right)\right) \mathrm{d} x
$$

Optimal(type 3, 201 leaves, 14 steps):

$$
\begin{aligned}
& \frac{b(2 d-3 e) x}{8 c^{3}}-\frac{2 b e x}{3 c^{3}}+\frac{b(2 d-e) x^{3}}{24 c}-\frac{b e x^{3}}{18 c}-\frac{b(2 d-3 e) \operatorname{arctanh}(c x)}{8 c^{4}}+\frac{2 b e \operatorname{arctanh}(c x)}{3 c^{4}}-\frac{e x^{2}(a+b \operatorname{arctanh}(c x))}{4 c^{2}} \\
& \quad-\frac{e x^{4}(a+b \operatorname{arctanh}(c x))}{8}+\frac{b e x \ln \left(-c^{2} x^{2}+1\right)}{4 c^{3}}+\frac{b e x^{3} \ln \left(-c^{2} x^{2}+1\right)}{12 c}-\frac{e(a+b \operatorname{arctanh}(c x)) \ln \left(-c^{2} x^{2}+1\right)}{4 c^{4}} \\
& \quad+\frac{x^{4}(a+b \operatorname{arctanh}(c x))\left(d+e \ln \left(-c^{2} x^{2}+1\right)\right)}{4}
\end{aligned}
$$

Result(type ?, 3783 leaves): Display of huge result suppressed!
Problem 141: Maple result simpler than optimal antiderivative, IF it can be verified!

$$
\int \frac{(a+b \operatorname{arctanh}(c x))\left(d+e \ln \left(-c^{2} x^{2}+1\right)\right)}{x^{2}} \mathrm{~d} x
$$

Optimal(type 4, 101 leaves, 6 steps):
$-\frac{c e(a+b \operatorname{arctanh}(c x))^{2}}{b}-\frac{(a+b \operatorname{arctanh}(c x))\left(d+e \ln \left(-c^{2} x^{2}+1\right)\right)}{x}+\frac{b c\left(d+e \ln \left(-c^{2} x^{2}+1\right)\right) \ln \left(1-\frac{1}{-c^{2} x^{2}+1}\right)}{2}$

$$
-\frac{b c e \text { polylog }\left(2, \frac{1}{-c^{2} x^{2}+1}\right)}{2}
$$

Result(type 3, 63 leaves):

$$
-\frac{\left(a-\frac{\mathrm{I} b \pi}{2}\right) e \ln \left(-c^{2} x^{2}+1\right)}{x}+\frac{\left(a-\frac{\mathrm{I} b \pi}{2}\right)(c e \ln (-c x+1) x-c e \ln (-c x-1) x-d)}{x}
$$

Problem 142: Maple result simpler than optimal antiderivative, IF it can be verified!

$$
\int \frac{(a+b \operatorname{arctanh}(c x))\left(d+e \ln \left(-c^{2} x^{2}+1\right)\right)}{x^{4}} \mathrm{~d} x
$$

Optimal(type 4, 183 leaves, 15 steps):
$\frac{2 c^{2} e(a+b \operatorname{arctanh}(c x))}{3 x}-\frac{c^{3} e(a+b \operatorname{arctanh}(c x))^{2}}{3 b}-b c^{3} e \ln (x)+\frac{b c^{3} e \ln \left(-c^{2} x^{2}+1\right)}{3}-\frac{b c\left(-c^{2} x^{2}+1\right)\left(d+e \ln \left(-c^{2} x^{2}+1\right)\right)}{6 x^{2}}$
$-\frac{(a+b \operatorname{arctanh}(c x))\left(d+e \ln \left(-c^{2} x^{2}+1\right)\right)}{3 x^{3}}+\frac{b c^{3}\left(d+e \ln \left(-c^{2} x^{2}+1\right)\right) \ln \left(1-\frac{1}{-c^{2} x^{2}+1}\right)}{6}-\frac{b c^{3} e \operatorname{poly} \log \left(2, \frac{1}{-c^{2} x^{2}+1}\right)}{6}$
Result (type 3, 81 leaves):

$$
-\frac{\left(a-\frac{\mathrm{I} b \pi}{2}\right) e \ln \left(-c^{2} x^{2}+1\right)}{3 x^{3}}+\frac{\left(a-\frac{\mathrm{I} b \pi}{2}\right)\left(c^{3} e \ln (-c x+1) x^{3}-c^{3} e \ln (-c x-1) x^{3}+2 e c^{2} x^{2}-d\right)}{3 x^{3}}
$$

Problem 143: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arctanh}(c x))\left(d+e \ln \left(g x^{2}+f\right)\right)}{x^{2}} \mathrm{~d} x
$$

Optimal(type 4, 493 leaves, 28 steps):
$-\frac{(a+b \operatorname{arctanh}(c x))\left(d+e \ln \left(g x^{2}+f\right)\right)}{x}+\frac{b c \ln \left(-\frac{g x^{2}}{f}\right)\left(d+e \ln \left(g x^{2}+f\right)\right)}{2}-\frac{b c \ln \left(\frac{g\left(-c^{2} x^{2}+1\right)}{f c^{2}+g}\right)\left(d+e \ln \left(g x^{2}+f\right)\right)}{2}$

$$
-\frac{b c e \text { polylog }\left(2, \frac{c^{2}\left(g x^{2}+f\right)}{f c^{2}+g}\right)}{2}+\frac{b c e \text { polylog }\left(2,1+\frac{g x^{2}}{f}\right)}{2}-\frac{b e \ln (-c x+1) \ln \left(\frac{c(\sqrt{-f}-x \sqrt{g})}{c \sqrt{-f}-\sqrt{g}}\right) \sqrt{g}}{2 \sqrt{-f}}
$$

$$
\begin{aligned}
& +\frac{b e \ln (c x+1) \ln \left(\frac{c(\sqrt{-f}-x \sqrt{g})}{c \sqrt{-f}+\sqrt{g}}\right) \sqrt{g}}{2 \sqrt{-f}}-\frac{b e \ln (c x+1) \ln \left(\frac{c(\sqrt{-f}+x \sqrt{g})}{c \sqrt{-f}-\sqrt{g}}\right) \sqrt{g}}{2 \sqrt{-f}}+\frac{b e \ln (-c x+1) \ln \left(\frac{c(\sqrt{-f}+x \sqrt{g})}{c \sqrt{-f}+\sqrt{g}}\right) \sqrt{g}}{2 \sqrt{-f}} \\
& -\frac{b e \operatorname{polylog}\left(2,-\frac{(-c x+1) \sqrt{g}}{c \sqrt{-f}-\sqrt{g}}\right) \sqrt{g}}{2 \sqrt{-f}}-\frac{b e \operatorname{polylog}\left(2,-\frac{(c x+1) \sqrt{g}}{c \sqrt{-f}-\sqrt{g}}\right) \sqrt{g}}{2 \sqrt{-f}}+\frac{b e \operatorname{polylog}\left(2, \frac{(-c x+1) \sqrt{g}}{c \sqrt{-f}+\sqrt{g}}\right) \sqrt{g}}{2 \sqrt{-f}} \\
& +\frac{b e \operatorname{polylog}\left(2, \frac{(c x+1) \sqrt{g}}{c \sqrt{-f}+\sqrt{g}}\right) \sqrt{g}}{2 \sqrt{-f}}+\frac{2 a e \arctan \left(\frac{x \sqrt{g}}{\sqrt{f}}\right) \sqrt{g}}{\sqrt{f}} \\
& \text { Result(type 8, } 26 \text { leaves): } \\
& \int \frac{(a+b \operatorname{arctanh}(c x))\left(d+e \ln \left(g x^{2}+f\right)\right)}{x^{2}} \mathrm{~d} x
\end{aligned}
$$

Test results for the 19 problems in "7.3.5u(a+b arctanh (c+d x))^p.txt"
Problem 1: Result more than twice size of optimal antiderivative.
$\int x^{3} \operatorname{arctanh}(b x+a)^{2} \mathrm{~d} x$
Optimal(type 4, 251 leaves, 19 steps):

$$
\begin{aligned}
-\frac{a x}{b^{3}} & +\frac{(b x+a)^{2}}{12 b^{4}}+\frac{a \operatorname{arctanh}(b x+a)}{b^{4}}+\frac{\left(6 a^{2}+1\right)(b x+a) \operatorname{arctanh}(b x+a)}{2 b^{4}}-\frac{a(b x+a)^{2} \operatorname{arctanh}(b x+a)}{b^{4}}+\frac{(b x+a)^{3} \operatorname{arctanh}(b x+a)}{6 b^{4}} \\
& -\frac{a\left(a^{2}+1\right) \operatorname{arctanh}(b x+a)^{2}}{b^{4}}-\frac{\left(a^{4}+6 a^{2}+1\right) \operatorname{arctanh}(b x+a)^{2}}{4 b^{4}}+\frac{x^{4} \operatorname{arctanh}(b x+a)^{2}}{4}+\frac{2 a\left(a^{2}+1\right) \operatorname{arctanh}(b x+a) \ln \left(\frac{2}{-b x-a+1}\right)}{b^{4}} \\
& +\frac{\ln \left(1-(b x+a)^{2}\right)}{12 b^{4}}+\frac{\left(6 a^{2}+1\right) \ln \left(1-(b x+a)^{2}\right)}{4 b^{4}}+\frac{a\left(a^{2}+1\right) \operatorname{polylog}\left(2, \frac{-b x-a-1}{-b x-a+1}\right)}{b^{4}}
\end{aligned}
$$

Result(type 4, 966 leaves):

$$
\begin{aligned}
& -\frac{11 a^{2}}{12 b^{4}}+\frac{x^{2}}{12 b^{2}}+\frac{\ln (b x+a-1)}{3 b^{4}}+\frac{\ln (b x+a+1)}{3 b^{4}}+\frac{\ln (b x+a+1)^{2}}{16 b^{4}}+\frac{\ln (b x+a-1)^{2}}{16 b^{4}}-\frac{\ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln (b x+a+1)}{8 b^{4}} \\
& +\frac{\ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{8 b^{4}}+\frac{13 \operatorname{arctanh}(b x+a) a^{3}}{6 b^{4}}+\frac{\operatorname{arctanh}(b x+a) x}{2 b^{3}}+\frac{\operatorname{arctanh}(b x+a) x^{3}}{6 b}+\frac{3 a^{2} \ln (b x+a-1)^{2}}{8 b^{4}} \\
& - \\
& -\frac{a^{3} \ln (b x+a-1)^{2}}{4 b^{4}}+\frac{a^{3} \operatorname{dilog}\left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{b^{4}}+\frac{a^{4} \ln (b x+a-1)^{2}}{16 b^{4}}+\frac{x^{4} \operatorname{arctanh}(b x+a)^{2}}{4}+\frac{3 a^{2} \ln (b x+a+1)^{2}}{8 b^{4}}+\frac{a^{3} \ln (b x+a+1)^{2}}{4 b^{4}}
\end{aligned}
$$

$-\frac{a \ln (b x+a-1)}{2 b^{4}}+\frac{a \ln (b x+a+1)}{2 b^{4}}+\frac{3 \ln (b x+a+1) a^{2}}{2 b^{4}}+\frac{3 \ln (b x+a-1) a^{2}}{2 b^{4}}-\frac{a \ln (b x+a-1)^{2}}{4 b^{4}}+\frac{a \operatorname{dilog}\left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{b^{4}}$
$+\frac{a \ln (b x+a+1)^{2}}{4 b^{4}}+\frac{a^{4} \ln (b x+a+1)^{2}}{16 b^{4}}-\frac{\ln (b x+a-1) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{8 b^{4}}+\frac{\operatorname{arctanh}(b x+a) \ln (b x+a-1)}{4 b^{4}}$
$-\frac{\operatorname{arctanh}(b x+a) \ln (b x+a+1)}{4 b^{4}}-\frac{5 a x}{6 b^{3}}+\frac{a^{3} \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2 b^{4}}-\frac{a^{4} \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln (b x+a+1)}{8 b^{4}}$
$+\frac{a^{4} \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{8 b^{4}}-\frac{\operatorname{arctanh}(b x+a) x^{2} a}{2 b^{2}}+\frac{3 \operatorname{arctanh}(b x+a) x a^{2}}{2 b^{3}}+\frac{\operatorname{arctanh}(b x+a) \ln (b x+a-1) a^{4}}{4 b^{4}}$
$-\frac{\operatorname{arctanh}(b x+a) \ln (b x+a-1) a^{3}}{b^{4}}+\frac{3 \operatorname{arctanh}(b x+a) \ln (b x+a-1) a^{2}}{2 b^{4}}-\frac{\operatorname{arctanh}(b x+a) \ln (b x+a-1) a}{b^{4}}$
$-\frac{\operatorname{arctanh}(b x+a) \ln (b x+a+1) a^{4}}{4 b^{4}}-\frac{\operatorname{arctanh}(b x+a) \ln (b x+a+1) a^{3}}{b^{4}}-\frac{3 \operatorname{arctanh}(b x+a) \ln (b x+a+1) a^{2}}{2 b^{4}}$
$-\frac{\operatorname{arctanh}(b x+a) \ln (b x+a+1) a}{b^{4}}+\frac{a \ln (b x+a-1) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2 b^{4}}-\frac{a \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln (b x+a+1)}{2 b^{4}}$
$+\frac{a \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2 b^{4}}-\frac{3 a^{2} \ln (b x+a-1) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{4 b^{4}}+\frac{a^{3} \ln (b x+a-1) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2 b^{4}}$
$-\frac{a^{4} \ln (b x+a-1) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{8 b^{4}}-\frac{3 a^{2} \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln (b x+a+1)}{4 b^{4}}+\frac{3 a^{2} \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{4 b^{4}}$
$-\frac{a^{3} \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln (b x+a+1)}{2 b^{4}}+\frac{a \operatorname{arctanh}(b x+a)}{2 b^{4}}$

Problem 2: Result more than twice size of optimal antiderivative.

$$
\int x \operatorname{arctanh}(b x+a)^{2} \mathrm{~d} x
$$

Optimal(type 4, 132 leaves, 12 steps):
$\frac{(b x+a) \operatorname{arctanh}(b x+a)}{b^{2}}-\frac{a \operatorname{arctanh}(b x+a)^{2}}{b^{2}}-\frac{\left(a^{2}+1\right) \operatorname{arctanh}(b x+a)^{2}}{2 b^{2}}+\frac{x^{2} \operatorname{arctanh}(b x+a)^{2}}{2}+\frac{2 a \operatorname{arctanh}(b x+a) \ln \left(\frac{2}{-b x-a+1}\right)}{b^{2}}$ $+\frac{\ln \left(1-(b x+a)^{2}\right)}{2 b^{2}}+\frac{a \operatorname{polylog}\left(2, \frac{-b x-a-1}{-b x-a+1}\right)}{b^{2}}$

Result (type 4, 364 leaves):
$\frac{x^{2} \operatorname{arctanh}(b x+a)^{2}}{2}-\frac{\operatorname{arctanh}(b x+a)^{2} a^{2}}{2 b^{2}}+\frac{\operatorname{arctanh}(b x+a) x}{b}+\frac{\operatorname{arctanh}(b x+a) a}{b^{2}}-\frac{\operatorname{arctanh}(b x+a) \ln (b x+a-1) a}{b^{2}}$

$$
\begin{aligned}
& +\frac{\operatorname{arctanh}(b x+a) \ln (b x+a-1)}{2 b^{2}}-\frac{\operatorname{arctanh}(b x+a) \ln (b x+a+1) a}{b^{2}}-\frac{\operatorname{arctanh}(b x+a) \ln (b x+a+1)}{2 b^{2}}+\frac{\ln (b x+a-1)^{2}}{8 b^{2}} \\
& -\frac{\ln (b x+a-1) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{4 b^{2}}+\frac{\ln (b x+a-1)}{2 b^{2}}+\frac{\ln (b x+a+1)}{2 b^{2}}+\frac{\ln (b x+a+1)^{2}}{8 b^{2}}-\frac{\ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln (b x+a+1)}{4 b^{2}} \\
& +\frac{\ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{4 b^{2}}-\frac{a \ln (b x+a-1)^{2}}{4 b^{2}}+\frac{a \operatorname{dilog}\left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{b^{2}}+\frac{a \ln (b x+a-1) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2 b^{2}} \\
& +\frac{a \ln (b x+a+1)^{2}}{4 b^{2}}-\frac{a \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln (b x+a+1)}{2 b^{2}}+\frac{\ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2 b^{2}}
\end{aligned}
$$

Problem 3: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(b x+a)^{2}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 4, 358 leaves, 21 steps):
$-\frac{b \operatorname{arctanh}(b x+a)}{\left(-a^{2}+1\right) x}-\frac{\operatorname{arctanh}(b x+a)^{2}}{2 x^{2}}+\frac{b^{2} \ln (x)}{\left(-a^{2}+1\right)^{2}}+\frac{b^{2} \operatorname{arctanh}(b x+a) \ln \left(\frac{2}{-b x-a+1}\right)}{2(1-a)^{2}}-\frac{b^{2} \ln (-b x-a+1)}{2(1-a)^{2}(1+a)}$

$$
\begin{aligned}
& -\frac{b^{2} \operatorname{arctanh}(b x+a) \ln \left(\frac{2}{b x+a+1}\right)}{2(1+a)^{2}}-\frac{2 a b^{2} \operatorname{arctanh}(b x+a) \ln \left(\frac{2}{b x+a+1}\right)}{2\left(-a^{2}+1\right)^{2}}+\frac{2 a b^{2} \operatorname{arctanh}(b x+a) \ln \left(\frac{2 b x}{(1-a)(b x+a+1)}\right)}{\left(-a^{2}+1\right)^{2}} \\
& -\frac{b^{2} \ln (b x+a+1)}{2(1-a)(1+a)^{2}}+\frac{b^{2} \operatorname{polylog}\left(2, \frac{-b x-a-1}{-b x-a+1}\right)}{\left(-a^{2}+1\right)^{2}}+\frac{b^{2} \operatorname{polylog}\left(2,1-\frac{2}{b x+a+1}\right)}{(1-a)^{2}}+\frac{a b^{2} \operatorname{polylog}\left(2,1-\frac{2}{b x+a+1}\right)}{4(1+a)^{2}} \\
& -\frac{a b^{2} \operatorname{polylog}\left(2,1-\frac{2 b x}{(1-a)(b x+a+1)}\right)}{\left(-a^{2}+1\right)^{2}}
\end{aligned}
$$

Result(type 4, 1612 leaves):
$-\frac{3 b^{2} a \operatorname{dilog}\left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(-2+2 a)}+\frac{b^{2} a \ln (b x+a+1)^{2}}{4(-1+a)^{2}(1+a)^{2}(2+2 a)}-\frac{2 b^{2} a \operatorname{dilog}\left(\frac{b x+a-1}{-1+a}\right)}{(-1+a)^{2}(1+a)^{2}(-2+2 a)}-\frac{2 b^{2} a \operatorname{dilog}\left(\frac{b x+a+1}{(-1+a)^{2}(1+a)^{2}(2+2 a)}\right.}{1+a}$

$$
\begin{aligned}
& +\frac{b^{2} \ln (b x+a-1) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(2+2 a)}+\frac{2 b^{2} \operatorname{arctanh}(b x+a) a \ln (b x)}{(-1+a)^{2}(1+a)^{2}}-\frac{b^{2} a^{3} \operatorname{dilog}\left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(-2+2 a)}-\frac{b^{2} a^{3} \ln (b x+a+1)^{2}}{4(-1+a)^{2}(1+a)^{2}(2+2 a)} \\
& -\frac{b^{2} a^{2} \ln (b x+a-1)^{2}}{4(-1+a)^{2}(1+a)^{2}(-2+2 a)}+\frac{3 b^{2} a^{2} \operatorname{dilog}\left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(2+2 a)}+\frac{b^{2} a \ln (b x+a-1)^{2}}{4(-1+a)^{2}(1+a)^{2}(-2+2 a)}+\frac{3 b^{2} a \operatorname{dilog}\left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(2+2 a)} \\
& +\frac{b^{2} a^{2} \ln (b x+a+1)}{(-1+a)^{2}(1+a)^{2}(2+2 a)}+\frac{b^{2} \operatorname{dilog}\left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(2+2 a)}+\frac{b \operatorname{arctanh}(b x+a)}{(-1+a)(1+a) x}-\frac{b^{2} \ln (b x+a+1)}{(-1+a)^{2}(1+a)^{2}(2+2 a)} \\
& +\frac{b^{2} \ln (b x+a-1)}{(-1+a)^{2}(1+a)^{2}(-2+2 a)}+\frac{b^{2} a^{2} \ln (b x)}{(-1+a)^{3}(1+a)^{3}}+\frac{b^{2} \operatorname{dilog}\left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(-2+2 a)}-\frac{b^{2} \ln (b x+a+1)^{2}}{4(-1+a)^{2}(1+a)^{2}(2+2 a)} \\
& -\frac{\operatorname{arctanh}(b x+a)^{2}}{2 x^{2}}-\frac{2 b^{2} a \ln (b x) \ln \left(\frac{b x+a-1}{-1+a}\right)}{(-1+a)^{2}(1+a)^{2}(-2+2 a)}-\frac{2 b^{2} a \ln (b x) \ln \left(\frac{b x+a+1}{1+a}\right)}{(-1+a)^{2}(1+a)^{2}(2+2 a)}+\frac{3 b^{2} a \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln (b x+a+1)}{2(-1+a)^{2}(1+a)^{2}(-2+2 a)} \\
& -\frac{3 b^{2} a \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(-2+2 a)}+\frac{b^{2} a^{3} \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln (b x+a+1)}{2(-1+a)^{2}(1+a)^{2}(-2+2 a)}-\frac{b^{2} a^{3} \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(-2+2 a)} \\
& +\frac{3 b^{2} a^{2} \ln (b x+a-1) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(2+2 a)}+\frac{3 b^{2} a \ln (b x+a-1) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(2+2 a)}+\frac{b^{2} a^{3} \ln (b x+a-1) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(2+2 a)} \\
& +\frac{2 b^{2} a^{2} \ln (b x) \ln \left(\frac{b x+a-1}{-1+a}\right)}{(-1+a)^{2}(1+a)^{2}(-2+2 a)}-\frac{2 b^{2} a^{2} \ln (b x) \ln \left(\frac{b x+a+1}{1+a}\right)}{(-1+a)^{2}(1+a)^{2}(2+2 a)}-\frac{3 b^{2} a^{2} \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln (b x+a+1)}{2(-1+a)^{2}(1+a)^{2}(-2+2 a)} \\
& +\frac{3 b^{2} a^{2} \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(-2+2 a)}+\frac{b^{2} \ln (b x+a-1)^{2}}{4(-1+a)^{2}(1+a)^{2}(-2+2 a)}-\frac{b^{2} a^{2} \ln (b x+a-1)}{(-1+a)^{2}(1+a)^{2}(-2+2 a)} \\
& +\frac{2 b^{2} a^{2} \operatorname{dilog}\left(\frac{b x+a-1}{-1+a}\right)}{(-1+a)^{2}(1+a)^{2}(-2+2 a)}-\frac{2 b^{2} a^{2} \operatorname{dilog}\left(\frac{b x+a+1}{1+a}\right)}{(-1+a)^{2}(1+a)^{2}(2+2 a)}-\frac{b^{2} a^{3} \ln (b x+a-1)^{2}}{4(-1+a)^{2}(1+a)^{2}(-2+2 a)}+\frac{b^{2} a^{3} \operatorname{dilog}\left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(2+2 a)} \\
& -\frac{b^{2} \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln (b x+a+1)}{2(-1+a)^{2}(1+a)^{2}(-2+2 a)}+\frac{b^{2} \ln \left(-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right) \ln \left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(-2+2 a)}+\frac{3 b^{2} a^{2} \operatorname{dilog}\left(\frac{b x}{2}+\frac{a}{2}+\frac{1}{2}\right)}{2(-1+a)^{2}(1+a)^{2}(-2+2 a)} \\
& +\frac{b^{2} a^{2} \ln (b x+a+1)^{2}}{4(-1+a)^{2}(1+a)^{2}(2+2 a)}-\frac{b^{2} \ln (b x)}{(-1+a)^{3}(1+a)^{3}}-\frac{b^{2} \operatorname{arctanh}(b x+a) \ln (b x+a-1)}{2(-1+a)^{2}}+\frac{b^{2} \operatorname{arctanh}(b x+a) \ln (b x+a+1)}{2(1+a)^{2}}
\end{aligned}
$$

Problem 4: Result more than twice size of optimal antiderivative.
$\int(d e x+c e)^{2}(a+b \operatorname{arctanh}(d x+c)) \mathrm{d} x$

Optimal(type 3, 63 leaves, 6 steps):

$$
\frac{b e^{2}(d x+c)^{2}}{6 d}+\frac{e^{2}(d x+c)^{3}(a+b \operatorname{arctanh}(d x+c))}{3 d}+\frac{b e^{2} \ln \left(1-(d x+c)^{2}\right)}{6 d}
$$

Result(type 3, 173 leaves) :
$\frac{d^{2} x^{3} a e^{2}}{3}+d x^{2} a c e^{2}+x a c^{2} e^{2}+\frac{a c^{3} e^{2}}{3 d}+\frac{d^{2} \operatorname{arctanh}(d x+c) x^{3} b e^{2}}{3}+d \operatorname{arctanh}(d x+c) x^{2} b c e^{2}+\operatorname{arctanh}(d x+c) x b c^{2} e^{2}+\frac{\operatorname{arctanh}(d x+c) b c^{3} e^{2}}{3 d}$

$$
+\frac{d b e^{2} x^{2}}{6}+\frac{x b c e^{2}}{3}+\frac{b c^{2} e^{2}}{6 d}+\frac{b e^{2} \ln (d x+c-1)}{6 d}+\frac{b e^{2} \ln (d x+c+1)}{6 d}
$$

Problem 5: Result more than twice size of optimal antiderivative.

$$
\int(d e x+c e)^{3}(a+b \operatorname{arctanh}(d x+c))^{2} \mathrm{~d} x
$$

Optimal (type 3, 145 leaves, 13 steps):

$$
\begin{aligned}
& \frac{a b e^{3} x}{2}+\frac{b^{2} e^{3}(d x+c)^{2}}{12 d}+\frac{b^{2} e^{3}(d x+c) \operatorname{arctanh}(d x+c)}{2 d}+\frac{b e^{3}(d x+c)^{3}(a+b \operatorname{arctanh}(d x+c))}{6 d}-\frac{e^{3}(a+b \operatorname{arctanh}(d x+c))^{2}}{4 d} \\
& \quad+\frac{e^{3}(d x+c)^{4}(a+b \operatorname{arctanh}(d x+c))^{2}}{4 d}+\frac{b^{2} e^{3} \ln \left(1-(d x+c)^{2}\right)}{3 d}
\end{aligned}
$$

Result(type 3, 731 leaves):
$2 d^{2} \operatorname{arctanh}(d x+c) x^{3} a b c e^{3}+3 d \operatorname{arctanh}(d x+c) x^{2} a b c^{2} e^{3}+d^{2} x^{3} a^{2} c e^{3}+\frac{a b c^{3} e^{3}}{6 d}+\frac{a b c e^{3}}{2 d}+\frac{3 d x^{2} a^{2} c^{2} e^{3}}{2}+\frac{d^{2} x^{3} a b e^{3}}{6}$

$$
\begin{aligned}
& +\frac{d^{3} \operatorname{arctanh}(d x+c)^{2} x^{4} b^{2} e^{3}}{4}+\frac{d^{2} \operatorname{arctanh}(d x+c) x^{3} b^{2} e^{3}}{6}+\frac{e^{3} a b \ln (d x+c-1)}{4 d}-\frac{e^{3} a b \ln (d x+c+1)}{4 d} \\
& -\frac{e^{3} b^{2} \ln \left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln (d x+c+1)}{8 d}+\frac{e^{3} b^{2} \operatorname{arctanh}(d x+c) \ln (d x+c-1)}{4 d}-\frac{e^{3} b^{2} \operatorname{arctanh}(d x+c) \ln (d x+c+1)}{4 d} \\
& -\frac{e^{3} b^{2} \ln (d x+c-1) \ln \left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{8 d}+\frac{e^{3} b^{2} \ln \left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln \left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{8 d}+\frac{\operatorname{arctanh}(d x+c)^{2} b^{2} c^{4} e^{3}}{4 d}+\frac{\operatorname{arctanh}(d x+c) b^{2} c^{3} e^{3}}{6 d}
\end{aligned}
$$

$$
+\frac{\operatorname{arctanh}(d x+c) b^{2} c e^{3}}{2 d}+\operatorname{arctanh}(d x+c)^{2} x b^{2} c^{3} e^{3}+\frac{\operatorname{arctanh}(d x+c) x b^{2} c^{2} e^{3}}{2}+\frac{x a b c^{2} e^{3}}{2}+2 \operatorname{arctanh}(d x+c) x a b c^{3} e^{3}
$$

$$
+\frac{\operatorname{arctanh}(d x+c) a b c^{4} e^{3}}{2 d}+\frac{d^{3} \operatorname{arctanh}(d x+c) x^{4} a b e^{3}}{2}+d^{2} \operatorname{arctanh}(d x+c)^{2} x^{3} b^{2} c e^{3}+\frac{3 d \operatorname{arctanh}(d x+c)^{2} x^{2} b^{2} c^{2} e^{3}}{2}
$$

$$
+\frac{d \operatorname{arctanh}(d x+c) x^{2} b^{2} c e^{3}}{2}+\frac{d x^{2} a b c e^{3}}{2}+\frac{a b e^{3} x}{2}+\frac{a^{2} c^{4} e^{3}}{4 d}+\frac{b^{2} c^{2} e^{3}}{12 d}+x a^{2} c^{3} e^{3}+\frac{x b^{2} c e^{3}}{6}+\frac{d^{3} x^{4} a^{2} e^{3}}{4}+\frac{d x^{2} b^{2} e^{3}}{12}
$$

$$
+\frac{\operatorname{arctanh}(d x+c) x b^{2} e^{3}}{2}+\frac{e^{3} b^{2} \ln (d x+c-1)}{3 d}+\frac{e^{3} b^{2} \ln (d x+c+1)}{3 d}+\frac{e^{3} b^{2} \ln (d x+c-1)^{2}}{16 d}+\frac{e^{3} b^{2} \ln (d x+c+1)^{2}}{16 d}
$$

[^0]$$
\int(d e x+c e)^{2}(a+b \operatorname{arctanh}(d x+c))^{2} \mathrm{~d} x
$$

Optimal(type 4, 167 leaves, 11 steps):

$$
\frac{b^{2} e^{2} x}{3}-\frac{b^{2} e^{2} \operatorname{arctanh}(d x+c)}{3 d}+\frac{b e^{2}(d x+c)^{2}(a+b \operatorname{arctanh}(d x+c))}{3 d}+\frac{e^{2}(a+b \operatorname{arctanh}(d x+c))^{2}}{3 d}+\frac{e^{2}(d x+c)^{3}(a+b \operatorname{arctanh}(d x+c))^{2}}{3 d}
$$

$$
-\frac{2 b e^{2}(a+b \operatorname{arctanh}(d x+c)) \ln \left(\frac{2}{-d x-c+1}\right)}{3 d}-\frac{b^{2} e^{2} \operatorname{polylog}\left(2, \frac{-d x-c-1}{-d x-c+1}\right)}{3 d}
$$

Result(type 4, 582 leaves):
$\frac{\operatorname{arctanh}(d x+c) b^{2} c^{2} e^{2}}{3 d}+\operatorname{arctanh}(d x+c)^{2} x b^{2} c^{2} e^{2}+\frac{2 \operatorname{arctanh}(d x+c) x b^{2} c e^{2}}{3}+\frac{d x^{2} a b e^{2}}{3}+d x^{2} a^{2} c e^{2}+\frac{d^{2} \operatorname{arctanh}(d x+c)^{2} x^{3} b^{2} e^{2}}{3}$
$+\frac{d \operatorname{arctanh}(d x+c) x^{2} b^{2} e^{2}}{3}+\frac{a b e^{2} \ln (d x+c-1)}{3 d}+\frac{a b e^{2} \ln (d x+c+1)}{3 d}+\frac{b^{2} e^{2} \operatorname{arctanh}(d x+c) \ln (d x+c-1)}{3 d}$
$+\frac{b^{2} e^{2} \operatorname{arctanh}(d x+c) \ln (d x+c+1)}{3 d}+\frac{b^{2} e^{2} \ln \left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln (d x+c+1)}{6 d}-\frac{b^{2} e^{2} \ln (d x+c-1) \ln \left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{6 d}$

$$
-\frac{b^{2} e^{2} \ln \left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln \left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{6 d}+\frac{2 x a b c e^{2}}{3}+\frac{a b c^{2} e^{2}}{3 d}+\frac{\operatorname{arctanh}(d x+c)^{2} b^{2} c^{3} e^{2}}{3 d}+\frac{2 \operatorname{arctanh}(d x+c) a b c^{3} e^{2}}{3 d}
$$

$+d \operatorname{arctanh}(d x+c) x^{2} b^{2} c e^{2}+\frac{2 d^{2} \operatorname{arctanh}(d x+c) x^{3} a b e^{2}}{3}+2 \operatorname{arctanh}(d x+c) x a b c^{2} e^{2}+\frac{a^{2} c^{3} e^{2}}{3 d}+\frac{b^{2} c e^{2}}{3 d}+x a^{2} c^{2} e^{2}+\frac{d^{2} x^{3} a^{2} e^{2}}{3}$
$+\frac{b^{2} e^{2} \ln (d x+c-1)}{6 d}-\frac{b^{2} e^{2} \ln (d x+c+1)}{6 d}+\frac{b^{2} e^{2} \ln (d x+c-1)^{2}}{12 d}-\frac{b^{2} e^{2} \operatorname{dilog}\left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{3 d}-\frac{b^{2} e^{2} \ln (d x+c+1)^{2}}{12 d}+\frac{b^{2} e^{2} x}{3}$
$+2 d \operatorname{arctanh}(d x+c) x^{2} a b c e^{2}$

Problem 7: Result more than twice size of optimal antiderivative.

$$
\int(d e x+c e)(a+b \operatorname{arctanh}(d x+c))^{2} \mathrm{~d} x
$$

Optimal(type 3, 89 leaves, 8 steps):

$$
a b e x+\frac{b^{2} e(d x+c) \operatorname{arctanh}(d x+c)}{d}-\frac{e(a+b \operatorname{arctanh}(d x+c))^{2}}{2 d}+\frac{e(d x+c)^{2}(a+b \operatorname{arctanh}(d x+c))^{2}}{2 d}+\frac{b^{2} e \ln \left(1-(d x+c)^{2}\right)}{2 d}
$$

Result(type 3, 389 leaves):
$\frac{d x^{2} a^{2} e}{2}+x a^{2} c e+\frac{a^{2} c^{2} e}{2 d}+\frac{d \operatorname{arctanh}(d x+c)^{2} x^{2} b^{2} e}{2}+\operatorname{arctanh}(d x+c)^{2} x b^{2} c e+\frac{\operatorname{arctanh}(d x+c)^{2} b^{2} c^{2} e}{2 d}+\operatorname{arctanh}(d x+c) x b^{2} e$
$+\frac{\operatorname{arctanh}(d x+c) b^{2} c e}{d}+\frac{e b^{2} \operatorname{arctanh}(d x+c) \ln (d x+c-1)}{2 d}-\frac{e b^{2} \operatorname{arctanh}(d x+c) \ln (d x+c+1)}{2 d}+\frac{e b^{2} \ln (d x+c-1)^{2}}{8 d}$
$-\frac{e b^{2} \ln (d x+c-1) \ln \left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{4 d}+\frac{e b^{2} \ln (d x+c-1)}{2 d}+\frac{e b^{2} \ln (d x+c+1)}{2 d}+\frac{e b^{2} \ln (d x+c+1)^{2}}{8 d}$
$-\frac{e b^{2} \ln \left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln (d x+c+1)}{4 d}+\frac{e b^{2} \ln \left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln \left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{4 d}+d \operatorname{arctanh}(d x+c) x^{2} a b e+2 \operatorname{arctanh}(d x+c) x a b c e$
$+\frac{\operatorname{arctanh}(d x+c) a b c^{2} e}{d}+a b e x+\frac{a b c e}{d}+\frac{e a b \ln (d x+c-1)}{2 d}-\frac{e a b \ln (d x+c+1)}{2 d}$

Problem 8: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(d x+c))^{2}}{d e x+c e} \mathrm{~d} x
$$

Optimal(type 4, 164 leaves, 8 steps):

$$
\begin{aligned}
&-\frac{2(a+b \operatorname{arctanh}(d x+c))^{2} \operatorname{arctanh}\left(-1+\frac{2}{-d x-c+1}\right)}{e d}-\frac{b(a+b \operatorname{arctanh}(d x+c)) \operatorname{polylog}\left(2,1-\frac{2}{-d x-c+1}\right)}{e d} \\
&+\frac{b(a+b \operatorname{arctanh}(d x+c)) \operatorname{polylog}\left(2,-1+\frac{2}{-d x-c+1}\right)}{e d}+\frac{b^{2} \operatorname{polylog}\left(3,1-\frac{2}{-d x-c+1}\right)}{2 e d}-\frac{b^{2} \operatorname{polylog}\left(3,-1+\frac{2}{-d x-c+1}\right)}{2 e d}
\end{aligned}
$$

Result(type 4, 892 leaves):
$\frac{a^{2} \ln (d x+c)}{d e}+\frac{b^{2} \ln (d x+c) \operatorname{arctanh}(d x+c)^{2}}{d e}-\frac{b^{2} \operatorname{arctanh}(d x+c) \operatorname{polylog}\left(2,-\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)}{d e}+\frac{b^{2} \operatorname{polylog}\left(3,-\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)}{2 d e}$
$-\frac{b^{2} \operatorname{arctanh}(d x+c)^{2} \ln \left(\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}-1\right)}{d e}+\frac{b^{2} \operatorname{arctanh}(d x+c)^{2} \ln \left(1+\frac{d x+c+1}{\sqrt{1-(d x+c)^{2}}}\right)}{d e}$
$+\frac{2 b^{2} \operatorname{arctanh}(d x+c) \operatorname{polylog}\left(2,-\frac{d x+c+1}{\sqrt{1-(d x+c)^{2}}}\right)}{d e}-\frac{2 b^{2} \operatorname{polylog}\left(3,-\frac{d x+c+1}{\sqrt{1-(d x+c)^{2}}}\right)}{d e}+\frac{b^{2} \operatorname{arctanh}(d x+c)^{2} \ln \left(1-\frac{d x+c+1}{\sqrt{1-(d x+c)^{2}}}\right)}{d e}$
$+\frac{2 b^{2} \operatorname{arctanh}(d x+c) \operatorname{polylog}\left(2, \frac{d x+c+1}{\sqrt{1-(d x+c)^{2}}}\right)}{d e}-\frac{2 b^{2} \text { polylog }\left(3, \frac{d x+c+1}{\sqrt{1-(d x+c)^{2}}}\right)}{d e}$
$+\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}-1\right)}{1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}}\right)^{3} \operatorname{arctanh}(d x+c)^{2}}{2 d e}$

$$
\begin{aligned}
& \left.+\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}-1\right)}{2 d e}\right) \operatorname{arctanh}(d x+c)^{2}}{1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}}\right) \\
& -\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\mathrm{I}\left(\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}-1\right)}{1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}}\right)^{2} \operatorname{arctanh}(d x+c)^{2}}{2 d e} \\
& -\frac{\mathrm{I} b^{2} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}-1\right)}{1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}}\right)^{2} \operatorname{arctanh}(d x+c)^{2}}{2 d e}+\frac{2 a b \ln (d x+c) \operatorname{arctanh}(d x+c)}{d e}-\frac{a b \operatorname{dilog}(d x+c+1)}{d e} \\
& -\frac{a b \ln (d x+c) \ln (d x+c+1)}{d e}-\frac{a b \operatorname{dilog}(d x+c)}{d e}
\end{aligned}
$$

Problem 9: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(d x+c))^{2}}{(d e x+c e)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 104 leaves, 6 steps):

$$
\frac{(a+b \operatorname{arctanh}(d x+c))^{2}}{d e^{2}}-\frac{(a+b \operatorname{arctanh}(d x+c))^{2}}{d e^{2}(d x+c)}+\frac{2 b(a+b \operatorname{arctanh}(d x+c)) \ln \left(2-\frac{2}{d x+c+1}\right)}{d e^{2}}-\frac{b^{2} \operatorname{polylog}\left(2,-1+\frac{2}{d x+c+1}\right)}{d e^{2}}
$$

Result(type 4, 395 leaves):

$$
\begin{aligned}
& -\frac{a^{2}}{d e^{2}(d x+c)}-\frac{b^{2} \operatorname{arctanh}(d x+c)^{2}}{d e^{2}(d x+c)}+\frac{2 b^{2} \ln (d x+c) \operatorname{arctanh}(d x+c)}{d e^{2}}-\frac{b^{2} \operatorname{arctanh}(d x+c) \ln (d x+c+1)}{d e^{2}}-\frac{b^{2} \operatorname{arctanh}(d x+c) \ln (d x+c-1)}{d e^{2}} \\
& -\frac{b^{2} \ln (d x+c-1)^{2}}{4 d e^{2}}+\frac{b^{2} \operatorname{dilog}\left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{d e^{2}}+\frac{b^{2} \ln (d x+c-1) \ln \left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2 d e^{2}}+\frac{b^{2} \ln (d x+c+1)^{2}}{4 d e^{2}} \\
& -\frac{b^{2} \ln \left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln (d x+c+1)}{2 d e^{2}}+\frac{b^{2} \ln \left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln \left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2 d e^{2}}-\frac{b^{2} \operatorname{dilog}(d x+c+1)}{d e^{2}}-\frac{b^{2} \ln (d x+c) \ln (d x+c+1)}{d e^{2}} \\
& -\frac{b^{2} \operatorname{dilog}(d x+c)}{d e^{2}}-\frac{2 a b \operatorname{arctanh}(d x+c)}{d e^{2}(d x+c)}+\frac{2 a b \ln (d x+c)}{d e^{2}}-\frac{a b \ln (d x+c+1)}{d e^{2}}-\frac{a b \ln (d x+c-1)}{d e^{2}}
\end{aligned}
$$

Problem 10: Result more than twice size of optimal antiderivative.

$$
\int(d e x+c e)^{2}(a+b \operatorname{arctanh}(d x+c))^{3} \mathrm{~d} x
$$

Optimal(type 4, 251 leaves, 14 steps):
$a b^{2} e^{2} x+\frac{b^{3} e^{2}(d x+c) \operatorname{arctanh}(d x+c)}{d}-\frac{b e^{2}(a+b \operatorname{arctanh}(d x+c))^{2}}{2 d}+\frac{b e^{2}(d x+c)^{2}(a+b \operatorname{arctanh}(d x+c))^{2}}{2 d}+\frac{e^{2}(a+b \operatorname{arctanh}(d x+c))^{3}}{3 d}$

$$
\begin{aligned}
& +\frac{e^{2}(d x+c)^{3}(a+b \operatorname{arctanh}(d x+c))^{3}}{3 d}-\frac{b e^{2}(a+b \operatorname{arctanh}(d x+c))^{2} \ln \left(\frac{2}{-d x-c+1}\right)}{d}+\frac{b^{3} e^{2} \ln \left(1-(d x+c)^{2}\right)}{2 d} \\
& -\frac{b^{2} e^{2}(a+b \operatorname{arctanh}(d x+c)) \operatorname{poly} \log \left(2,1-\frac{2}{-d x-c+1}\right)}{d}+\frac{b^{3} e^{2} \operatorname{polylog}\left(3,1-\frac{2}{-d x-c+1}\right)}{2 d}
\end{aligned}
$$

Result (type 4, 1785 leaves):
$-\frac{e^{2} a b^{2} \ln \left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln \left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2 d}+\frac{\operatorname{arctanh}(d x+c)^{2} a b^{2} c^{3} e^{2}}{d}+\frac{\operatorname{arctanh}(d x+c) a b^{2} c^{2} e^{2}}{d}+\frac{\operatorname{arctanh}(d x+c) a^{2} b c^{3} e^{2}}{d}+3 \operatorname{arctanh}(d x$ $+c)^{2} x a b^{2} c^{2} e^{2}+2 \operatorname{arctanh}(d x+c) x a b^{2} c e^{2}+3 \operatorname{arctanh}(d x+c) x a^{2} b c^{2} e^{2}-\frac{\mathrm{I} e^{2} b^{3} \pi \operatorname{arctanh}(d x+c)^{2}}{2 d}+d^{2} \operatorname{arctanh}(d x+c)^{2} x^{3} a b^{2} e^{2}+d \operatorname{arctanh}(d x$ $+c) x^{2} a b^{2} e^{2}+d^{2} \operatorname{arctanh}(d x+c) x^{3} a^{2} b e^{2}+d \operatorname{arctanh}(d x+c)^{3} x^{2} b^{3} c e^{2}+\frac{e^{2} a b^{2} \operatorname{arctanh}(d x+c) \ln (d x+c-1)}{d}$
$+\frac{e^{2} a b^{2} \operatorname{arctanh}(d x+c) \ln (d x+c+1)}{d}-\frac{e^{2} a b^{2} \ln (d x+c-1) \ln \left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2 d}+\frac{e^{2} a b^{2} \ln \left(-\frac{d x}{2}-\frac{c}{2}+\frac{1}{2}\right) \ln (d x+c+1)}{2 d}+\frac{a b^{2} c e^{2}}{d}$
$+\frac{a^{2} b c^{2} e^{2}}{2 d}+d x^{2} a^{3} c e^{2}+\operatorname{arctanh}(d x+c)^{3} x b^{3} c^{2} e^{2}+\operatorname{arctanh}(d x+c)^{2} x b^{3} c e^{2}+\frac{d^{2} \operatorname{arctanh}(d x+c)^{3} x^{3} b^{3} e^{2}}{3}+\frac{d \operatorname{arctanh}(d x+c)^{2} x^{2} b^{3} e^{2}}{2}$
$+\frac{a^{2} b e^{2} \ln (d x+c+1)}{2 d}+\frac{e^{2} b^{3} \operatorname{arctanh}(d x+c)^{2} \ln (d x+c-1)}{2 d}+\frac{e^{2} b^{3} \operatorname{arctanh}(d x+c)^{2} \ln (d x+c+1)}{2 d}$
$-\frac{e^{2} b^{3} \operatorname{arctanh}(d x+c)^{2} \ln \left(\frac{d x+c+1}{\sqrt{1-(d x+c)^{2}}}\right)}{d}-\frac{e^{2} b^{3} \operatorname{arctanh}(d x+c) \operatorname{polylog}\left(2,-\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)}{d}+\frac{\operatorname{arctanh}(d x+c) b^{3} c e^{2}}{d}$
$+\frac{\operatorname{arctanh}(d x+c)^{3} b^{3} c^{3} e^{2}}{3 d}+\frac{\operatorname{arctanh}(d x+c)^{2} b^{3} c^{2} e^{2}}{2 d}-\frac{e^{2} b^{3} \ln (2) \operatorname{arctanh}(d x+c)^{2}}{d}+\frac{e^{2} a b^{2} \ln (d x+c-1)}{2 d}$

$$
\left.\left.\begin{array}{l}
\mathrm{I} e^{2} b^{3} \pi \operatorname{arctanh}(d x+c)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(d x+c+1)^{2}}{4 d}\left(1-(d x+c)^{2}\right)\left(1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)\right.
\end{array}\right)^{2}-\frac{e^{2} a b^{2} \ln (d x+c+1)}{2 d}\right)=-\frac{e^{2} a b^{2} \ln (d x+c-1)^{2}}{4 d}-\frac{e^{2} a b^{2} \operatorname{dilog}\left(\frac{d x}{2}+\frac{c}{2}+\frac{1}{2}\right)}{d}-\frac{e^{2} a b^{2} \ln (d x+c+1)^{2}}{4 d}+\frac{a^{2} b e^{2} \ln (d x+c-1)}{2 d}+\frac{a^{2} b x^{2} d e^{2}}{2}+a b^{2} e^{2} x+\frac{d^{2} x^{3} a^{3} e^{2}}{3} .
$$

$$
\begin{aligned}
& +\operatorname{arctanh}(d x+c) x b^{3} e^{2}+\frac{e^{2} b^{3} \operatorname{polylog}\left(3,-\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)}{2 d}+\frac{e^{2} b^{3} \operatorname{arctanh}(d x+c)^{3}}{3 d}-\frac{e^{2} b^{3} \operatorname{arctanh}(d x+c)^{2}}{2 d}+\frac{e^{2} b^{3} \operatorname{arctanh}(d x+c)}{d} \\
& -\frac{e^{2} b^{3} \ln \left(1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)}{d}+\frac{a^{3} c^{3} e^{2}}{3 d}+x a^{3} c^{2} e^{2} \\
& +\frac{\mathrm{I} e^{2} b^{3} \pi \operatorname{arctanh}(d x+c)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(d x+c+1)^{2}}{1-(d x+c)^{2}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(d x+c+1)^{2}}{\left(1-(d x+c)^{2}\right)\left(1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)}\right)}{(1)} \\
& 4 d \\
& \frac{\mathrm{I} e^{2} b^{3} \pi \operatorname{arctanh}(d x+c)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}}\right)^{3}}{2 d} \\
& +\frac{\mathrm{I} e^{2} b^{3} \pi \operatorname{arctanh}(d x+c)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)^{3}}{4 d}+\frac{\mathrm{I} e^{2} b^{3} \pi \operatorname{arctanh}(d x+c)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(d x+c+1)^{2}}{\left(1-(d x+c)^{2}\right)\left(1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)}\right)^{3}}{4 d} \\
& +\frac{\mathrm{I} e^{2} b^{3} \pi \operatorname{arctanh}(d x+c)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}}{1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}}\right)^{2}}{2 d}-\frac{\mathrm{I} e^{2} b^{3} \pi \operatorname{arctanh}(d x+c)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(d x+c+1)}{\sqrt{1-(d x+c)^{2}}}\right)}{2 d} \\
& -\underline{\mathrm{I} e^{2} b^{3} \pi \operatorname{arctanh}(d x+c)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(d x+c+1)^{2}}{1-(d x+c)^{2}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(d x+c+1)^{2}}{\left(1-(d x+c)^{2}\right)\left(1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)}\right)^{2}} \\
& 4 d \\
& +\frac{\mathrm{I} e^{2} b^{3} \pi \operatorname{arctanh}(d x+c)^{2} \operatorname{csgn}\left(\frac{\mathrm{I}(d x+c+1)^{2}}{1-(d x+c)^{2}}\right) \operatorname{csgn}\left(\frac{\mathrm{I}(d x+c+1)}{\sqrt{1-(d x+c)^{2}}}\right)^{2}}{4 d}+x a^{2} b c e^{2}
\end{aligned}
$$

Problem 13: Result more than twice size of optimal antiderivative.

$$
\int(f x+e)^{2}(a+b \operatorname{arctanh}(d x+c))^{2} \mathrm{~d} x
$$

Optimal(type 4, 360 leaves, 16 steps):
$\frac{b^{2} f^{2} x}{3 d^{2}}+\frac{2 a b f(-c f+e d) x}{d^{2}}-\frac{b^{2} f^{2} \operatorname{arctanh}(d x+c)}{3 d^{3}}+\frac{2 b^{2} f(-c f+e d)(d x+c) \operatorname{arctanh}(d x+c)}{d^{3}}+\frac{b f^{2}(d x+c)^{2}(a+b \operatorname{arctanh}(d x+c))}{3 d^{3}}$

$$
\begin{aligned}
& -\frac{(-c f+e d)\left(d^{2} e^{2}-2 c d e f+\left(c^{2}+3\right) f^{2}\right)(a+b \operatorname{arctanh}(d x+c))^{2}}{3 d^{3} f}+\frac{\left(3 d^{2} e^{2}-6 c d e f+\left(3 c^{2}+1\right) f^{2}\right)(a+b \operatorname{arctanh}(d x+c))^{2}}{3 d^{3}} \\
& +\frac{(f x+e)^{3}(a+b \operatorname{arctanh}(d x+c))^{2}}{3 f}-\frac{2 b\left(3 d^{2} e^{2}-6 c d e f+\left(3 c^{2}+1\right) f^{2}\right)(a+b \operatorname{arctanh}(d x+c)) \ln \left(\frac{2}{-d x-c+1}\right)}{3 d^{3}} \\
& \quad+\frac{b^{2} f(-c f+e d) \ln \left(1-(d x+c)^{2}\right)}{d^{3}}-\frac{b^{2}\left(3 d^{2} e^{2}-6 c d e f+\left(3 c^{2}+1\right) f^{2}\right) \operatorname{polylog}\left(2, \frac{-d x-c-1}{-d x-c+1}\right)}{3 d^{3}} \\
& \text { Result(type ?, } 2693 \text { leaves): Display of huge result suppressed! } \\
& \text { Problem 15: Result more than twice size of optimal antiderivative. }
\end{aligned}
$$

$$
\int(f x+e)^{2}(a+b \operatorname{arctanh}(d x+c))^{3} \mathrm{~d} x
$$

Optimal(type 4, 534 leaves, 21 steps):

$$
\begin{aligned}
& \frac{a b^{2} f^{2} x}{d^{2}}+\frac{b^{3} f^{2}(d x+c) \operatorname{arctanh}(d x+c)}{d^{3}}-\frac{b f^{2}(a+b \operatorname{arctanh}(d x+c))^{2}}{2 d^{3}}+\frac{3 b f(-c f+e d)(a+b \operatorname{arctanh}(d x+c))^{2}}{d^{3}} \\
& +\frac{3 b f(-c f+e d)(d x+c)(a+b \operatorname{arctanh}(d x+c))^{2}}{d^{3}}+\frac{b f^{2}(d x+c)^{2}(a+b \operatorname{arctanh}(d x+c))^{2}}{2 d^{3}} \\
& \quad-\frac{(-c f+e d)\left(d^{2} e^{2}-2 c d e f+\left(c^{2}+3\right) f^{2}\right)(a+b \operatorname{arctanh}(d x+c))^{3}}{3 d^{3} f}+\frac{\left(3 d^{2} e^{2}-6 c d e f+\left(3 c^{2}+1\right) f^{2}\right)(a+b \operatorname{arctanh}(d x+c))^{3}}{3 d^{3}}
\end{aligned}
$$

$$
+\frac{(f x+e)^{3}(a+b \operatorname{arctanh}(d x+c))^{3}}{3 f}-\frac{6 b^{2} f(-c f+e d)(a+b \operatorname{arctanh}(d x+c)) \ln \left(\frac{2}{-d x-c+1}\right)}{d^{3}}
$$

$$
-\frac{b\left(3 d^{2} e^{2}-6 c d e f+\left(3 c^{2}+1\right) f^{2}\right)(a+b \operatorname{arctanh}(d x+c))^{2} \ln \left(\frac{2}{-d x-c+1}\right)}{d^{3}}+\frac{b^{3} f^{2} \ln \left(1-(d x+c)^{2}\right)}{2 d^{3}}
$$

$$
-\frac{3 b^{3} f(-c f+e d) \operatorname{polylog}\left(2, \frac{-d x-c-1}{-d x-c+1}\right)}{d^{3}}-\frac{b^{2}\left(3 d^{2} e^{2}-6 c d e f+\left(3 c^{2}+1\right) f^{2}\right)(a+b \operatorname{arctanh}(d x+c)) \operatorname{polylog}\left(2,1-\frac{2}{-d x-c+1}\right)}{d^{3}}
$$

$$
+\frac{b^{3}\left(3 d^{2} e^{2}-6 c d e f+\left(3 c^{2}+1\right) f^{2}\right) \operatorname{polylog}\left(3,1-\frac{2}{-d x-c+1}\right)}{2 d^{3}}
$$

Result(type ?, 12290 leaves): Display of huge result suppressed!
Problem 16: Result more than twice size of optimal antiderivative.

$$
\int(a+b \operatorname{arctanh}(d x+c))^{3} \mathrm{~d} x
$$

Optimal(type 4, 130 leaves, 6 steps):
$\frac{(a+b \operatorname{arctanh}(d x+c))^{3}}{d}+\frac{(d x+c)(a+b \operatorname{arctanh}(d x+c))^{3}}{d}-\frac{3 b(a+b \operatorname{arctanh}(d x+c))^{2} \ln \left(\frac{2}{-d x-c+1}\right)}{d}$

$$
-\frac{3 b^{2}(a+b \operatorname{arctanh}(d x+c)) \operatorname{polylog}\left(2,1-\frac{2}{-d x-c+1}\right)}{d}+\frac{3 b^{3} \operatorname{polylog}\left(3,1-\frac{2}{-d x-c+1}\right)}{2 d}
$$

Result(type 4, 345 leaves):
$a^{3} x+\frac{a^{3} c}{d}+\operatorname{arctanh}(d x+c)^{3} x b^{3}+\frac{\operatorname{arctanh}(d x+c)^{3} b^{3} c}{d}+\frac{b^{3} \operatorname{arctanh}(d x+c)^{3}}{d}-\frac{3 b^{3} \operatorname{arctanh}(d x+c)^{2} \ln \left(1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)}{d}$

$$
\begin{aligned}
& -\frac{3 b^{3} \operatorname{arctanh}(d x+c) \operatorname{polylog}\left(2,-\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)}{d}+\frac{3 b^{3} \operatorname{polylog}\left(3,-\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right)}{2 d}+3 \operatorname{arctanh}(d x+c)^{2} x a b^{2}+\frac{3 \operatorname{arctanh}(d x+c)^{2} a b^{2} c}{d} \\
& +\frac{3 a b^{2} \operatorname{arctanh}(d x+c)^{2}}{d}-\frac{6 \operatorname{arctanh}(d x+c) \ln \left(1+\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right) a b^{2}}{d}-\frac{3 \operatorname{polylog}\left(2,-\frac{(d x+c+1)^{2}}{1-(d x+c)^{2}}\right) a b^{2}}{d}+3 \operatorname{arctanh}(d x+c) x a^{2} b \\
& +\frac{3 \operatorname{arctanh}(d x+c) a^{2} b c}{d}+\frac{3 a^{2} b \ln \left(1-(d x+c)^{2}\right)}{2 d}
\end{aligned}
$$

Problem 17: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arctanh}(d x+c))^{3}}{(f x+e)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 1067 leaves, 33 steps):

$$
\begin{aligned}
& -\frac{(a+b \operatorname{arctanh}(d x+c))^{3}}{f(f x+e)}+\frac{3 a b^{2} d \operatorname{arctanh}(d x+c) \ln \left(\frac{2}{-d x-c+1}\right)}{f(-c f+e d+f)}+\frac{3 b^{3} d \operatorname{arctanh}(d x+c)^{2} \ln \left(\frac{2}{-d x-c+1}\right)}{2 f(-c f+e d+f)}-\frac{3 a^{2} b d \ln (-d x-c+1)}{2 f(-c f+e d+f)} \\
& -\frac{3 a b^{2} d \operatorname{arctanh}(d x+c) \ln \left(\frac{2}{d x+c+1}\right)}{f(-c f+e d-f)}+\frac{6 a b^{2} d \operatorname{arctanh}(d x+c) \ln \left(\frac{2}{d x+c+1}\right)}{(-c f+e d+f)(e d-(c+1) f)}-\frac{3 b^{3} d \operatorname{arctanh}(d x+c)^{2} \ln \left(\frac{2}{d x+c+1}\right)}{2 f(-c f+e d-f)} \\
& +\frac{3 b^{3} d \operatorname{arctanh}(d x+c)^{2} \ln \left(\frac{2}{d x+c+1}\right)}{(-c f+e d+f)(e d-(c+1) f)}+\frac{3 a^{2} b d \ln (d x+c+1)}{2 f(-c f+e d-f)}+\frac{3 a^{2} b d \ln (f x+e)}{f^{2}-(-c f+e d)^{2}} \\
& -\frac{6 a b^{2} d \operatorname{arctanh}(d x+c) \ln \left(\frac{2 d(f x+e)}{(-c f+e d+f)(d x+c+1)}\right)}{(-c f+e d+f)(e d-(c+1) f)}-\frac{3 b^{3} d \operatorname{arctanh}(d x+c)^{2} \ln \left(\frac{2 d(f x+e)}{(-c f+e d+f)(d x+c+1)}\right)}{(-c f+e d+f)(e d-(c+1) f)} \\
& +\frac{3 a b^{2} d \operatorname{polylog}\left(2, \frac{-d x-c-1}{-d x-c+1}\right)}{2 f(-c f+e d+f)}+\frac{3 b^{3} d \operatorname{arctanh}(d x+c) \operatorname{polylog}\left(2,1-\frac{2}{-d x-c+1}\right)}{2 f(-c f+e d+f)}+\frac{3 a b^{2} d \operatorname{polylog}\left(2,1-\frac{2 f(-c f+e d-f)}{d x+c+1}\right)}{2}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{3 a b^{2} d \operatorname{polylog}\left(2,1-\frac{2}{d x+c+1}\right)}{(-c f+e d+f)(e d-(c+1) f)}+\frac{3 b^{3} d \operatorname{arctanh}(d x+c) \operatorname{polylog}\left(2,1-\frac{2}{d x+c+1}\right)}{2 f(-c f+e d-f)}-\frac{3 b^{3} d \operatorname{arctanh}(d x+c) \operatorname{polylog}\left(2,1-\frac{2}{d x+c+1}\right)}{(-c f+e d+f)(e d-(c+1) f)} \\
& +\frac{3 a b^{2} d \operatorname{poly} \log \left(2,1-\frac{2 d(f x+e)}{(-c f+e d+f)(d x+c+1)}\right)}{(-c f+e d+f)(e d-(c+1) f)}+\frac{3 b^{3} d \operatorname{arctanh}(d x+c) \operatorname{polylog}\left(2,1-\frac{2 d(f x+e)}{(-c f+e d+f)(d x+c+1)}\right)}{(-c f+e d+f)(e d-(c+1) f)} \\
& -\frac{3 b^{3} d \operatorname{polylog}\left(3,1-\frac{2}{-d x-c+1}\right)}{4 f(-c f+e d+f)}+\frac{3 b^{3} d \operatorname{polylog}\left(3,1-\frac{2}{d x+c+1}\right)}{4 f(-c f+e d-f)}-\frac{3 b^{3} d \operatorname{polylog}\left(3,1-\frac{2}{d x+c+1}\right)}{2(-c f+e d+f)(e d-(c+1) f)} \\
& +\frac{3 b^{3} d \operatorname{polylog}\left(3,1-\frac{2 d(f x+e)}{(-c f+e d+f)(d x+c+1)}\right)}{2(-c f+o d+f)(o d-(d+1) f)}
\end{aligned}
$$

Result(type ?, 6293 leaves): Display of huge result suppressed!
Test results for the 351 problems in "7.3.6 Exponentials of inverse hyperbolic tangent functions.txt"
Problem 12: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2} \sqrt{-x^{2} a^{2}+1}}{a x+1} \mathrm{~d} x
$$

Optimal(type 3, 61 leaves, 7 steps):

$$
-\frac{\left(-x^{2} a^{2}+1\right)^{3 / 2}}{3 a^{3}}+\frac{\arcsin (a x)}{2 a^{3}}+\frac{\sqrt{-x^{2} a^{2}+1}}{a^{3}}-\frac{x \sqrt{-x^{2} a^{2}+1}}{2 a^{2}}
$$

Result(type 3, 133 leaves):

$$
-\frac{\left(-x^{2} a^{2}+1\right)^{3 / 2}}{3 a^{3}}+\frac{\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}}{a^{3}}+\frac{\arctan \left(\frac{\sqrt{a^{2}} x}{\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}}\right)}{a^{2} \sqrt{a^{2}}}-\frac{x \sqrt{-x^{2} a^{2}+1}}{2 a^{2}}-\frac{\arctan \left(\frac{\sqrt{a^{2}} x}{\sqrt{-x^{2} a^{2}+1}}\right)}{2 a^{2} \sqrt{a^{2}}}
$$

Problem 13: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{-x^{2} a^{2}+1}}{(a x+1) x^{2}} d x
$$

Optimal(type 3, 33 leaves, 5 steps):

$$
a \operatorname{arctanh}\left(\sqrt{-x^{2} a^{2}+1}\right)-\frac{\sqrt{-x^{2} a^{2}+1}}{x}
$$

Result(type 3, 161 leaves):

$$
\begin{gathered}
-\frac{\left(-x^{2} a^{2}+1\right)^{3 / 2}}{x}-\sqrt{-x^{2} a^{2}+1} x a^{2}-\frac{a^{2} \arctan \left(\frac{\sqrt{a^{2}} x}{\sqrt{-x^{2} a^{2}+1}}\right)}{\sqrt{a^{2}}}+a \sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)} \\
+\frac{a^{2} \arctan \left(\frac{\sqrt{a^{2}} x}{\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}}\right)}{\sqrt{a^{2}}}+a \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2} a^{2}+1}}\right)-a \sqrt{-x^{2} a^{2}+1}
\end{gathered}
$$

Problem 14: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{-x^{2} a^{2}+1}}{(a x+1) x^{4}} \mathrm{~d} x
$$

Optimal(type 3, 74 leaves, 7 steps):

$$
\frac{a^{3} \operatorname{arctanh}\left(\sqrt{-x^{2} a^{2}+1}\right)}{2}-\frac{\sqrt{-x^{2} a^{2}+1}}{3 x^{3}}+\frac{a \sqrt{-x^{2} a^{2}+1}}{2 x^{2}}-\frac{2 a^{2} \sqrt{-x^{2} a^{2}+1}}{3 x}
$$

Result(type 3, 206 leaves):

$$
\begin{aligned}
& -\frac{\left(-x^{2} a^{2}+1\right)^{3 / 2}}{3 x^{3}}-\frac{a^{2}\left(-x^{2} a^{2}+1\right)^{3 / 2}}{x}-a^{4} x \sqrt{-x^{2} a^{2}+1}-\frac{a^{4} \arctan \left(\frac{\sqrt{a^{2} x}}{\sqrt{-x^{2} a^{2}+1}}\right)}{\sqrt{a^{2}}}+a^{3} \sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)} \\
& +a^{4} \arctan \left(\frac{\sqrt{a^{2}} x}{\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}}\right) \\
& +\frac{a\left(-x^{2} a^{2}+1\right)^{3 / 2}}{2 x^{2}}+\frac{a^{3} \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2} a^{2}+1}}\right)}{2}-\frac{a^{3} \sqrt{-x^{2} a^{2}+1}}{2}
\end{aligned}
$$

Problem 18: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(-x^{2} a^{2}+1\right)^{3 / 2}}{(a x+1)^{3} x^{2}} \mathrm{~d} x
$$

Optimal(type 3, 56 leaves, 8 steps):

$$
3 a \operatorname{arctanh}\left(\sqrt{-x^{2} a^{2}+1}\right)-\frac{\sqrt{-x^{2} a^{2}+1}}{x}-\frac{4 a \sqrt{-x^{2} a^{2}+1}}{a x+1}
$$

Result(type 3, 260 leaves):

$$
\begin{aligned}
& -\frac{\left(-x^{2} a^{2}+1\right)^{5 / 2}}{x}-a^{2} x\left(-x^{2} a^{2}+1\right)^{3 / 2}-\frac{3 \sqrt{-x^{2} a^{2}+1} x a^{2}}{2}-\frac{3 a^{2} \arctan \left(\frac{\sqrt{a^{2} x}}{\sqrt{-x^{2} a^{2}+1}}\right)}{2 \sqrt{a^{2}}}-\frac{\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{5 / 2}}{a^{2}\left(x+\frac{1}{a}\right)^{3}}-a\left(-x^{2} a^{2}\right. \\
& \left.+1)^{3 / 2}+3 a \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2} a^{2}+1}}\right)-3 a \sqrt{-x^{2} a^{2}+1}+a\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{3 / 2}+\frac{3 a^{2} \sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)} x}{2}\right) \\
& \quad 3 a^{2} \arctan \left(\frac{\sqrt{a^{2}} x}{2 \sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}}\right)
\end{aligned}
$$

Problem 19: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(-x^{2} a^{2}+1\right)^{3 / 2}}{(a x+1)^{3} x^{5}} \mathrm{~d} x
$$

Optimal(type 3, 117 leaves, 19 steps):

$$
-\frac{51 a^{4} \operatorname{arctanh}\left(\sqrt{-x^{2} a^{2}+1}\right)}{8}-\frac{\sqrt{-x^{2} a^{2}+1}}{4 x^{4}}+\frac{a \sqrt{-x^{2} a^{2}+1}}{x^{3}}-\frac{19 a^{2} \sqrt{-x^{2} a^{2}+1}}{8 x^{2}}+\frac{6 a^{3} \sqrt{-x^{2} a^{2}+1}}{x}+\frac{4 a^{4} \sqrt{-x^{2} a^{2}+1}}{a x+1}
$$

Result(type 3, 358 leaves):

$$
\begin{aligned}
& -\frac{\left(-x^{2} a^{2}+1\right)^{5 / 2}}{4 x^{4}}-\frac{23 a^{2}\left(-x^{2} a^{2}+1\right)^{5 / 2}}{8 x^{2}}+\frac{17 a^{4}\left(-x^{2} a^{2}+1\right)^{3 / 2}}{8}+\frac{51 a^{4} \sqrt{-x^{2} a^{2}+1}}{8}-\frac{51 a^{4} \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2} a^{2}+1}}\right)}{8}+\frac{a\left(-x^{2} a^{2}+1\right)^{5} / 2}{x^{3}} \\
& +\frac{8 a^{3}\left(-x^{2} a^{2}+1\right)^{5 / 2}}{x}+8 a^{5} x\left(-x^{2} a^{2}+1\right)^{3 / 2}+12 a^{5} x \sqrt{-x^{2} a^{2}+1}+\frac{12 a^{5} \arctan \left(\frac{\sqrt{a^{2} x}}{\sqrt{-x^{2} a^{2}+1}}\right)}{\sqrt{a^{2}}}+\frac{a\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{5 / 2}}{\left(x+\frac{1}{a}\right)^{3}} \\
& -\frac{3 a^{2}\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{5 / 2}}{\left(x+\frac{1}{a}\right)^{2}}-8 a^{4}\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{3 / 2}-12 a^{5} \sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)} x
\end{aligned}
$$



Problem 20: Unable to integrate problem.

$$
\int \frac{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}}{x} d x
$$

Optimal(type 3, 180 leaves, 17 steps):
$-2 \arctan \left(\frac{(a x+1)^{1 / 4}}{(-a x+1)^{1 / 4}}\right)-2 \operatorname{arctanh}\left(\frac{(a x+1)^{1 / 4}}{(-a x+1)^{1 / 4}}\right)-\frac{\ln \left(1-\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}{2}$
$\quad+\frac{\ln \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}{2}-\arctan \left(-1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}-\arctan \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}$
Result(type 8, 26 leaves):

$$
\int \frac{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}}{x} \mathrm{~d} x
$$

Problem 21: Unable to integrate problem.

$$
\int \frac{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}}{x^{2}} d x
$$

Optimal(type 3, 61 leaves, 6 steps):

$$
-\frac{(-a x+1)^{3 / 4}(a x+1)^{1 / 4}}{x}-a \arctan \left(\frac{(a x+1)^{1 / 4}}{(-a x+1)^{1 / 4}}\right)-a \operatorname{arctanh}\left(\frac{(a x+1)^{1 / 4}}{(-a x+1)^{1 / 4}}\right)
$$

Result(type 8, 26 leaves):

$$
\int \frac{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}}{x^{2}} d x
$$

Problem 22: Unable to integrate problem.

$$
\int \frac{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}}{x^{3}} d x
$$

Optimal(type 3, 86 leaves, 7 steps):

$$
-\frac{a(-a x+1)^{3 / 4}(a x+1)^{1 / 4}}{4 x}-\frac{(-a x+1)^{3 / 4}(a x+1)^{5 / 4}}{2 x^{2}}-\frac{a^{2} \arctan \left(\frac{(a x+1)^{1 / 4}}{(-a x+1)^{1 / 4}}\right)}{4}-\frac{a^{2} \operatorname{arctanh}\left(\frac{(a x+1)^{1 / 4}}{\left.(-a x+1)^{1 / 4}\right)}\right.}{4}
$$

Result(type 8, 26 leaves):

$$
\int \frac{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}}{x^{3}} d x
$$

Problem 23: Unable to integrate problem.

$$
\int\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{3 / 2} x^{m} \mathrm{~d} x
$$

Optimal(type 6, 27 leaves, 2 steps):

$$
\frac{x^{1+m} \text { AppellF1 }\left(1+m, \frac{3}{4},-\frac{3}{4}, 2+m, a x,-a x\right)}{1+m}
$$

Result(type 8, 26 leaves):

$$
\int\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{3 / 2} x^{m} \mathrm{~d} x
$$

Problem 24: Unable to integrate problem.

$$
\int\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{5 / 2} x^{m} \mathrm{~d} x
$$

Optimal(type 6, 27 leaves, 2 steps):

$$
\frac{x^{1+m} \text { AppellF } 1\left(1+m, \frac{5}{4},-\frac{5}{4}, 2+m, a x,-a x\right)}{1+m}
$$

Result(type 8, 26 leaves):

$$
\int\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{5 / 2} x^{m} \mathrm{~d} x
$$

Problem 25: Unable to integrate problem.

$$
\int\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{5 / 2} x^{3} \mathrm{~d} x
$$

Optimal(type 3, 246 leaves, 16 steps):
$\frac{475(-a x+1)^{3 / 4}(a x+1)^{1 / 4}}{64 a^{4}}+\frac{4 x^{3}(a x+1)^{5 / 4}}{a(-a x+1)^{1 / 4}}+\frac{17 x^{2}(-a x+1)^{3 / 4}(a x+1)^{5 / 4}}{4 a^{2}}+\frac{(-a x+1)^{3 / 4}(a x+1)^{5 / 4}(452 a x+521)}{96 a^{4}}$
$+\frac{475 \arctan \left(-1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}}{128 a^{4}}+\frac{475 \arctan \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}}{128 a^{4}}+\frac{475 \ln \left(1-\frac{(-a x+1)^{1 / 4} \sqrt{2}}{\left.(a x+1)^{1 / 4}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}\right.}{256 a^{4}}$

$$
-\frac{475 \ln \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}{256 a^{4}}
$$

Result(type 8, 26 leaves):

$$
\int\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{5 / 2} x^{3} \mathrm{~d} x
$$

Problem 26: Unable to integrate problem.

$$
\int \frac{x^{3}}{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}} d x
$$

Optimal(type 3, 223 leaves, 15 steps):
$-\frac{11(-a x+1)^{1 / 4}(a x+1)^{3 / 4}}{64 a^{4}}-\frac{x^{2}(-a x+1)^{5 / 4}(a x+1)^{3 / 4}}{4 a^{2}}-\frac{(-4 a x+25)(-a x+1)^{5 / 4}(a x+1)^{3 / 4}}{96 a^{4}}$

$$
\begin{aligned}
& +\frac{11 \arctan \left(-1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}}{128 a^{4}}+\frac{11 \arctan \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}}{128 a^{4}}-\frac{11 \ln \left(1-\frac{(-a x+1)^{1 / 4} \sqrt{2}}{\left.(a x+1)^{1 / 4}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}\right.}{256 a^{4}} \\
& +\frac{11 \ln \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}{256 a^{4}}
\end{aligned}
$$

Result(type 8, 26 leaves):

$$
\int \frac{x^{3}}{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}} \mathrm{~d} x
$$

Problem 27: Unable to integrate problem.

$$
\int \frac{x^{2}}{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}} \mathrm{~d} x
$$

Optimal(type 3, 215 leaves, 15 steps):

$$
\begin{aligned}
& \frac{3(-a x+1)^{1 / 4}(a x+1)^{3 / 4}}{8 a^{3}}+\frac{(-a x+1)^{5 / 4}(a x+1)^{3 / 4}}{12 a^{3}}-\frac{x(-a x+1)^{5 / 4}(a x+1)^{3 / 4}}{3 a^{2}}-\frac{3 \arctan \left(-1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}}{16 a^{3}} \\
& \quad-\frac{3 \arctan \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}}{16 a^{3}}+\frac{3 \ln \left(1-\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}{32 a^{3}}-\frac{3 \ln \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}{32 a^{3}}
\end{aligned}
$$

Result(type 8, 26 leaves):

$$
\int \frac{x^{2}}{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}} \mathrm{~d} x
$$

Problem 28: Unable to integrate problem.

$$
\int \frac{x^{m}}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 6, 27 leaves, 2 steps):

$$
\frac{x^{1+m} \text { AppellF1 }\left(1+m,-\frac{3}{4}, \frac{3}{4}, 2+m, a x,-a x\right)}{1+m}
$$

Result(type 8, 26 leaves):

$$
\int \frac{x^{m}}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{3 / 2}} d x
$$

Problem 29: Unable to integrate problem.

$$
\int \frac{x^{3}}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{3 / 2}} d x
$$

Optimal(type 3, 223 leaves, 15 steps):
$-\frac{41(-a x+1)^{3 / 4}(a x+1)^{1 / 4}}{64 a^{4}}-\frac{x^{2}(-a x+1)^{7 / 4}(a x+1)^{1 / 4}}{4 a^{2}}-\frac{(-4 a x+11)(-a x+1)^{7 / 4}(a x+1)^{1 / 4}}{32 a^{4}}$

$$
+\frac{123 \arctan \left(-1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}}{128 a^{4}}+\frac{123 \arctan \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}}{128 a^{4}}+\frac{123 \ln \left(1-\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}{256 a^{4}}
$$

$$
-\frac{123 \ln \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}{256 a^{4}}
$$

Result(type 8, 26 leaves):

$$
\int \frac{x^{3}}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{3 / 2}} \mathrm{~d} x
$$

Problem 30: Unable to integrate problem.

$$
\int \frac{x^{2}}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 215 leaves, 15 steps):
$\frac{17(-a x+1)^{3 / 4}(a x+1)^{1 / 4}}{24 a^{3}}+\frac{(-a x+1)^{7 / 4}(a x+1)^{1 / 4}}{4 a^{3}}-\frac{x(-a x+1)^{7 / 4}(a x+1)^{1 / 4}}{3 a^{2}}-\frac{17 \arctan \left(-1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{\left.(a x+1)^{1 / 4}\right) \sqrt{2}}\right.}{16 a^{3}}$
$-\frac{17 \arctan \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}}{16 a^{3}}-\frac{17 \ln \left(1-\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}{32 a^{3}}$
$+\frac{17 \ln \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}{32 a^{3}}$
Result(type 8, 26 leaves):

$$
\int \frac{x^{2}}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{3 / 2}} \mathrm{~d} x
$$

Problem 31: Unable to integrate problem.

$$
\int \frac{x}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{5 / 2}} d x
$$

Optimal(type 3, 214 leaves, 15 steps):

$$
\begin{aligned}
& \frac{2(-a x+1)^{9 / 4}}{a^{2}(a x+1)^{1 / 4}}+\frac{25(-a x+1)^{1 / 4}(a x+1)^{3 / 4}}{4 a^{2}}+\frac{5(-a x+1)^{5 / 4}(a x+1)^{3} / 4}{2 a^{2}}-\frac{25 \arctan \left(-1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}}{8 a^{2}} \\
& \quad-\frac{25 \arctan \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}\right) \sqrt{2}}{8 a^{2}}+\frac{25 \ln \left(1-\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}{16 a^{2}} \\
& \quad-\frac{25 \ln \left(1+\frac{(-a x+1)^{1 / 4} \sqrt{2}}{(a x+1)^{1 / 4}}+\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right) \sqrt{2}}{16 a^{2}}
\end{aligned}
$$

Result(type 8, 24 leaves):

$$
\int \frac{x}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{5 / 2}} \mathrm{~d} x
$$

Problem 32: Unable to integrate problem.

$$
\int \frac{1}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{5 / 2} x^{3}} \mathrm{~d} x
$$

Optimal(type 3, 106 leaves, 8 steps):

$$
\frac{25 a^{2}(-a x+1)^{1 / 4}}{2(a x+1)^{1 / 4}}+\frac{5 a(-a x+1)^{5 / 4}}{4 x(a x+1)^{1 / 4}}-\frac{(-a x+1)^{9 / 4}}{2 x^{2}(a x+1)^{1 / 4}}+\frac{25 a^{2} \arctan \left(\frac{(a x+1)^{1 / 4}}{(-a x+1)^{1 / 4}}\right)}{4}-\frac{25 a^{2} \operatorname{arctanh}\left(\frac{(a x+1)^{1 / 4}}{(-a x+1)^{1 / 4}}\right)}{4}
$$

Result(type 8, 26 leaves):

$$
\int \frac{1}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{5 / 2} x^{3}} d x
$$

Problem 33: Unable to integrate problem.

$$
\int \frac{1}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{5 / 2} x^{4}} \mathrm{~d} x
$$

Optimal(type 3, 129 leaves, 10 steps):

$$
\begin{aligned}
& -\frac{287 a^{3}(-a x+1)^{1 / 4}}{24(a x+1)^{1 / 4}}-\frac{(-a x+1)^{1 / 4}}{3 x^{3}(a x+1)^{1 / 4}}+\frac{13 a(-a x+1)^{1 / 4}}{12 x^{2}(a x+1)^{1 / 4}}-\frac{61 a^{2}(-a x+1)^{1 / 4}}{24 x(a x+1)^{1 / 4}}-\frac{55 a^{3} \arctan \left(\frac{(a x+1)^{1 / 4}}{(-a x+1)^{1 / 4}}\right)}{8} \\
& \quad+\frac{55 a^{3} \operatorname{arctanh}\left(\frac{(a x+1)^{1 / 4}}{(-a x+1)^{1 / 4}}\right)}{8}
\end{aligned}
$$

Result(type 8, 26 leaves):

$$
\int \frac{1}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{5 / 2} x^{4}} \mathrm{~d} x
$$

Problem 34: Unable to integrate problem.

$$
\int \frac{1}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{5 / 2} x^{5}} \mathrm{~d} x
$$

Optimal(type 3, 152 leaves, 11 steps):

$$
\begin{aligned}
& \frac{2467 a^{4}(-a x+1)^{1 / 4}}{192(a x+1)^{1 / 4}}-\frac{(-a x+1)^{1 / 4}}{4 x^{4}(a x+1)^{1 / 4}}+\frac{17 a(-a x+1)^{1 / 4}}{24 x^{3}(a x+1)^{1 / 4}}-\frac{113 a^{2}(-a x+1)^{1 / 4}}{96 x^{2}(a x+1)^{1 / 4}}+\frac{521 a^{3}(-a x+1)^{1 / 4}}{192 x(a x+1)^{1 / 4}}+\frac{475 a^{4} \arctan \left(\frac{(a x+1)^{1 / 4}}{(-a x+1)^{1 / 4}}\right)}{64} \\
& \quad-\frac{475 a^{4} \operatorname{arctanh}\left(\frac{(a x+1)^{1 / 4}}{(-a x+1)^{1 / 4}}\right)}{64}
\end{aligned}
$$

Result(type 8, 26 leaves):

$$
\int \frac{1}{\left(\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}\right)^{5 / 2} x^{5}} \mathrm{~d} x
$$

Problem 35: Unable to integrate problem.

$$
\int\left(\frac{1+x}{\sqrt{-x^{2}+1}}\right)^{1 / 3} x^{m} \mathrm{~d} x
$$

Optimal(type 6, 24 leaves, 2 steps):

$$
\frac{x^{1+m} \text { AppellF1 }\left(1+m, \frac{1}{6},-\frac{1}{6}, 2+m, x,-x\right)}{1+m}
$$

Result(type 8, 21 leaves):

$$
\int\left(\frac{1+x}{\sqrt{-x^{2}+1}}\right)^{1 / 3} x^{m} \mathrm{~d} x
$$

Problem 36: Unable to integrate problem.

$$
\int\left(\frac{1+x}{\sqrt{-x^{2}+1}}\right)^{1 / 3} x \mathrm{~d} x
$$

Optimal(type 3, 164 leaves, 15 steps):

$$
\begin{aligned}
& -\frac{(1-x)^{5 / 6}(1+x)^{1 / 6}}{6}-\frac{(1-x)^{5 / 6}(1+x)^{7 / 6}}{2}-\frac{\arctan \left(\frac{(1-x)^{1 / 6}}{(1+x)^{1 / 6}}\right)}{9}-\frac{\arctan \left(\frac{2(1-x)^{1 / 6}}{(1+x)^{1 / 6}}-\sqrt{3}\right)}{18}-\frac{\arctan \left(\frac{2(1-x)^{1 / 6}}{(1+x)^{1 / 6}}+\sqrt{3}\right)}{18} \\
& -\frac{\ln \left(1+\frac{(1-x)^{1 / 3}}{(1+x)^{1 / 3}}-\frac{(1-x)^{1 / 6} \sqrt{3}}{(1+x)^{1 / 6}}\right) \sqrt{3}}{36}+\frac{\ln \left(1+\frac{(1-x)^{1 / 3}}{(1+x)^{1 / 3}}+\frac{(1-x)^{1 / 6} \sqrt{3}}{(1+x)^{1 / 6}}\right) \sqrt{3}}{36}
\end{aligned}
$$

Result(type 8, 19 leaves):

$$
\int\left(\frac{1+x}{\sqrt{-x^{2}+1}}\right)^{1 / 3} x \mathrm{~d} x
$$

Problem 37: Unable to integrate problem.

$$
\int \frac{\left(\frac{1+x}{\sqrt{-x^{2}+1}}\right)^{1 / 3}}{x} \mathrm{~d} x
$$

Optimal(type 3, 268 leaves, 25 steps):

$$
\begin{aligned}
& -2 \arctan \left(\frac{(1-x)^{1 / 6}}{(1+x)^{1 / 6}}\right)-\arctan \left(\frac{2(1-x)^{1 / 6}}{(1+x)^{1 / 6}}-\sqrt{3}\right)-\arctan \left(\frac{2(1-x)^{1 / 6}}{(1+x)^{1 / 6}}+\sqrt{3}\right)-2 \operatorname{arctanh}\left(\frac{(1+x)^{1 / 6}}{\left.(1-x)^{1 / 6}\right)}\right. \\
& \quad+\frac{\ln \left(1-\frac{(1+x)^{1 / 6}}{(1-x)^{1 / 6}}+\frac{(1+x)^{1 / 3}}{(1-x)^{1 / 3}}\right)}{2}-\frac{\ln \left(1+\frac{(1+x)^{1 / 6}}{\left.(1-x)^{1 / 6}+\frac{(1+x)^{1 / 3}}{(1-x)^{1 / 3}}\right)}\right.}{2}+\arctan \left(\frac{\left(1-\frac{2(1+x)^{1 / 6}}{\left.(1-x)^{1 / 6}\right) \sqrt{3}}\right)}{3}\right) \sqrt{3} \\
& \quad-\arctan \left(\frac{\left(1+\frac{2(1+x)^{1 / 6}}{(1-x)^{1 / 6}}\right) \sqrt{3}}{3}\right)-\frac{\ln \left(1+\frac{(1-x)^{1 / 3}}{(1+x)^{1 / 3}}-\frac{(1-x)^{1 / 6} \sqrt{3}}{(1+x)^{1 / 6}}\right) \sqrt{3}}{2}+\frac{\ln \left(1+\frac{(1-x)^{1 / 3}}{\left.(1+x)^{1 / 3}+\frac{(1-x)^{1 / 6} \sqrt{3}}{(1+x)^{1 / 6}}\right) \sqrt{3}}\right.}{2}
\end{aligned}
$$

Result(type 8, 21 leaves):

$$
\int \frac{\left(\frac{1+x}{\sqrt{-x^{2}+1}}\right)^{1 / 3}}{x} \mathrm{~d} x
$$

[^1]Optimal(type 3, 168 leaves, 14 steps):

$$
\begin{aligned}
& -\frac{(1-x)^{5 / 6}(1+x)^{1 / 6}}{6 x}-\frac{(1-x)^{5 / 6}(1+x)^{7 / 6}}{2 x^{2}}-\frac{\operatorname{arctanh}\left(\frac{(1+x)^{1 / 6}}{(1-x)^{1 / 6}}\right)}{9}+\frac{\ln \left(1-\frac{(1+x)^{1 / 6}}{(1-x)^{1 / 6}}+\frac{(1+x)^{1 / 3}}{(1-x)^{1 / 3}}\right)}{36} \\
& -\frac{\ln \left(1+\frac{(1+x)^{1 / 6}}{(1-x)^{1 / 6}}+\frac{(1+x)^{1 / 3}}{(1-x)^{1 / 3}}\right)}{36}+\frac{\arctan \left(\frac{\left(1-\frac{2(1+x)^{1 / 6}}{(1-x)^{1 / 6}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{18}-\frac{\arctan \left(\frac{\left(1+\frac{2(1+x)^{1 / 6}}{(1-x)^{1 / 6}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{18}
\end{aligned}
$$

Result(type 8, 21 leaves):

$$
\int \frac{\left(\frac{1+x}{\sqrt{-x^{2}+1}}\right)^{1 / 3}}{x^{3}} \mathrm{~d} x
$$

Problem 39: Unable to integrate problem.

$$
\int \frac{\left(\frac{1+x}{\sqrt{-x^{2}+1}}\right)^{2 / 3}}{x} \mathrm{~d} x
$$

Optimal(type 3, 104 leaves, 4 steps):

$$
\begin{aligned}
& -\frac{\ln (x)}{2}+\frac{\ln (1+x)}{2}+\frac{3 \ln \left(1+\frac{(1-x)^{1 / 3}}{(1+x)^{1 / 3}}\right)}{2}+\frac{3 \ln \left((1-x)^{1 / 3}-(1+x)^{1 / 3}\right)}{2}-\arctan \left(-\frac{\sqrt{3}}{3}+\frac{2(1-x)^{1 / 3} \sqrt{3}}{3(1+x)^{1 / 3}}\right) \sqrt{3}+\arctan \left(\frac{\sqrt{3}}{3}\right. \\
& \left.\quad+\frac{2(1-x)^{1 / 3} \sqrt{3}}{3(1+x)^{1 / 3}}\right) \sqrt{3}
\end{aligned}
$$

Result(type 8, 21 leaves):

$$
\int \frac{\left(\frac{1+x}{\sqrt{-x^{2}+1}}\right)^{2 / 3}}{x} \mathrm{~d} x
$$

[^2]$$
\int \frac{\left(\frac{1+x}{\sqrt{-x^{2}+1}}\right)^{2 / 3}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 3, 86 leaves, 4 steps):

$$
-\frac{(1-x)^{2 / 3}(1+x)^{1 / 3}}{3 x}-\frac{(1-x)^{2 / 3}(1+x)^{4 / 3}}{2 x^{2}}-\frac{\ln (x)}{9}+\frac{\ln \left((1-x)^{1 / 3}-(1+x)^{1 / 3}\right)}{3}+\frac{2 \arctan \left(\frac{\sqrt{3}}{3}+\frac{2(1-x)^{1 / 3} \sqrt{3}}{\left.3(1+x)^{1 / 3}\right) \sqrt{3}}\right.}{9}
$$

Result(type 8, 21 leaves):

$$
\int \frac{\left(\frac{1+x}{\sqrt{-x^{2}+1}}\right)^{2 / 3}}{x^{3}} \mathrm{~d} x
$$

Problem 42: Unable to integrate problem.

$$
\int \frac{x^{m}\left(-x^{2} a^{2}+1\right)^{3 / 2}}{(a x+1)^{3}} \mathrm{~d} x
$$

Optimal(type 5, 134 leaves, 9 steps):

$$
\begin{aligned}
-\frac{3 x^{1+m} \text { hypergeom }\left(\left[\frac{1}{2}, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right], x^{2} a^{2}\right)}{1+m}+\frac{a x^{2+m} \text { hypergeom }\left(\left[\frac{1}{2}, 1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right], x^{2} a^{2}\right)}{2+m} \\
+\frac{4 x^{1+m} \operatorname{hypergeom}\left(\left[\frac{3}{2}, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right], x^{2} a^{2}\right)}{1+m}-\frac{4 a x^{2+m} \operatorname{hypergeom}\left(\left[\frac{3}{2}, 1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right], x^{2} a^{2}\right)}{2+m}
\end{aligned}
$$

Result(type 8, 25 leaves):

$$
\int \frac{x^{m}\left(-x^{2} a^{2}+1\right)^{3 / 2}}{(a x+1)^{3}} \mathrm{~d} x
$$

Problem 43: Unable to integrate problem.

$$
\int \mathrm{e}^{n} \operatorname{arctanh}(a x) x^{m} \mathrm{~d} x
$$

Optimal(type 6, 31 leaves, 2 steps):

$$
\frac{x^{1+m} \text { AppellF1 }\left(1+m, \frac{n}{2},-\frac{n}{2}, 2+m, a x,-a x\right)}{1+m}
$$

Result(type 8, 13 leaves):

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)} x^{m} \mathrm{~d} x
$$

Problem 44: Unable to integrate problem.

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)} \mathrm{d} x
$$

Optimal(type 5, 53 leaves, 2 steps):

$$
-\frac{2^{1+\frac{n}{2}}(-a x+1)^{1-\frac{n}{2}} \text { hypergeom }\left(\left[-\frac{n}{2}, 1-\frac{n}{2}\right],\left[2-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right)}{a(2-n)}
$$

Result(type 8, 9 leaves):

$$
\int \mathrm{e}^{n} \operatorname{arctanh}(a x) \mathrm{d} x
$$

Problem 45: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 5, 93 leaves, 3 steps):

$$
-\frac{(-a x+1)^{1-\frac{n}{2}}(a x+1)^{1+\frac{n}{2}}}{2 x^{2}}-\frac{2 a^{2} n(-a x+1)^{1-\frac{n}{2}}(a x+1)^{-1+\frac{n}{2}} \operatorname{hypergeom}\left(\left[2,1-\frac{n}{2}\right],\left[2-\frac{n}{2}\right], \frac{-a x+1}{a x+1}\right)}{2-n}
$$

Result(type 8, 13 leaves):

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{x^{3}} \mathrm{~d} x
$$

Problem 46: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{x^{4}} \mathrm{~d} x
$$

Optimal(type 5, 127 leaves, 5 steps):
$-\frac{(-a x+1)^{1-\frac{n}{2}}(a x+1)^{1+\frac{n}{2}}}{3 x^{3}}-\frac{a n(-a x+1)^{1-\frac{n}{2}}(a x+1)^{1+\frac{n}{2}}}{6 x^{2}}$

$$
-\frac{2 a^{3}\left(n^{2}+2\right)(-a x+1)^{1-\frac{n}{2}}(a x+1)^{-1+\frac{n}{2}} \operatorname{hypergeom}\left(\left[2,1-\frac{n}{2}\right],\left[2-\frac{n}{2}\right], \frac{-a x+1}{a x+1}\right)}{3(2-n)}
$$

Result(type 8, 13 leaves):

Problem 51: Unable to integrate problem.

$$
\int \frac{(-a c x+c)^{p} \sqrt{-x^{2} a^{2}+1}}{a x+1} \mathrm{~d} x
$$

Optimal(type 5, 53 leaves, 3 steps):

$$
-\frac{(-a c x+c)^{1+p} \text { hypergeom }\left(\left[\frac{1}{2}, \frac{3}{2}+p\right],\left[\frac{5}{2}+p\right],-\frac{a x}{2}+\frac{1}{2}\right) \sqrt{2} \sqrt{-a x+1}}{a c(3+2 p)}
$$

Result(type 8, 31 leaves):

$$
\int \frac{(-a c x+c)^{p} \sqrt{-x^{2} a^{2}+1}}{a x+1} \mathrm{~d} x
$$

Problem 52: Result more than twice size of optimal antiderivative.

$$
\int \frac{(-a c x+c) \sqrt{-x^{2} a^{2}+1}}{a x+1} \mathrm{~d} x
$$

Optimal(type 3, 53 leaves, 4 steps):

$$
\frac{3 c \arcsin (a x)}{2 a}+\frac{3 c \sqrt{-x^{2} a^{2}+1}}{2 a}+\frac{c(-a x+1) \sqrt{-x^{2} a^{2}+1}}{2 a}
$$

Result(type 3, 113 leaves):

$$
-\frac{c x \sqrt{-x^{2} a^{2}+1}}{2}-\frac{c \arctan \left(\frac{\sqrt{a^{2} x}}{\sqrt{-x^{2} a^{2}+1}}\right)}{2 \sqrt{a^{2}}}+\frac{2 c \sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}}{a}+\frac{2 c \arctan \left(\frac{\sqrt{a^{2}} x}{\left.\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}\right)}\right.}{\sqrt{a^{2}}}
$$

Problem 57: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(-x^{2} a^{2}+1\right)^{3 / 2}}{(a x+1)^{3}(-a c x+c)} \mathrm{d} x
$$

Optimal(type 3, 39 leaves, 3 steps):

$$
-\frac{\arcsin (a x)}{c a}-\frac{2(-a x+1)}{a c \sqrt{-x^{2} a^{2}+1}}
$$

Result(type 3, 291 leaves):

$$
\begin{aligned}
& -\frac{\left(-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)\right)^{3 / 2}}{24 c a}+\frac{\sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)} x}{16 c}+\frac{\arctan \left(\frac{\sqrt{a^{2} x} x}{\left.\sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)}\right)}\right.}{16 c \sqrt{a^{2}}} \\
& -\frac{\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{5 / 2}}{2 c a^{4}\left(x+\frac{1}{a}\right)^{3}}-\frac{3\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{5 / 2}}{4 c a^{3}\left(x+\frac{1}{a}\right)^{2}}-\frac{17\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{3 / 2}}{24 c a} \\
& -\frac{17 \arctan \left(\frac{\sqrt{a^{2}} x}{\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}}\right.}{16 c} x
\end{aligned}
$$

Problem 74: Unable to integrate problem.

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)}(-a c x+c)^{7 / 2} \mathrm{~d} x
$$

Optimal(type 5, 65 leaves, 3 steps):

$$
-\frac{2^{1+\frac{n}{2}}(-a c x+c)^{9 / 2} \text { hypergeom }\left(\left[-\frac{n}{2}, \frac{9}{2}-\frac{n}{2}\right],\left[\frac{11}{2}-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right)}{a c(9-n)(-a x+1)^{\frac{n}{2}}}
$$

Result(type 8, 19 leaves):

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)}(-a c x+c)^{7 / 2} \mathrm{~d} x
$$

Problem 75: Unable to integrate problem.

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)} \sqrt{-a c x+c} \mathrm{~d} x
$$

Optimal(type 5, 65 leaves, 3 steps):

$$
-\frac{2^{1+\frac{n}{2}}(-a c x+c)^{3 / 2} \text { hypergeom }\left(\left[-\frac{n}{2}, \frac{3}{2}-\frac{n}{2}\right],\left[\frac{5}{2}-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right)}{a c(3-n)(-a x+1)^{\frac{n}{2}}}
$$

Result(type 8, 19 leaves):
$\int \mathrm{e}^{n \operatorname{arctanh}(a x)} \sqrt{-a c x+c} \mathrm{~d} x$

Problem 76: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{\sqrt{-a c x+c}} \mathrm{~d} x
$$

Optimal(type 5, 65 leaves, 3 steps):

$$
-\frac{2^{1+\frac{n}{2}} \text { hypergeom }\left(\left[-\frac{n}{2}, \frac{1}{2}-\frac{n}{2}\right],\left[\frac{3}{2}-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right) \sqrt{-a c x+c}}{a c(1-n)(-a x+1)^{\frac{n}{2}}}
$$

Result(type 8, 19 leaves):

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{\sqrt{-a c x+c}} \mathrm{~d} x
$$

Problem 77: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{(-a c x+c)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 62 leaves, 3 steps):

$$
\frac{2^{1+\frac{n}{2}} \text { hypergeom }\left(\left[-\frac{n}{2},-\frac{3}{2}-\frac{n}{2}\right],\left[-\frac{1}{2}-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right)}{a c(3+n)(-a x+1)^{\frac{n}{2}}(-a c x+c)^{3 / 2}}
$$

Result(type 8, 19 leaves):

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{(-a c x+c)^{5 / 2}} \mathrm{~d} x
$$

Problem 102: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{1+x}}{\sqrt{-x^{2}+1}} d x
$$

Optimal(type 2, 9 leaves, 2 steps):

$$
-2 \sqrt{1-x}
$$

Result(type 2, 19 leaves):

$$
\frac{2(-1+x) \sqrt{1+x}}{\sqrt{-x^{2}+1}}
$$

Problem 103: Result more than twice size of optimal antiderivative.

$$
\int \frac{1+x}{\sqrt{-x^{2}+1} \sqrt{1-x}} \mathrm{~d} x
$$

Optimal(type 3, 24 leaves, 5 steps):

$$
2 \operatorname{arctanh}\left(\frac{\sqrt{1+x} \sqrt{2}}{2}\right) \sqrt{2}-2 \sqrt{1+x}
$$

Result (type 3, 51 leaves):

$$
-\frac{2 \sqrt{-x^{2}+1} \sqrt{1-x}\left(\operatorname{arctanh}\left(\frac{\sqrt{1+x} \sqrt{2}}{2}\right) \sqrt{2}-\sqrt{1+x}\right)}{(-1+x) \sqrt{1+x}}
$$

Problem 118: Unable to integrate problem.

$$
\int \frac{(-a c x+c)^{p}}{\mathrm{e}^{2 p \operatorname{arctanh}(a x)}} \mathrm{d} x
$$

Optimal(type 5, 59 leaves, 3 steps):

$$
-\frac{(-a x+1)^{p}(-a c x+c)^{1+p} \text { hypergeom }\left([p, 1+2 p],[2+2 p],-\frac{a x}{2}+\frac{1}{2}\right)}{2^{p} a c(1+2 p)}
$$

Result(type 8, 22 leaves):

$$
\int \frac{(-a c x+c)^{p}}{\mathrm{e}^{2 p \operatorname{arctanh}(a x)}} \mathrm{d} x
$$

Problem 119: Unable to integrate problem.

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)}(-a c x+c)^{p} \mathrm{~d} x
$$

Optimal(type 5, 72 leaves, 3 steps):

$$
-\frac{2^{1+\frac{n}{2}}(-a c x+c)^{1+p} \text { hypergeom }\left(\left[-\frac{n}{2}, 1-\frac{n}{2}+p\right],\left[2-\frac{n}{2}+p\right],-\frac{a x}{2}+\frac{1}{2}\right)}{a c(2-n+2 p)(-a x+1)^{\frac{n}{2}}}
$$

Result (type 8, 19 leaves):

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)}(-a c x+c)^{p} \mathrm{~d} x
$$

Problem 121: Unable to integrate problem.

$$
\int \frac{(a x+1)\left(c-\frac{c}{a x}\right)^{p}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Optimal(type 6, 56 leaves, 3 steps):

$$
\frac{\left(c-\frac{c}{a x}\right)^{p} x \text { AppellF1 }\left(1-p, \frac{1}{2}-p,-\frac{1}{2}, 2-p, a x,-a x\right)}{(1-p)(-a x+1)^{p}}
$$

Result(type 8, 33 leaves):

$$
\int \frac{(a x+1)\left(c-\frac{c}{a x}\right)^{p}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Problem 128: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{3}\left(c-\frac{c}{a x}\right)^{2}}{\left(-x^{2} a^{2}+1\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 63 leaves, 9 steps):

$$
-\frac{c^{2} \arcsin (a x)}{a}-\frac{c^{2} \operatorname{arctanh}\left(\sqrt{-x^{2} a^{2}+1}\right)}{a}-\frac{c^{2}(-a x+1) \sqrt{-x^{2} a^{2}+1}}{x a^{2}}
$$

Result(type 3, 132 leaves):

$$
-\frac{c^{2}}{a^{2} x \sqrt{-x^{2} a^{2}+1}}+\frac{c^{2} x}{\sqrt{-x^{2} a^{2}+1}}+\frac{c^{2}}{a \sqrt{-x^{2} a^{2}+1}}-\frac{c^{2} \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2} a^{2}+1}}\right)}{a}-\frac{c^{2} \arctan \left(\frac{\sqrt{a^{2} x}}{\sqrt{-x^{2} a^{2}+1}}\right)}{\sqrt{a^{2}}}-\frac{c^{2} a x^{2}}{\sqrt{-x^{2} a^{2}+1}}
$$

Problem 131: Unable to integrate problem.

$$
\int \frac{(a x+1)^{4}\left(c-\frac{c}{a x}\right)^{p}}{\left(-x^{2} a^{2}+1\right)^{2}} \mathrm{~d} x
$$

Optimal(type 5, 95 leaves, 7 steps):

$$
-\frac{c(5-p)\left(c-\frac{c}{a x}\right)^{-1+p}}{a(1-p)}+c\left(c-\frac{c}{a x}\right)^{-1+p} x+\frac{(4-p)\left(c-\frac{c}{a x}\right)^{p} \text { hypergeom }\left([1, p],[1+p], 1-\frac{1}{a x}\right)}{a p}
$$

Result(type 8, 35 leaves):

$$
\int \frac{(a x+1)^{4}\left(c-\frac{c}{a x}\right)^{p}}{\left(-x^{2} a^{2}+1\right)^{2}} \mathrm{~d} x
$$

Problem 133: Unable to integrate problem.

$$
\int \frac{\left(c-\frac{c}{a x}\right)^{p} \sqrt{-x^{2} a^{2}+1}}{a x+1} \mathrm{~d} x
$$

Optimal (type 6, 56 leaves, 3 steps):

$$
\frac{\left(c-\frac{c}{a x}\right)^{p} x \operatorname{AppellF1}\left(1-p,-\frac{1}{2}-p, \frac{1}{2}, 2-p, a x,-a x\right)}{(1-p)(-a x+1)^{p}}
$$

Result(type 8, 35 leaves):

$$
\int \frac{\left(c-\frac{c}{a x}\right)^{p} \sqrt{-x^{2} a^{2}+1}}{a x+1} \mathrm{~d} x
$$

Problem 134: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{-x^{2} a^{2}+1}}{(a x+1)\left(c-\frac{c}{a x}\right)^{3}} \mathrm{~d} x
$$

Optimal(type 3, 84 leaves, 7 steps):

$$
-\frac{(a x+1)^{2}}{3 a c^{3}\left(-x^{2} a^{2}+1\right)^{3 / 2}}-\frac{2 \arcsin (a x)}{a c^{3}}+\frac{8(a x+1)}{3 a c^{3} \sqrt{-x^{2} a^{2}+1}}+\frac{\sqrt{-x^{2} a^{2}+1}}{a c^{3}}
$$

Result(type 3, 241 leaves):
$\frac{5\left(-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)\right)^{3 / 2}}{4 a^{3} c^{3}\left(x-\frac{1}{a}\right)^{2}}+$

$\left.\frac{17 \arctan \left(\frac{\sqrt{a^{2}} x}{\sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)}}\right.}{8 c^{3} \sqrt{a^{2}}}\right)$

$$
+\frac{\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}}{8 a c^{3}}+\frac{\arctan \left(\frac{\sqrt{a^{2}} x}{\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}}\right)}{8 c^{3} \sqrt{a^{2}}}+\frac{\left(-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)\right)^{3 / 2}}{6 a^{4} c^{3}\left(x-\frac{1}{a}\right)^{3}}
$$

Problem 135: Unable to integrate problem.

$$
\int \frac{\left(c-\frac{c}{a x}\right)^{p}\left(-x^{2} a^{2}+1\right)}{(a x+1)^{2}} \mathrm{~d} x
$$

Optimal(type 5, 114 leaves, 8 steps):

$$
-\frac{\left(c-\frac{c}{a x}\right)^{2+p} x}{c^{2}}-\frac{\left(c-\frac{c}{a x}\right)^{2+p} \text { hypergeom }\left([1,2+p],[3+p], \frac{a-\frac{1}{x}}{2 a}\right)}{2 a c^{2}(2+p)}+\frac{\left(c-\frac{c}{a x}\right)^{2+p} \text { hypergeom }\left([1,2+p],[3+p], 1-\frac{1}{a x}\right)}{a c^{2}}
$$

Result(type 8, 33 leaves):

$$
\int \frac{\left(c-\frac{c}{a x}\right)^{p}\left(-x^{2} a^{2}+1\right)}{(a x+1)^{2}} \mathrm{~d} x
$$

Problem 137: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(-x^{2} a^{2}+1\right)^{3 / 2}}{(a x+1)^{3}\left(c-\frac{c}{a x}\right)} d x
$$

Optimal(type 3, 61 leaves, 5 steps):

$$
-\frac{2 \arcsin (a x)}{c a}-\frac{(-a x+1)^{2}}{a c \sqrt{-x^{2} a^{2}+1}}-\frac{2 \sqrt{-x^{2} a^{2}+1}}{c a}
$$

Result(type 3, 291 leaves):
$\frac{\left(-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)\right)^{3 / 2}}{24 c a}-\frac{\sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)} x}{16 c}-\frac{\arctan \left(\frac{\sqrt{a^{2} x}}{\sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)}}\right)}{16 c \sqrt{a^{2}}}$

$$
\begin{aligned}
& -\frac{\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{5 / 2}}{2 c a^{4}\left(x+\frac{1}{a}\right)^{3}}-\frac{5\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{5 / 2}}{4 c a^{3}\left(x+\frac{1}{a}\right)^{2}}-\frac{31\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{3 / 2}}{24 c a} \\
& -\frac{31 \sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)} x}{16 c}-\frac{31 \arctan \left(\frac{\sqrt{a^{2} x}}{\left.\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}\right)}\right.}{16 c \sqrt{a^{2}}}
\end{aligned}
$$

Problem 138: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(-x^{2} a^{2}+1\right)^{3 / 2}}{(a x+1)^{3}\left(c-\frac{c}{a x}\right)^{5}} \mathrm{~d} x
$$

Optimal(type 3, 111 leaves, 8 steps):

$$
-\frac{(a x+1)^{2}}{5 a c^{5}\left(-x^{2} a^{2}+1\right)^{5 / 2}}+\frac{22(a x+1)}{15 a c^{5}\left(-x^{2} a^{2}+1\right)^{3 / 2}}+\frac{2 \arcsin (a x)}{a c^{5}}-\frac{2(23 a x+30)}{15 a c^{5} \sqrt{-x^{2} a^{2}+1}}-\frac{\sqrt{-x^{2} a^{2}+1}}{a c^{5}}
$$

Result(type 3, 467 leaves):

$$
\begin{aligned}
& \frac{\left(-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)\right)^{5 / 2}}{40 a^{6} c^{5}\left(x-\frac{1}{a}\right)^{5}}+\frac{7\left(-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)\right)^{5 / 2}}{48 a^{5} c^{5}\left(x-\frac{1}{a}\right)^{4}}+\frac{31\left(-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)\right)^{5 / 2}}{48 a^{4} c^{5}\left(x-\frac{1}{a}\right)^{3}} \\
& -\frac{139\left(-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)\right)^{5 / 2}}{96 a^{3} c^{5}\left(x-\frac{1}{a}\right)^{2}}-\frac{187\left(-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)\right)^{3 / 2}}{128 a c^{5}}+\frac{561 \sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right) x}}{256 c^{5}}
\end{aligned}
$$

$$
+\frac{561 \arctan \left(\frac{\sqrt{a^{2}} x}{\sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)}}\right)}{256 c^{5} \sqrt{a^{2}}}-\frac{\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{5 / 2}}{32 a^{4} c^{5}\left(x+\frac{1}{a}\right)^{3}}-\frac{9\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{5 / 2}}{64 a^{3} c^{5}\left(x+\frac{1}{a}\right)^{2}}
$$

$$
\begin{aligned}
& -\frac{49\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{3 / 2}}{384 a c^{5}}-\frac{49 \sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)} x}{256 c^{5}}- \\
& \text { Problem 140: Result more than twice size of optimal antiderivative. } \\
& \int \frac{(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right) \sqrt{c-\frac{c}{a x}}} \mathrm{~d} x
\end{aligned}
$$

Optimal(type 3, 61 leaves, 8 steps):

$$
-\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}}}{\sqrt{c}}\right)}{a \sqrt{c}}+\frac{5}{a \sqrt{c-\frac{c}{a x}}}-\frac{x}{\sqrt{c-\frac{c}{a x}}}
$$

Result(type 3, 196 leaves):

$$
\begin{aligned}
& -\frac{1}{2 \sqrt{(a x-1) x} c a^{3 / 2}(a x-1)^{2}}\left(\sqrt { \frac { c ( a x - 1 ) } { a x } } x \left(10 a^{7 / 2} \sqrt{(a x-1) x} x^{2}-8 a^{5 / 2}((a x-1) x)^{3 / 2}+5 \ln \left(\frac{2 \sqrt{(a x-1) x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) x^{2} a^{3}\right.\right. \\
& \left.\left.-20 a^{5 / 2} \sqrt{(a x-1) x} x-10 \ln \left(\frac{2 \sqrt{(a x-1) x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) x a^{2}+10 \sqrt{(a x-1) x} a^{3} / 2+5 \ln \left(\frac{2 \sqrt{(a x-1) x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) a\right)\right)
\end{aligned}
$$

Problem 146: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(c-\frac{c}{a x}\right)^{3 / 2}\left(-x^{2} a^{2}+1\right)}{(a x+1)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 94 leaves, 11 steps):

$$
-\left(c-\frac{c}{a x}\right)^{3 / 2} x+\frac{7 c^{3 / 2} \operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}}}{\sqrt{c}}\right)}{a}-\frac{8 c^{3 / 2} \operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}} \sqrt{2}}{2 \sqrt{c}}\right) \sqrt{2}}{a}+\frac{c \sqrt{c-\frac{c}{a x}}}{a}
$$

Result(type 3, 222 leaves):

$$
\begin{aligned}
& -\frac{1}{2 x a \sqrt{(a x-1) x} \sqrt{\frac{1}{a}}}\left(\sqrt { \frac { c ( a x - 1 ) } { a x } } c \left(5 \sqrt{a} \ln \left(\frac{2 \sqrt{a x^{2}-x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) x^{2} \sqrt{\frac{1}{a}}-12 \sqrt{a} \ln \left(\frac{2 \sqrt{(a x-1) x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) x^{2} \sqrt{\frac{1}{a}}\right.\right. \\
& \left.\left.\quad-10 a \sqrt{a x^{2}-x} x^{2} \sqrt{\frac{1}{a}}+8 a \sqrt{(a x-1) x} x^{2} \sqrt{\frac{1}{a}}-8 \sqrt{2} \ln \left(\frac{2 \sqrt{2} \sqrt{\frac{1}{a} \sqrt{(a x-1) x} a-3 a x+1}}{a x+1}\right) x^{2}+4\left(a x^{2}-x\right)^{3 / 2} \sqrt{\frac{1}{a}}\right)\right)
\end{aligned}
$$

Problem 147: Result more than twice size of optimal antiderivative.

$$
\int \frac{-x^{2} a^{2}+1}{(a x+1)^{2}\left(c-\frac{c}{a x}\right)^{9 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 142 leaves, 13 steps):

$$
\frac{6}{5 a c^{2}\left(c-\frac{c}{a x}\right)^{5 / 2}}+\frac{11}{6 a c^{3}\left(c-\frac{c}{a x}\right)^{3 / 2}}-\frac{x}{c^{2}\left(c-\frac{c}{a x}\right)^{5 / 2}}-\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}}}{\sqrt{c}}\right)}{a c^{9 / 2}}-\frac{\operatorname{arctanh}\left(\frac{\left.\sqrt{c-\frac{c}{a x} \sqrt{2}}\right) \sqrt{2}}{2 \sqrt{c}}\right)}{8 a c^{9 / 2}}+\frac{21}{4 a c^{4} \sqrt{c-\frac{c}{a x}}}
$$

Result(type 3, 627 leaves):

$$
\frac{1}{240 a^{13 / 2} \sqrt{(a x-1) x} c^{5}(a x-1)^{4} \sqrt{\frac{1}{a}}}\left(\sqrt { \frac { c ( a x - 1 ) } { a x } } x \left(-1260 \sqrt{\frac{1}{a}} a^{21 / 2} \sqrt{(a x-1) x} x^{4}\right.\right.
$$

$$
+15 \ln \left(\frac{2 \sqrt{2} \sqrt{\frac{1}{a}} \sqrt{(a x-1) x} a-3 a x+1}{a x+1}\right) a^{19 / 2} \sqrt{2} x^{4}+1020 \sqrt{\frac{1}{a}} a^{19 / 2}((a x-1) x)^{3 / 2} x^{2}+5040 \sqrt{\frac{1}{a}} a^{19} / 2 \sqrt{(a x-1) x} x^{3}
$$

$$
-60 \ln \left(\frac{2 \sqrt{2} \sqrt{\frac{1}{a}} \sqrt{(a x-1) x} a-3 a x+1}{a x+1}\right) a^{17 / 2} \sqrt{2} x^{3}-1792 \sqrt{\frac{1}{a}} a^{17 / 2}((a x-1) x)^{3 / 2} x-7560 \sqrt{\frac{1}{a}} a^{17 / 2} \sqrt{(a x-1) x} x^{2}
$$

$$
+90 \ln \left(\frac{2 \sqrt{2} \sqrt{\frac{1}{a}} \sqrt{(a x-1) x} a-3 a x+1}{a x+1}\right) a^{15 / 2} \sqrt{2} x^{2}+820 a^{15 / 2}((a x-1) x)^{3 / 2} \sqrt{\frac{1}{a}}
$$

$$
-600 \sqrt{\frac{1}{a}} \ln \left(\frac{2 \sqrt{(a x-1) x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) x^{4} a^{10}+5040 \sqrt{\frac{1}{a}} a^{15 / 2} \sqrt{(a x-1) x} x
$$

$-60 \ln \left(\frac{2 \sqrt{2} \sqrt{\frac{1}{a}} \sqrt{(a x-1) x} a-3 a x+1}{a x+1}\right) a^{13 / 2} \sqrt{2} x+2400 \sqrt{\frac{1}{a}} \ln \left(\frac{2 \sqrt{(a x-1) x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) x^{3} a^{9}-1260 \sqrt{(a x-1) x} a^{13} / 2 \sqrt{\frac{1}{a}}$
$+15 \sqrt{2} \ln \left(\frac{2 \sqrt{2} \sqrt{\frac{1}{a}} \sqrt{(a x-1) x} a-3 a x+1}{a x+1}\right) a^{11 / 2}-3600 \sqrt{\frac{1}{a}} \ln \left(\frac{2 \sqrt{(a x-1) x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) x^{2} a^{8}$
$\left.\left.+2400 \sqrt{\frac{1}{a}} \ln \left(\frac{2 \sqrt{(a x-1) x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) x a^{7}-600 \ln \left(\frac{2 \sqrt{(a x-1) x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) a^{6} \sqrt{\frac{1}{a}}\right)\right)$

Problem 159: Result more than twice size of optimal antiderivative.

$$
\int \frac{x \sqrt{c-\frac{c}{a x}}\left(-x^{2} a^{2}+1\right)}{(a x+1)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 97 leaves, 11 steps):

$$
-\frac{23 \operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}}}{\sqrt{c}}\right) \sqrt{c}}{4 a^{2}}+\frac{4 \operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}} \sqrt{2}}{2 \sqrt{c}}\right) \sqrt{2} \sqrt{c}}{a^{2}}+\frac{9 x \sqrt{c-\frac{c}{a x}}}{4 a}-\frac{x^{2} \sqrt{c-\frac{c}{a x}}}{2}
$$

Result(type 3, 214 leaves):

$$
\begin{aligned}
& \frac{1}{8 \sqrt{(a x-1) x} a^{7 / 2} \sqrt{\frac{1}{a}}}\left(\sqrt { \frac { c ( a x - 1 ) } { a x } } x \left(-4 \sqrt{\frac{1}{a}} \sqrt{a x^{2}-x} a^{7 / 2} x+2 \sqrt{a x^{2}-x} a^{5} / 2 \sqrt{\frac{1}{a}}+16 \sqrt{(a x-1) x} a^{5 / 2} \sqrt{\frac{1}{a}}\right.\right. \\
& -16 \sqrt{2} \ln \left(\frac{2 \sqrt{2} \sqrt{\frac{1}{a}} \sqrt{(a x-1) x} a-3 a x+1}{a x+1}\right) a^{3 / 2}+\ln \left(\frac{2 \sqrt{a x^{2}-x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) a^{2} \sqrt{\frac{1}{a}} \\
& \left.-24 \ln \left(\frac{2 \sqrt{(a x-1) x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) a^{2} \sqrt{\frac{1}{a}}\right)
\end{aligned}
$$

[^3]$$
\int \frac{\sqrt{c-\frac{c}{a x}}\left(-x^{2} a^{2}+1\right)}{(a x+1)^{2} x^{2}} \mathrm{~d} x
$$

Optimal(type 3, 67 leaves, 8 steps):

$$
\frac{2 a\left(c-\frac{c}{a x}\right)^{3 / 2}}{3 c}-4 a \operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}} \sqrt{2}}{2 \sqrt{c}}\right) \sqrt{2} \sqrt{c}+4 a \sqrt{c-\frac{c}{a x}}
$$

Result(type 3, 243 leaves):

$$
\begin{aligned}
& -\frac{1}{3 x^{2} \sqrt{(a x-1) x} \sqrt{\frac{1}{a}}}\left(\sqrt { \frac { c ( a x - 1 ) } { a x } } \left(9 a^{3 / 2} \ln \left(\frac{2 \sqrt{a x^{2}-x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) x^{3} \sqrt{\frac{1}{a}}-9 a^{3} / 2 \ln \left(\frac{2 \sqrt{(a x-1) x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) x^{3} \sqrt{\frac{1}{a}}\right.\right. \\
& \quad-18 a^{2} \sqrt{a x^{2}-x} x^{3} \sqrt{\frac{1}{a}}+6 a^{2} \sqrt{(a x-1) x} x^{3} \sqrt{\frac{1}{a}}-6 a \sqrt{2} \ln \left(\frac{2 \sqrt{2} \sqrt{\frac{1}{a}} \sqrt{(a x-1) x} a-3 a x+1}{a x+1}\right) x^{3}+12 a\left(a x^{2}-x\right)^{3 / 2} x \sqrt{\frac{1}{a}} \\
& \left.-2\left(a x^{2}-x\right)^{3 / 2} \sqrt{\frac{1}{a}}\right)
\end{aligned}
$$

Problem 161: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{c-\frac{c}{a x}}\left(-x^{2} a^{2}+1\right)}{(a x+1)^{2} x^{3}} \mathrm{~d} x
$$

Optimal(type 3, 94 leaves, 9 steps):

$$
-\frac{2 a^{2}\left(c-\frac{c}{a x}\right)^{3 / 2}}{3 c}-\frac{2 a^{2}\left(c-\frac{c}{a x}\right)^{5 / 2}}{5 c^{2}}+4 a^{2} \operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a x}} \sqrt{2}}{2 \sqrt{c}}\right) \sqrt{2} \sqrt{c}-4 a^{2} \sqrt{c-\frac{c}{a x}}
$$

Result(type 3, 269 leaves):
$\frac{1}{15 x^{3} \sqrt{(a x-1) x} \sqrt{\frac{1}{a}}}\left(\sqrt{\frac{c(a x-1)}{a x}}\left(45 a^{5} / 2 \ln \left(\frac{2 \sqrt{a x^{2}-x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) x^{4} \sqrt{\frac{1}{a}}-45 a^{5} / 2 \ln \left(\frac{2 \sqrt{(a x-1) x} \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right) x^{4} \sqrt{\frac{1}{a}}\right.\right.$
$-90 a^{3} \sqrt{a x^{2}-x} x^{4} \sqrt{\frac{1}{a}}+30 a^{3} \sqrt{(a x-1) x} x^{4} \sqrt{\frac{1}{a}}-30 a^{2} \sqrt{2} \ln \left(\frac{2 \sqrt{2} \sqrt{\frac{1}{a}} \sqrt{(a x-1) x} a-3 a x+1}{a x+1}\right) x^{4}+60 a^{2}\left(a x^{2}-x\right)^{3 / 2} x^{2} \sqrt{\frac{1}{a}}$
$\left.\left.-16 a\left(a x^{2}-x\right)^{3 / 2} x \sqrt{\frac{1}{a}}+6\left(a x^{2}-x\right)^{3 / 2} \sqrt{\frac{1}{a}}\right)\right)$

Problem 165: Unable to integrate problem.

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)}\left(c-\frac{c}{a x}\right)^{p} \mathrm{~d} x
$$

Optimal(type 6, 60 leaves, 3 steps):

$$
\frac{\left(c-\frac{c}{a x}\right)^{p} x \operatorname{AppellF1}\left(1-p, \frac{n}{2}-p,-\frac{n}{2}, 2-p, a x,-a x\right)}{(1-p)(-a x+1)^{p}}
$$

Result(type 8, 23 leaves):

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)}\left(c-\frac{c}{a x}\right)^{p} \mathrm{~d} x
$$

Problem 166: Unable to integrate problem.

$$
\int \frac{\left(c-\frac{c}{a x}\right)^{p}}{\mathrm{e}^{2 p \operatorname{arctanh}(a x)}} \mathrm{d} x
$$

Optimal(type 6, 54 leaves, 3 steps):

$$
\frac{\left(c-\frac{c}{a x}\right)^{p} x \operatorname{AppellF1}(1-p,-2 p, p, 2-p, a x,-a x)}{(1-p)(-a x+1)^{p}}
$$

Result(type 8, 26 leaves):

$$
\int \frac{\left(c-\frac{c}{a x}\right)^{p}}{\mathrm{e}^{2 p \operatorname{arctanh}(a x)}} \mathrm{d} x
$$

Problem 167: Unable to integrate problem.

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)}\left(c-\frac{c}{a x}\right) \mathrm{d} x
$$

Optimal(type 5, 163 leaves, 6 steps):

$$
\begin{gathered}
\frac{c(-a x+1)^{2-\frac{n}{2}}(a x+1)^{-1+\frac{n}{2}}}{a(2-n)}-\frac{2 c(-a x+1)^{1-\frac{n}{2}}(a x+1)^{-1+\frac{n}{2}} \operatorname{hypergeom}\left(\left[1,-1+\frac{n}{2}\right],\left[\frac{n}{2}\right], \frac{a x+1}{-a x+1}\right)}{a(2-n)} \\
\quad+\frac{2^{\frac{n}{2}} c(1-n)(-a x+1)^{2-\frac{n}{2}} \operatorname{hypergeom}\left(\left[1-\frac{n}{2}, 2-\frac{n}{2}\right],\left[3-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right)}{a(2-n)(4-n)}
\end{gathered}
$$

Result(type 8, 21 leaves):

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)}\left(c-\frac{c}{a x}\right) \mathrm{d} x
$$

Problem 168: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{c-\frac{c}{a x}} \mathrm{~d} x
$$

Optimal(type 5, 97 leaves, 4 steps):

$$
-\frac{(a x+1)^{1+\frac{n}{2}}}{\operatorname{acn}(-a x+1)^{\frac{n}{2}}}-\frac{2^{1+\frac{n}{2}}(1+n)(-a x+1)^{1-\frac{n}{2}} \text { hypergeom }\left(\left[-\frac{n}{2}, 1-\frac{n}{2}\right],\left[2-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right)}{\operatorname{ac(2-n)n}}
$$

Result(type 8, 23 leaves):

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{c-\frac{c}{a x}} \mathrm{~d} x
$$

Problem 169: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{\left(c-\frac{c}{a x}\right)^{2}} \mathrm{~d} x
$$

Optimal(type 5, 121 leaves, 5 steps):

$$
\frac{(3+n)(-a x+1)^{-1-\frac{n}{2}}(a x+1)^{1+\frac{n}{2}}}{a c^{2}(2+n)}-\frac{x(-a x+1)^{-1-\frac{n}{2}}(a x+1)^{1+\frac{n}{2}}}{c^{2}}-\frac{2^{1+\frac{n}{2}}(2+n) \text { hypergeom }\left(\left[-\frac{n}{2},-\frac{n}{2}\right],\left[1-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right)}{a c^{2} n(-a x+1)^{\frac{n}{2}}}
$$

Result(type 8, 23 leaves):

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{\left(c-\frac{c}{a x}\right)^{2}} \mathrm{~d} x
$$

Problem 170: Unable to integrate problem.

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)}\left(c-\frac{c}{a x}\right)^{3 / 2} \mathrm{~d} x
$$

Optimal(type 6, 42 leaves, 3 steps):

$$
-\frac{2\left(c-\frac{c}{a x}\right)^{3 / 2} x \operatorname{AppellF} 1\left(-\frac{1}{2},-\frac{3}{2}+\frac{n}{2},-\frac{n}{2}, \frac{1}{2}, a x,-a x\right)}{(-a x+1)^{3 / 2}}
$$

Result(type 8, 23 leaves):

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)}\left(c-\frac{c}{a x}\right)^{3 / 2} \mathrm{~d} x
$$

Problem 180: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{-x^{2} a^{2}+1}}{(a x+1)\left(c-\frac{c}{x^{2} a^{2}}\right)^{3}} d x
$$

Optimal(type 3, 114 leaves, 7 steps):

$$
-\frac{a^{4} x^{5}(-a x+1)}{5 c^{3}\left(-x^{2} a^{2}+1\right)^{5 / 2}}+\frac{a^{2} x^{3}(-6 a x+5)}{15 c^{3}\left(-x^{2} a^{2}+1\right)^{3 / 2}}+\frac{\arcsin (a x)}{a c^{3}}-\frac{x(-8 a x+5)}{5 c^{3} \sqrt{-x^{2} a^{2}+1}}+\frac{16 \sqrt{-x^{2} a^{2}+1}}{5 a c^{3}}
$$

Result(type 3, 355 leaves):

$$
\begin{aligned}
& \frac{\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{3 / 2}}{40 a^{5} c^{3}\left(x+\frac{1}{a}\right)^{4}}-\frac{43\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{3 / 2}}{240 a^{4} c^{3}\left(x+\frac{1}{a}\right)^{3}}+\frac{\left(-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)\right)^{3 / 2}}{48 a^{4} c^{3}\left(x-\frac{1}{a}\right)^{3}} \\
& \quad+\frac{\left(-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)\right)^{3 / 2}}{4 a^{3} c^{3}\left(x-\frac{1}{a}\right)^{2}}+\frac{19 \sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)}}{32 a c^{3}}-\frac{19 \arctan \left(\frac{\sqrt{a^{2}} x}{\left.\sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)}\right)}\right.}{32 c^{3} \sqrt{a^{2}}}
\end{aligned}
$$

$$
+\frac{15\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{3 / 2}}{16 a^{3} c^{3}\left(x+\frac{1}{a}\right)^{2}}+\frac{51 \sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}}{32 a c^{3}}+\frac{51 \arctan \left(\frac{\sqrt{a^{2}} x}{\left.\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}\right)}\right.}{32 c^{3} \sqrt{a^{2}}}
$$

Problem 184: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(c-\frac{c}{x^{2} a^{2}}\right)^{2}\left(-x^{2} a^{2}+1\right)^{3 / 2}}{(a x+1)^{3}} \mathrm{~d} x
$$

Optimal(type 3, 109 leaves, 10 steps):

$$
-\frac{c^{2}\left(-x^{2} a^{2}+1\right)^{3 / 2}}{3 a^{4} x^{3}}+\frac{3 c^{2}\left(-x^{2} a^{2}+1\right)^{3 / 2}}{2 a^{3} x^{2}}-\frac{3 c^{2} \arcsin (a x)}{a}-\frac{c^{2} \operatorname{arctanh}\left(\sqrt{-x^{2} a^{2}+1}\right)}{2 a}-\frac{c^{2}(-a x+6) \sqrt{-x^{2} a^{2}+1}}{2 x a^{2}}
$$

Result(type 3, 298 leaves):

$$
\begin{aligned}
& -\frac{c^{2}\left(-x^{2} a^{2}+1\right)^{5 / 2}}{3 a^{4} x^{3}}-\frac{10 c^{2}\left(-x^{2} a^{2}+1\right)^{5} / 2}{3 a^{2} x}-\frac{10 c^{2}\left(-x^{2} a^{2}+1\right)^{3 / 2} x}{3}-5 c^{2} \sqrt{-x^{2} a^{2}+1} x-\frac{5 c^{2} \arctan \left(\frac{\sqrt{a^{2} x}}{\left.\sqrt{-x^{2} a^{2}+1}\right)}\right.}{\sqrt{a^{2}}}+\frac{3 c^{2}\left(-x^{2} a^{2}+1\right)^{5} / 2}{2 a^{3} x^{2}} \\
& \quad+\frac{c^{2}\left(-x^{2} a^{2}+1\right)^{3 / 2}}{6 a}+\frac{c^{2} \sqrt{-x^{2} a^{2}+1}}{2 a}-\frac{c^{2} \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2} a^{2}+1}}\right)}{2 a}+\frac{4 c^{2}\left(-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)\right)^{3 / 2}}{3 a} \\
& +2 c^{2} \sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}
\end{aligned}
$$

Problem 189: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{9 / 2}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 3, 396 leaves, 16 steps):

$$
\begin{aligned}
& \frac{295 a^{4}\left(c-\frac{c}{x^{2} a^{2}}\right)^{9 / 2} x^{5}}{1344(-a x+1)^{4}}-\frac{501 a^{8}\left(c-\frac{c}{x^{2} a^{2}}\right)^{9 / 2} x^{9}}{128(-a x+1)^{4}(a x+1)^{4}}+\frac{373 a^{7}\left(c-\frac{c}{x^{2} a^{2}}\right)^{9 / 2} x^{8}}{192(-a x+1)^{4}(a x+1)^{3}}+\frac{501 a^{6}\left(c-\frac{c}{x^{2} a^{2}}\right)^{9 / 2} x^{7}}{640(-a x+1)^{4}(a x+1)^{2}}+\frac{661 a^{5}\left(c-\frac{c}{x^{2} a^{2}}\right)^{9 / 2} x^{6}}{1680(-a x+1)^{4}(a x+1)} \\
& -\frac{127 a^{3}\left(c-\frac{c}{x^{2} a^{2}}\right)^{9 / 2} x^{4}(a x+1)}{420(-a x+1)^{4}}+\frac{71 a^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{9 / 2} x^{3}(a x+1)}{336(-a x+1)^{3}}-\frac{a\left(c-\frac{c}{x^{2} a^{2}}\right)^{9 / 2} x^{2}(a x+1)}{28(-a x+1)^{2}}-\frac{\left(c-\frac{c}{x^{2} a^{2}}\right)^{9 / 2} x(a x+1)}{8(-a x+1)} \\
& \quad+\frac{2 a^{8}\left(c-\frac{c}{x^{2} a^{2}}\right)^{9 / 2} x^{9} \arcsin (a x)}{(-a x+1)^{9 / 2}(a x+1)^{9 / 2}}+\frac{245 a^{8}\left(c-\frac{c}{x^{2} a^{2}}\right)^{9 / 2} x^{9} \operatorname{arctanh}(\sqrt{-a x+1} \sqrt{a x+1})}{128(-a x+1)^{9 / 2}(a x+1)^{9 / 2}}
\end{aligned}
$$

Result(type 3, 964 leaves):

$$
-\frac{1}{40320\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{9 / 2} a^{2} c \sqrt{-\frac{c}{a^{2}}}}\left(( \frac { c ( x ^ { 2 } a ^ { 2 } - 1 ) } { x ^ { 2 } a ^ { 2 } } ) ^ { 9 / 2 } x \left(-5040 a^{4}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{11 / 2} \sqrt{-\frac{c}{a^{2}}}\right.\right.
$$

$$
+77175 c^{6} \ln \left(\frac{2\left(\sqrt{-\frac{c}{a^{2}}} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} a^{2}-c\right)}{x a^{2}}\right) x^{8}+22050 c^{11 / 2} \ln \left(\frac{\sqrt{c} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}}+c x}{\sqrt{c}}\right) a x^{8} \sqrt{-\frac{c}{a^{2}}}+58590 c^{11 / 2} \ln (x \sqrt{c}
$$

$$
\left.+\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}}\right) a x^{8} \sqrt{-\frac{c}{a^{2}}}-11760 a^{7} c^{3}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2} x^{9} \sqrt{-\frac{c}{a^{2}}}-31248 a^{7} c^{3} x^{9}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} \sqrt{-\frac{c}{a^{2}}}
$$

$$
+15435 a^{6} c^{3}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} x^{8} \sqrt{-\frac{c}{a^{2}}}+14700 a^{5} c^{4}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2} x^{9} \sqrt{-\frac{c}{a^{2}}}+39060 a^{5} c^{4} x^{9}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} \sqrt{-\frac{c}{a^{2}}}
$$

$$
-25725 a^{4} c^{4}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{8} \sqrt{-\frac{c}{a^{2}}}-22050 a^{3} c^{5} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}} x^{9} \sqrt{-\frac{c}{a^{2}}}-58590 a^{3} c^{5} x^{9} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} \sqrt{-\frac{c}{a^{2}}}
$$

$$
+77175 c^{5} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} a^{2} x^{8} \sqrt{-\frac{c}{a^{2}}}-23808 a^{11} x^{9}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{9 / 2} c \sqrt{-\frac{c}{a^{2}}}+8960 a^{10}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{9 / 2} c x^{8} \sqrt{-\frac{c}{a^{2}}}
$$

$$
+8575 a^{10}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{9 / 2} c x^{8} \sqrt{-\frac{c}{a^{2}}}+10080 a^{9} c^{2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{7 / 2} x^{9} \sqrt{-\frac{c}{a^{2}}}+26784 a^{9} c^{2} x^{9}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{7 / 2} \sqrt{-\frac{c}{a^{2}}}
$$

$-11025 a^{8} c^{2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{7 / 2} x^{8} \sqrt{-\frac{c}{a^{2}}}+23808 a^{11}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{11 / 2} x^{7} \sqrt{-\frac{c}{a^{2}}}-17535 a^{10}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{11 / 2} x^{6} \sqrt{-\frac{c}{a^{2}}}$
$-13056 a^{9}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{11 / 2} x^{5} \sqrt{-\frac{c}{a^{2}}}-6510 a^{8}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{11 / 2} x^{4} \sqrt{-\frac{c}{a^{2}}}-6912 a^{7}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{11 / 2} x^{3} \sqrt{-\frac{c}{a^{2}}}$
$\left.\left.-10920 a^{6}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{11 / 2} x^{2} \sqrt{-\frac{c}{a^{2}}}-11520 a^{5}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{11 / 2} x \sqrt{-\frac{c}{a^{2}}}\right)\right)$

Problem 190: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{3 / 2}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 3, 186 leaves, 10 steps):

$$
\begin{aligned}
& -\frac{a\left(c-\frac{c}{x^{2} a^{2}}\right)^{3 / 2} x^{2}}{-a x+1}+\frac{5 a^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{3 / 2} x^{3}}{2(-a x+1)(a x+1)}-\frac{\left(c-\frac{c}{x^{2} a^{2}}\right)^{3 / 2} x(a x+1)}{2(-a x+1)}-\frac{2 a^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{3 / 2} x^{3} \arcsin (a x)}{(-a x+1)^{3 / 2}(a x+1)^{3 / 2}} \\
& -\frac{a^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{3 / 2} x^{3} \operatorname{arctanh}(\sqrt{-a x+1} \sqrt{a x+1})}{2(-a x+1)^{3 / 2}(a x+1)^{3 / 2}}
\end{aligned}
$$

Result(type 3, 453 leaves):

$$
\begin{aligned}
& \frac{1}{6\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} c a^{2} \sqrt{-\frac{c}{a^{2}}}}\left(( \frac { c ( x ^ { 2 } a ^ { 2 } - 1 ) } { x ^ { 2 } a ^ { 2 } } ) ^ { 3 / 2 } x \left(-12 a^{5} x^{3}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} c \sqrt{-\frac{c}{a^{2}}}+12 a^{5}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} x \sqrt{-\frac{c}{a^{2}}}\right.\right. \\
& -4 a^{4}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2} x^{2} c \sqrt{-\frac{c}{a^{2}}}+a^{4}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{2} c \sqrt{-\frac{c}{a^{2}}}-6 a^{3} c^{2} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}} x^{3} \sqrt{-\frac{c}{a^{2}}} \\
& +3 a^{4}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} \sqrt{-\frac{c}{a^{2}}}+18 a^{3} c^{2} x^{3} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} \sqrt{-\frac{c}{a^{2}}}+6 c^{5 / 2} \ln \left(\frac{\sqrt{c} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}}+c x}{x^{2} a} \sqrt{-\frac{c}{a^{2}}}\right.
\end{aligned}
$$

$\left.\left.-18 c^{5 / 2} \ln \left(x \sqrt{c}+\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}}\right) x^{2} a \sqrt{-\frac{c}{a^{2}}}-3 c^{2} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} x^{2} a^{2} \sqrt{-\frac{c}{a^{2}}}-3 c^{3} \ln \left(\frac{2\left(\sqrt{-\frac{c}{a^{2}}} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} a^{2}-c\right)}{x a^{2}}\right) x^{2}\right)\right)$

Problem 192: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2}} d x
$$

Optimal(type 3, 181 leaves, 8 steps):
$\frac{(a x+1)^{2}}{5 a^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x}-\frac{2(-a x+1)(a x+1)^{2}}{3 a^{3}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{2}}+\frac{58(-a x+1)^{2}(a x+1)^{2}}{15 a^{4}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{3}}+\frac{2(-a x+1)^{3}(a x+1)^{2}(43 a x+28)}{15 a^{6}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{5}}$
$-\frac{2(-a x+1)^{5 / 2}(a x+1)^{5 / 2} \arcsin (a x)}{a^{6}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{5}}$
Result(type 3, 465 leaves):
$-\frac{1}{15\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2} x^{5}\left(\frac{c\left(x^{2} a^{2}-1\right)}{x^{2} a^{2}}\right)^{5 / 2} a^{6} c^{7 / 2}}\left(\left(15 c^{7 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2} x^{5} a^{5}\right.\right.$
$-45 x^{4} c^{7 / 2} a^{4}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2}-16 c^{7 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{4} a^{4}-60 x^{3} c^{7 / 2} a^{3}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2}$
$+16 c^{7 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{3} a^{3}+90 c^{7 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2} x^{2} a^{2}+24 c^{7 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{2} a^{2}+30 \ln (x \sqrt{c}$
$\left.+\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}}\right)\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x a^{4} c^{2}+50 c^{7 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2} x a$
$-24 c^{7 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x a-30 \ln \left(x \sqrt{c}+\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}}\right) c^{2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} a^{3}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2}$
$\left.\left.-50 c^{7 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2}-6 c^{7 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2}\right)(a x+1)\right)$

Problem 193: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right)\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 253 leaves, 10 steps):

$$
\begin{gathered}
\left.\frac{(a x+1)^{2}}{7 a^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x}-\frac{2(-a x+1)(a x+1)^{2}}{5 a^{3}\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x^{2}}+\frac{124(-a x+1)^{2}(a x+1)^{2}}{105 a^{4}\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x^{3}}-\frac{782(-a x+1)^{3}(a x+1)^{2}}{105 a^{5}\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x^{4}}-\frac{142(-a x+1)^{4}(a x+1)^{2}}{x^{4}}+\frac{2(-a x+1)^{7 / 2}(a x+1)^{7 / 2} \arcsin (a x)}{x^{2} a^{2}}\right)^{x^{7}\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x^{7}}
\end{gathered}
$$

Result (type 3, 575 leaves):

$$
\begin{aligned}
& -\frac{1}{105\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2} x^{7}\left(\frac{c\left(x^{2} a^{2}-1\right)}{x^{2} a^{2}}\right)^{7 / 2} a^{8} c^{11 / 2}}\left(\left(105 c^{11 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2} x^{7} a^{7}\right.\right. \\
& -553 x^{6} c^{11 / 2} a^{6}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2}+96 c^{11 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} x^{6} a^{6}-392 x^{5} c^{11 / 2} a^{5}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2} \\
& -96 c^{11 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} x^{5} a^{5}+1540 c^{11 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2} x^{4} a^{4}-240 c^{11 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} x^{4} a^{4} \\
& +350 c^{11 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2} x^{3} a^{3}+240 c^{11 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} x^{3} a^{3}+210 \ln (x \sqrt{c} \\
& \left.+\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}}\right)\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} x a^{6} c^{3}-1470 c^{11 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2} x^{2} a^{2} \\
& +180 c^{11 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} x^{2} a^{2}-210 \ln \left(x \sqrt{c}+\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}}\right) c^{3}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} a^{5}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2} \\
& -42 c^{11 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2} x a-180 c^{11 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} x a+462 c^{11 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2} \\
& \left.\left.-30 c^{11 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2}\right)(a x+1)\right)
\end{aligned}
$$

Problem 199: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2}\left(-x^{2} a^{2}+1\right)}{(a x+1)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 329 leaves, 14 steps):

$$
\begin{aligned}
& -\frac{7 a^{6}\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x^{7}}{16(-a x+1)^{3}(a x+1)^{3}}-\frac{3 a^{5}\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x^{6}}{8(-a x+1)^{3}(a x+1)^{2}}+\frac{a\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x^{2}}{15(a x+1)}+\frac{19 a^{4}\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x^{5}}{16(-a x+1)^{3}(a x+1)}-\frac{2 a^{3}\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x^{4}}{3(-a x+1)^{2}(a x+1)} \\
& \quad+\frac{23 a^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x^{3}}{120(-a x+1)(a x+1)}-\frac{\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x(-a x+1)}{6(a x+1)}+\frac{2 a^{6}\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x^{7} \arcsin (a x)}{(-a x+1)^{7 / 2}(a x+1)^{7 / 2}} \\
& \quad-\frac{25 a^{6}\left(c-\frac{c}{x^{2} a^{2}}\right)^{7 / 2} x^{7} \operatorname{arctanh}(\sqrt{-a x+1} \sqrt{a x+1})}{16(-a x+1)^{7 / 2}(a x+1)^{7 / 2}}
\end{aligned}
$$

Result(type 3, 794 leaves):

$$
\begin{aligned}
& -\frac{1}{1680\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{7 / 2} c a^{2} \sqrt{-\frac{c}{a^{2}}}}\left(( \frac { c ( x ^ { 2 } a ^ { 2 } - 1 ) } { x ^ { 2 } a ^ { 2 } } ) ^ { 7 / 2 } x \left(-2016 a^{9} x^{7}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{7 / 2} c \sqrt{-\frac{c}{a^{2}}}+2016 a^{9}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{9 / 2} x^{5} \sqrt{-\frac{c}{a^{2}}}\right.\right. \\
& +480 a^{8}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{7 / 2} x^{6} c \sqrt{-\frac{c}{a^{2}}}-375 a^{8}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{7 / 2} x^{6} c \sqrt{-\frac{c}{a^{2}}}-560 a^{7} c^{2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2} x^{7} \sqrt{-\frac{c}{a^{2}}} \\
& -105 a^{8}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{9 / 2} x^{4} \sqrt{-\frac{c}{a^{2}}}+2352 a^{7} c^{2} x^{7}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} \sqrt{-\frac{c}{a^{2}}}+224 a^{7}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{9 / 2} x^{3} \sqrt{-\frac{c}{a^{2}}} \\
& +525 a^{6} c^{2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} x^{6} \sqrt{-\frac{c}{a^{2}}}+700 a^{5} c^{3}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2} x^{7} \sqrt{-\frac{c}{a^{2}}}-2940 a^{5} c^{3} x^{7}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} \sqrt{-\frac{c}{a^{2}}} \\
& -630 a^{6}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{9 / 2} x^{2} \sqrt{-\frac{c}{a^{2}}}+1050 c^{9 / 2} \ln \left(\frac{\sqrt{c} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}}+c x}{\sqrt{c}}\right) x^{6} a \sqrt{-\frac{c}{a^{2}}}-4410 c^{9} / 2 \ln (x \sqrt{c} \\
& \left.+\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}}\right) x^{6} a \sqrt{-\frac{c}{a^{2}}}-875 a^{4} c^{3}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{6} \sqrt{-\frac{c}{a^{2}}}-1050 a^{3} c^{4} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}} x^{7} \sqrt{-\frac{c}{a^{2}}}
\end{aligned}
$$

$$
\begin{aligned}
& +672 a^{5}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{9 / 2} x \sqrt{-\frac{c}{a^{2}}}+4410 a^{3} c^{4} x^{7} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}} \sqrt{-\frac{c}{a^{2}}}-280 a^{4}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{9 / 2} \sqrt{-\frac{c}{a^{2}}}} \\
& \left.\left.+2625 c^{4} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} x^{6} a^{2} \sqrt{-\frac{c}{a^{2}}}+2625 c^{5} \ln \left(\frac{2\left(\sqrt{\left.-\frac{c}{a^{2}} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} a^{2}-c\right)}\right.}{x a^{2}}\right) x^{6}\right)\right)
\end{aligned}
$$

Problem 200: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2}\left(-x^{2} a^{2}+1\right)}{(a x+1)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 257 leaves, 12 steps):

$$
\begin{aligned}
& \frac{7 a^{4}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{5}}{8(-a x+1)^{2}(a x+1)^{2}}+\frac{a\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{2}}{6(a x+1)}-\frac{2 a^{3}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{4}}{(-a x+1)^{2}(a x+1)}+\frac{7 a^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{3}}{24(-a x+1)(a x+1)}-\frac{\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x(-a x+1)}{4(a x+1)} \\
& -\frac{2 a^{4}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{5} \arcsin (a x)}{(-a x+1)^{5 / 2}(a x+1)^{5 / 2}}+\frac{9 a^{4}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{5} \operatorname{arctanh}(\sqrt{-a x+1} \sqrt{a x+1})}{8(-a x+1)^{5 / 2}(a x+1)^{5 / 2}}
\end{aligned}
$$

Result(type 3, 624 leaves):

$$
\begin{aligned}
& \frac{1}{120\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} c a^{2} \sqrt{-\frac{c}{a^{2}}}}\left(( \frac { c ( x ^ { 2 } a ^ { 2 } - 1 ) } { x ^ { 2 } a ^ { 2 } } ) ^ { 5 / 2 } x \left(-80 a^{7} x^{5}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} c \sqrt{-\frac{c}{a^{2}}}+80 a^{7}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{7 / 2} x^{3} \sqrt{-\frac{c}{a^{2}}}\right.\right. \\
& \quad-48 a^{6}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{5 / 2} x^{4} c \sqrt{-\frac{c}{a^{2}}}-27 a^{6}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{5 / 2} x^{4} c \sqrt{-\frac{c}{a^{2}}}+60 a^{5} c^{2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2} x^{5} \sqrt{-\frac{c}{a^{2}}} \\
& \quad+75 a^{6}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{7 / 2} x^{2} \sqrt{-\frac{c}{a^{2}}}+100 a^{5} c^{2} x^{5}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} \sqrt{-\frac{c}{a^{2}}}-80 a^{5}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{7 / 2} x \sqrt{-\frac{c}{a^{2}}} \\
& \quad+45 c^{2} a^{4}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{4} \sqrt{-\frac{c}{a^{2}}}+90 c^{7 / 2} \ln \left(\frac{\left.\sqrt{c} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}}+c x\right)}{x^{4} a \sqrt{-\frac{c}{a^{2}}}+150 c^{7 / 2} \ln (x \sqrt{c}}\right.
\end{aligned}
$$

$\left.+\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}}\right) x^{4} a \sqrt{-\frac{c}{a^{2}}}-90 a^{3} c^{3} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}} x^{5} \sqrt{-\frac{c}{a^{2}}}-150 a^{3} c^{3} x^{5} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} \sqrt{-\frac{c}{a^{2}}}$

$$
\left.\left.+30 a^{4}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{7 / 2} \sqrt{-\frac{c}{a^{2}}}-135 c^{3} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} x^{4} a^{2} \sqrt{-\frac{c}{a^{2}}}-135 c^{4} \ln \left(\frac{2\left(\sqrt{-\frac{c}{a^{2}} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} a^{2}-c}\right)}{x a^{2}}\right) x^{4}\right)\right)
$$

Problem 201: Result more than twice size of optimal antiderivative.

$$
\int \frac{-x^{2} a^{2}+1}{(a x+1)^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2}} d x
$$

Optimal(type 3, 174 leaves, 8 steps):

$$
\begin{aligned}
& \frac{(-a x+1)^{2}}{a^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x}+\frac{2(-a x+1)^{3}}{5 a^{3}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{2}}-\frac{2(-a x+1)^{3}(a x+1)}{15 a^{4}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{3}}+\frac{2(-a x+1)^{3}(a x+1)^{2}(13 a x+28)}{15 a^{6}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{5}} \\
& \quad+\frac{2(-a x+1)^{5 / 2}(a x+1)^{5 / 2} \arcsin (a x)}{a^{6}\left(c-\frac{c}{x^{2} a^{2}}\right)^{5 / 2} x^{5}}
\end{aligned}
$$

Result(type 3, 465 leaves):

$$
\begin{aligned}
& -\frac{1}{15\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2} x^{5}\left(\frac{c\left(x^{2} a^{2}-1\right)}{x^{2} a^{2}}\right)^{5 / 2} a^{6} c^{7 / 2}}\left(\left(15 c^{7 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2} x^{5} a^{5}\right.\right. \\
& \quad+45 x^{4} c^{7 / 2} a^{4}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2}+16 c^{7 / 2\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{4} a^{4}-60 x^{3} c^{7 / 2} a^{3}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2}} \\
& \quad+16 c^{7 / 2\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{3} a^{3}-90 c^{7 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2} x^{2} a^{2}-24 c^{7 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{2} a^{2}-30 \ln (x \sqrt{c}} \\
& \quad+\sqrt{\left.\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x a^{4} c^{2}+50 c^{7 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2} x a} \\
& \quad-24 c^{7 / 2\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x a-30 \ln \left(x \sqrt{c}+\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}}\right) c^{2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} a^{3}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2}} \\
& \left.\left.\quad+50 c^{7 / 2}\left(\frac{(a x-1)(a x+1) c}{a^{2}}\right)^{3 / 2}+6 c^{7 / 2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2}\right)(a x-1)\right)
\end{aligned}
$$

Problem 206: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2} \sqrt{c-\frac{c}{x^{2} a^{2}}}}{\left(-x^{2} a^{2}+1\right) x^{5}} \mathrm{~d} x
$$

Optimal(type 3, 149 leaves, 10 steps):

$$
-\frac{6 a^{4} \sqrt{c-\frac{c}{x^{2} a^{2}}}}{5}-\frac{\sqrt{c-\frac{c}{x^{2} a^{2}}}}{5 x^{4}}-\frac{a \sqrt{c-\frac{c}{x^{2} a^{2}}}}{2 x^{3}}-\frac{3 a^{2} \sqrt{c-\frac{c}{x^{2} a^{2}}}}{5 x^{2}}-\frac{3 a^{3} \sqrt{c-\frac{c}{x^{2} a^{2}}}}{4 x}-\frac{3 a^{5} x \operatorname{arctanh}(\sqrt{-a x+1} \sqrt{a x+1}) \sqrt{c-\frac{c}{x^{2} a^{2}}}}{4 \sqrt{-a x+1} \sqrt{a x+1}}
$$

Result(type 3, 446 leaves):

$$
\begin{aligned}
& -\frac{1}{20 x^{4} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} c \sqrt{-\frac{c}{a^{2}}}}\left(\sqrt { \frac { c ( x ^ { 2 } a ^ { 2 } - 1 ) } { x ^ { 2 } a ^ { 2 } } } a ^ { 2 } \left(40 a^{4} x^{6} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} c \sqrt{-\frac{c}{a^{2}}}-40 a^{4}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{4} \sqrt{-\frac{c}{a^{2}}}\right.\right. \\
& +40 a^{2} c^{3 / 2} \ln \left(\frac{\sqrt{c} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}}+c x}{\sqrt{c}}\right) x^{5} \sqrt{-\frac{c}{a^{2}}}-40 a^{2} c^{3 / 2} \ln \left(x \sqrt{c}+\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}}\right) x^{5} \sqrt{-\frac{c}{a^{2}}} \\
& +40 a^{3} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}} c x^{5} \sqrt{-\frac{c}{a^{2}}}-15 a^{3} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} c x^{5} \sqrt{-\frac{c}{a^{2}}}-25 a^{3}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{3} \sqrt{-\frac{c}{a^{2}}} \\
& -15 a c^{2} \ln \left(\frac{2\left(\sqrt{-\frac{c}{a^{2}}} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} a^{2}-c\right)}{x a^{2}}\right) x^{5}-16 a^{2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{2} \sqrt{-\frac{c}{a^{2}}}-10 a\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x \sqrt{-\frac{c}{a^{2}}} \\
& \left.\left.-4\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} \sqrt{-\frac{c}{a^{2}}}\right)\right)
\end{aligned}
$$

Problem 213: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{c-\frac{c}{x^{2} a^{2}}}\left(-x^{2} a^{2}+1\right)}{(a x+1)^{2} x} \mathrm{~d} x
$$

Optimal(type 3, 100 leaves, 8 steps):

$$
-\sqrt{c-\frac{c}{x^{2} a^{2}}}+\frac{a x \arcsin (a x) \sqrt{c-\frac{c}{x^{2} a^{2}}}}{\sqrt{-a x+1} \sqrt{a x+1}}+\frac{2 a x \operatorname{arctanh}(\sqrt{-a x+1} \sqrt{a x+1}) \sqrt{c-\frac{c}{x^{2} a^{2}}}}{\sqrt{-a x+1} \sqrt{a x+1}}
$$

Result(type 3, 306 leaves):

$$
\begin{aligned}
& -\frac{1}{\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} c a \sqrt{-\frac{c}{a^{2}}}}\left(\sqrt { \frac { c ( x ^ { 2 } a ^ { 2 } - 1 ) } { x ^ { 2 } a ^ { 2 } } } \left(a^{3} x^{2} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} c \sqrt{-\frac{c}{a^{2}}}-a^{3}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} \sqrt{-\frac{c}{a^{2}}}\right.\right. \\
& +2 c^{3 / 2} \ln \left(\frac{\sqrt{c} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}}+c x}{\sqrt{c}}\right) x a \sqrt{-\frac{c}{a^{2}}-c^{3} / 2} \ln \left(x \sqrt{c}+\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}}\right) x a \sqrt{-\frac{c}{a^{2}}} \\
& -2 a^{2} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}} c x \sqrt{\frac{c}{a^{2}}}+2 a^{2} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} c x \sqrt{-\frac{c}{a^{2}}}+2 c^{2} \ln \left(\frac{\left.2\left(\sqrt{-\frac{c}{a^{2}} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} a^{2}-c}\right) x\right)}{x a^{2}}\right)
\end{aligned}
$$

Problem 214: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{c-\frac{c}{x^{2} a^{2}}}\left(-x^{2} a^{2}+1\right)}{(a x+1)^{2} x^{4}} d x
$$

Optimal(type 3, 128 leaves, 9 steps):

$$
\frac{4 a^{3} \sqrt{c-\frac{c}{x^{2} a^{2}}}}{3}-\frac{\sqrt{c-\frac{c}{x^{2} a^{2}}}}{4 x^{3}}+\frac{2 a \sqrt{c-\frac{c}{x^{2} a^{2}}}}{3 x^{2}}-\frac{7 a^{2} \sqrt{c-\frac{c}{x^{2} a^{2}}}}{8 x}-\frac{7 a^{4} x \operatorname{arctanh}(\sqrt{-a x+1} \sqrt{a x+1}) \sqrt{c-\frac{c}{x^{2} a^{2}}}}{8 \sqrt{-a x+1} \sqrt{a x+1}}
$$

Result(type 3, 409 leaves):
$\frac{1}{24 x^{3} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} c \sqrt{-\frac{c}{a^{2}}}}\left(\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{x^{2} a^{2}}} a^{2}\left(48 a^{3} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} c x^{5} \sqrt{-\frac{c}{a^{2}}}-48 a^{3}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{3} \sqrt{-\frac{c}{a^{2}}}\right.\right.$
$+48 a c^{3 / 2} \ln \left(\frac{\sqrt{c} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}}+c x}{\sqrt{c}}\right) x^{4} \sqrt{-\frac{c}{a^{2}}}-48 a c^{3 / 2} \ln \left(x \sqrt{c}+\sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}}\right) x^{4} \sqrt{-\frac{c}{a^{2}}}$

$$
\begin{aligned}
& -48 a^{2} \sqrt{\frac{(a x-1)(a x+1) c}{a^{2}}} x^{4} c \sqrt{-\frac{c}{a^{2}}}+21 a^{2} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} x^{4} c \sqrt{-\frac{c}{a^{2}}}+27 a^{2}\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x^{2} \sqrt{-\frac{c}{a^{2}}} \\
& \left.\left.+21 c^{2} \ln \left(\frac{2\left(\sqrt{-\frac{c}{a^{2}}} \sqrt{\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}} a^{2}-c\right)}{x a^{2}}\right) x^{4}-16 a\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} x \sqrt{-\frac{c}{a^{2}}}+6\left(\frac{c\left(x^{2} a^{2}-1\right)}{a^{2}}\right)^{3 / 2} \sqrt{-\frac{c}{a^{2}}}\right)\right)
\end{aligned}
$$

Problem 217: Unable to integrate problem.

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)} \sqrt{c-\frac{c}{x^{2} a^{2}}} \mathrm{~d} x
$$

Optimal(type 5, 226 leaves, 6 steps):

$$
\begin{aligned}
& -\frac{x(-a x+1)^{\frac{3}{2}-\frac{n}{2}}(a x+1)^{-\frac{1}{2}+\frac{n}{2}} \sqrt{c-\frac{c}{x^{2} a^{2}}}}{(1-n) \sqrt{-x^{2} a^{2}+1}} \\
& +\frac{2 x(-a x+1)^{\frac{1}{2}-\frac{n}{2}}(a x+1)^{-\frac{1}{2}+\frac{n}{2}} \operatorname{hypergeom}\left(\left[1,-\frac{1}{2}+\frac{n}{2}\right],\left[\frac{1}{2}+\frac{n}{2}\right], \frac{a x+1}{-a x+1}\right) \sqrt{c-\frac{c}{x^{2} a^{2}}}}{(1-n) \sqrt{-x^{2} a^{2}+1}} \\
& \quad+\frac{2^{\frac{1}{2}+\frac{n}{2}} n x(-a x+1)^{\frac{3}{2}-\frac{n}{2}} \operatorname{hypergeom}\left(\left[\frac{3}{2}-\frac{n}{2}, \frac{1}{2}-\frac{n}{2}\right],\left[\frac{5}{2}-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right) \sqrt{c-\frac{c}{x^{2} a^{2}}}}{\left(n^{2}-4 n+3\right) \sqrt{-x^{2} a^{2}+1}}
\end{aligned}
$$

Result(type 8, 23 leaves):

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)} \sqrt{c-\frac{c}{x^{2} a^{2}}} \mathrm{~d} x
$$

Problem 218: Unable to integrate problem.

$$
\int \frac{(a x+1)^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{p}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 5, 207 leaves, 10 steps):

$$
\frac{\left(c-\frac{c}{x^{2} a^{2}}\right)^{p} x \text { hypergeom }\left(\left[1-p, \frac{1}{2}-p\right],\left[\frac{3}{2}-p\right], x^{2} a^{2}\right)}{(1-2 p)(-a x+1)^{p}(a x+1)^{p}}+\frac{a^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{p} x^{3} \text { hypergeom }\left(\left[1-p, \frac{3}{2}-p\right],\left[\frac{5}{2}-p\right], x^{2} a^{2}\right)}{(3-2 p)(-a x+1)^{p}(a x+1)^{p}}
$$

$$
+\frac{a\left(c-\frac{c}{x^{2} a^{2}}\right)^{p} x^{2} \text { hypergeom }\left([1-p, 1-p],[2-p], x^{2} a^{2}\right)}{(1-p)(-a x+1)^{p}(a x+1)^{p}}
$$

Result(type 8, 35 leaves):

$$
\int \frac{(a x+1)^{2}\left(c-\frac{c}{x^{2} a^{2}}\right)^{p}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Problem 219: Unable to integrate problem.

$$
\int \frac{\left(c-\frac{c}{x^{2} a^{2}}\right)^{p} \sqrt{-x^{2} a^{2}+1}}{a x+1} \mathrm{~d} x
$$

Optimal(type 5, 127 leaves, 5 steps):

$$
\frac{\left(c-\frac{c}{x^{2} a^{2}}\right)^{p} x \text { hypergeom }\left(\left[\frac{1}{2}-p, \frac{1}{2}-p\right],\left[\frac{3}{2}-p\right], x^{2} a^{2}\right)}{(1-2 p)\left(-x^{2} a^{2}+1\right)^{p}}-\frac{a\left(c-\frac{c}{x^{2} a^{2}}\right)^{p} x^{2} \text { hypergeom }\left(\left[1-p, \frac{1}{2}-p\right],[2-p], x^{2} a^{2}\right)}{2(1-p)\left(-x^{2} a^{2}+1\right)^{p}}
$$

Result(type 8, 35 leaves):

$$
\int \frac{\left(c-\frac{c}{x^{2} a^{2}}\right)^{p} \sqrt{-x^{2} a^{2}+1}}{a x+1} \mathrm{~d} x
$$

Problem 220: Unable to integrate problem.

$$
\int \frac{(1+x)^{3 / 2} x \sin (x)}{\sqrt{-x^{2}+1}} \mathrm{~d} x
$$

Optimal(type 4, 134 leaves, 16 steps):

$$
\begin{aligned}
& -(1-x)^{3 / 2} \cos (x)-\frac{3 \cos (1) \text { FresnelC }\left(\frac{\sqrt{2} \sqrt{1-x}}{\sqrt{\pi}}\right) \sqrt{2} \sqrt{\pi}}{2}+\frac{5 \cos (1) \operatorname{FresnelS}\left(\frac{\sqrt{2} \sqrt{1-x}}{\sqrt{\pi}}\right) \sqrt{2} \sqrt{\pi}}{4} \\
& -\frac{5 \text { FresnelC }\left(\frac{\sqrt{2} \sqrt{1-x}}{\sqrt{\pi}}\right) \sin (1) \sqrt{2} \sqrt{\pi}}{4}-\frac{3 \text { FresnelS }\left(\frac{\sqrt{2} \sqrt{1-x}) \sin (1) \sqrt{2} \sqrt{\pi}}{\sqrt{\pi}}\right)}{2}+3 \cos (x) \sqrt{1-x}-\frac{3 \sin (x) \sqrt{1-x}}{2}
\end{aligned}
$$

Result(type 8, 20 leaves):

$$
\int \frac{(1+x)^{3 / 2} x \sin (x)}{\sqrt{-x^{2}+1}} \mathrm{~d} x
$$

Problem 221: Unable to integrate problem.

$$
\int \frac{\sqrt{1+x} \sin (x)}{\sqrt{-x^{2}+1}} \mathrm{~d} x
$$

Optimal(type 4, 50 leaves, 6 steps):

$$
\cos (1) \text { FresnelS }\left(\frac{\sqrt{2} \sqrt{1-x}}{\sqrt{\pi}}\right) \sqrt{2} \sqrt{\pi}-\operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{1-x}}{\sqrt{\pi}}\right) \sin (1) \sqrt{2} \sqrt{\pi}
$$

Result(type 8, 19 leaves):

$$
\int \frac{\sqrt{1+x} \sin (x)}{\sqrt{-x^{2}+1}} d x
$$

Problem 222: Result more than twice size of optimal antiderivative.

$$
\int \frac{b x+a+1}{\sqrt{1-(b x+a)^{2}} x^{3}} d x
$$

Optimal(type 3, 140 leaves, 5 steps):

$$
-\frac{(2 a+1) b^{2} \operatorname{arctanh}\left(\frac{\sqrt{1-a} \sqrt{b x+a+1}}{\sqrt{1+a} \sqrt{-b x-a+1}}\right)}{(1-a)^{2}(1+a) \sqrt{-a^{2}+1}}-\frac{(b x+a+1)^{3} / 2 \sqrt{-b x-a+1}}{2\left(-a^{2}+1\right) x^{2}}-\frac{(2 a+1) b \sqrt{-b x-a+1} \sqrt{b x+a+1}}{2(1-a)^{2}(1+a) x}
$$

Result(type 3, 452 leaves):

$$
\begin{aligned}
& -\frac{b \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{\left(-a^{2}+1\right) x}-\frac{3 a b^{2} \ln \left(\frac{-2 a^{2}+2-2 a b x+2 \sqrt{-a^{2}+1} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{x}\right)}{2\left(-a^{2}+1\right)^{3 / 2}}-\frac{\sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{2\left(-a^{2}+1\right) x^{2}} \\
& -\frac{3 a b \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{2\left(-a^{2}+1\right)^{2} x}-\frac{3 a^{2} b^{2} \ln \left(\frac{-2 a^{2}+2-2 a b x+2 \sqrt{-a^{2}+1} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{x}\right)}{2\left(-a^{2}+1\right)^{5 / 2}} \\
& -\frac{b^{2} \ln \left(\frac{-2 a^{2}+2-2 a b x+2 \sqrt{-a^{2}+1} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{x}\right)}{2\left(-a^{2}+1\right)^{3 / 2}}-\frac{a \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{2\left(-a^{2}+1\right) x^{2}}-\frac{3 a^{2} b \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{2\left(-a^{2}+1\right)^{2} x}
\end{aligned}
$$



Problem 223: Result more than twice size of optimal antiderivative.

$$
\int \frac{(b x+a+1)^{2} x^{4}}{1-(b x+a)^{2}} d x
$$

Optimal(type 3, 77 leaves, 3 steps):

$$
-\frac{2(1-a)^{3} x}{b^{4}}-\frac{(1-a)^{2} x^{2}}{b^{3}}-\frac{2(1-a) x^{3}}{3 b^{2}}-\frac{x^{4}}{2 b}-\frac{x^{5}}{5}-\frac{2(1-a)^{4} \ln (-b x-a+1)}{b^{5}}
$$

Result(type 3, 160 leaves):
$-\frac{x^{5}}{5}-\frac{x^{4}}{2 b}+\frac{2 a x^{3}}{3 b^{2}}-\frac{2 x^{3}}{3 b^{2}}-\frac{a^{2} x^{2}}{b^{3}}+\frac{2 x^{2} a}{b^{3}}+\frac{2 a^{3} x}{b^{4}}-\frac{x^{2}}{b^{3}}-\frac{6 x a^{2}}{b^{4}}+\frac{6 a x}{b^{4}}-\frac{2 x}{b^{4}}-\frac{2 \ln (b x+a-1) a^{4}}{b^{5}}+\frac{8 \ln (b x+a-1) a^{3}}{b^{5}}$

$$
-\frac{12 \ln (b x+a-1) a^{2}}{b^{5}}+\frac{8 \ln (b x+a-1) a}{b^{5}}-\frac{2 \ln (b x+a-1)}{b^{5}}
$$

Problem 227: Result more than twice size of optimal antiderivative.

$$
\int \frac{(b x+a+1)^{3} x}{\left(1-(b x+a)^{2}\right)^{3 / 2}} d x
$$

Optimal(type 3, 103 leaves, 7 steps):

$$
-\frac{3(3-2 a) \arcsin (b x+a)}{2 b^{2}}+\frac{(1-a)(b x+a+1)^{5 / 2}}{b^{2} \sqrt{-b x-a+1}}+\frac{(3-2 a)(b x+a+1)^{3 / 2} \sqrt{-b x-a+1}}{2 b^{2}}+\frac{3(3-2 a) \sqrt{-b x-a+1} \sqrt{b x+a+1}}{2 b^{2}}
$$

Result(type 3, 380 leaves):

Problem 228: Result more than twice size of optimal antiderivative.

$$
\int \frac{(b x+a+1)^{3}}{\left(1-(b x+a)^{2}\right)^{3 / 2} x^{2}} \mathrm{~d} x
$$

Optimal(type 3, 116 leaves, 5 steps):

$$
-\frac{6(1+a) b \operatorname{arctanh}\left(\frac{\sqrt{1-a} \sqrt{b x+a+1}}{\sqrt{1+a} \sqrt{-b x-a+1}}\right)}{(1-a)^{2} \sqrt{-a^{2}+1}}-\frac{(b x+a+1)^{3 / 2}}{(1-a) x \sqrt{-b x-a+1}}+\frac{6 b \sqrt{b x+a+1}}{(1-a)^{2} \sqrt{-b x-a+1}}
$$

Result(type 3, 1519 leaves):

$$
\begin{aligned}
& \frac{3 a^{5} b^{2} x}{\left(-a^{2}+1\right)^{2} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{9 a^{4} b^{2} x}{\left(-a^{2}+1\right)^{2} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{9 a^{3} b^{2} x}{\left(-a^{2}+1\right)^{2} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}} \\
& +\frac{5 b^{2} x a^{3}}{\left(-a^{2}+1\right) \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{9 b^{2} x a^{2}}{\left(-a^{2}+1\right) \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{9 b^{2} x a}{\left(-a^{2}+1\right) \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}} \\
& +\frac{6 a b^{2}\left(-2 b^{2} x-2 a b\right)}{\left(-4 b^{2}\left(-a^{2}+1\right)-4 a^{2} b^{2}\right) \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{3 a^{2} b^{2} x}{\left(-a^{2}+1\right)^{2} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{12 a^{4} b}{\left(-a^{2}+1\right)^{2} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}} \\
& +\frac{9 a^{2} b}{\left(-a^{2}+1\right)^{2} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{9 a^{5} b}{\left(-a^{2}+1\right)^{2} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{9 a^{3}}{\left(-a^{2}+1\right)^{2} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}} \\
& \\
& -\frac{3 a^{4} b \ln \left(\frac{-2 a^{2}+2-2 a b x+2 \sqrt{-a^{2}+1} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{x}\right)}{\left(-a^{2}+1\right)^{5 / 2}}
\end{aligned}
$$

$$
-\frac{9 a^{2} b \ln \left(\frac{-2 a^{2}+2-2 a b x+2 \sqrt{-a^{2}+1} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{x}\right)}{\left(-a^{2}+1\right)^{5 / 2}}+
$$

$$
+\frac{5 a^{4} b}{\left(-a^{2}+1\right) \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}
$$

$$
+\frac{12 a^{3} b}{\left(-a^{2}+1\right) \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{12 a^{2} b}{\left(-a^{2}+1\right) \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{6 b^{2}\left(-2 b^{2} x-2 a b\right)}{\left(-4 b^{2}\left(-a^{2}+1\right)-4 a^{2} b^{2}\right) \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}
$$

$$
-\frac{3 a b \ln \left(\frac{-2 a^{2}+2-2 a b x+2 \sqrt{-a^{2}+1} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{x}\right)}{\left(-a^{2}+1\right)^{5 / 2}}-\frac{6 a b \ln \left(\frac{-2 a^{2}+2-2 a b x+2 \sqrt{-a^{2}+1} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{x}\right)}{\left(-a^{2}+1\right)^{3 / 2}}
$$

$$
-\frac{b^{2} a x}{\sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}-\frac{3 b \ln \left(\frac{-2 a^{2}+2-2 a b x+2 \sqrt{-a^{2}+1} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{x}\right) a^{2}}{\left(-a^{2}+1\right)^{3 / 2}}+\frac{3 a b}{\left(-a^{2}+1\right)^{2} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}
$$

$$
+\frac{12 a^{3} b}{\left(-a^{2}+1\right)^{2} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{2 b^{2} x}{\left(-a^{2}+1\right) \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{8 a b}{\left(-a^{2}+1\right) \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}
$$

$$
\begin{aligned}
& -\frac{a^{3}}{\left(-a^{2}+1\right) x \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}-\frac{3 a^{2}}{\left(-a^{2}+1\right) x \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}-\frac{3 a}{\left(-a^{2}+1\right) x \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}} \\
& -\frac{1}{\left(-a^{2}+1\right) x \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}-\frac{3 a^{2}}{\sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}+\frac{3 b}{\left(-a^{2}+1\right) \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}} \\
& -\frac{3 b \ln \left(\frac{-2 a^{2}+2-2 a b x+2 \sqrt{-a^{2}+1} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{x}\right)}{\left(-a^{2}+1\right)^{3 / 2}}+\frac{b}{\sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}
\end{aligned}
$$

Problem 229: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3} \sqrt{1-(b x+a)^{2}}}{b x+a+1} \mathrm{~d} x
$$

Optimal (type 3, 136 leaves, 7 steps):

$$
\begin{aligned}
& -\frac{\left(8 a^{3}+12 a^{2}+12 a+3\right) \arcsin (b x+a)}{8 b^{4}}-\frac{x^{2}(-b x-a+1)^{3 / 2} \sqrt{b x+a+1}}{4 b^{2}}-\frac{(-b x-a+1)^{3 / 2}\left(7+10 a+18 a^{2}-2(1+6 a) b x\right) \sqrt{b x+a+1}}{24 b^{4}} \\
& \quad-\frac{\left(8 a^{3}+12 a^{2}+12 a+3\right) \sqrt{-b x-a+1} \sqrt{b x+a+1}}{8 b^{4}}
\end{aligned}
$$

Result(type 3, 808 leaves):
$\frac{3 a^{2} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1} x}{2 b^{3}}+\frac{3 a^{2} \arctan \left(\frac{\sqrt{b^{2}}\left(x+\frac{a}{b}\right)}{\sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}\right)}{2 b^{3} \sqrt{b^{2}}}+\frac{3 \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1} x a}{2 b^{3}}$
$-\frac{\sqrt{-b^{2}\left(x+\frac{1+a}{b}\right)^{2}+2 b\left(x+\frac{1+a}{b}\right)}}{b^{4}}-\frac{\sqrt{-b^{2}\left(x+\frac{1+a}{b}\right)^{2}+2 b\left(x+\frac{1+a}{b}\right)} a^{3}}{b^{4}}-\frac{3 \sqrt{-b^{2}\left(x+\frac{1+a}{b}\right)^{2}+2 b\left(x+\frac{1+a}{b}\right)} a^{2}}{b^{4}}$
$-\frac{3 \sqrt{-b^{2}\left(x+\frac{1+a}{b}\right)^{2}+2 b\left(x+\frac{1+a}{b}\right)} a}{b^{4}}-\frac{\arctan \left(\frac{b}{\sqrt{-b^{2}\left(x+\frac{1+a}{b}\right)^{2}+2 b\left(x+\frac{1+a}{b}\right)}}\right)}{b^{3} \sqrt{b^{2}}}+\frac{3 \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1} a^{2}}{2 b^{4}}$
$-\frac{x\left(-b^{2} x^{2}-2 a b x-a^{2}+1\right)^{3 / 2}}{4 b^{3}}+\frac{3 a\left(-b^{2} x^{2}-2 a b x-a^{2}+1\right)^{3 / 2}}{4 b^{4}}+\frac{3 a^{3} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{2 b^{4}}+\frac{5 \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1} x}{8 b^{3}}$

$$
\begin{aligned}
& +\frac{5 \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1} a}{8 b^{4}}+\frac{5 \arctan \left(\frac{\sqrt{b^{2}}\left(x+\frac{a}{b}\right)}{\sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}\right)}{8 b^{3} \sqrt{b^{2}}}+\frac{\left(-b^{2} x^{2}-2 a b x-a^{2}+1\right)^{3 / 2}}{3 b^{4}} \\
& +\frac{3 a \arctan \left(\frac{\sqrt{b^{2}}\left(x+\frac{a}{b}\right)}{\sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}\right)}{2 b^{3} \sqrt{b^{2}}}-\frac{\arctan \left(\frac{\sqrt{b^{2}}\left(x+\frac{1+a}{b}-\frac{1}{b}\right)}{\sqrt{-b^{2}\left(x+\frac{1+a}{b}\right)^{2}+2 b\left(x+\frac{1+a}{b}\right)}}\right) a^{3}}{b^{3} \sqrt{b^{2}}} \\
& -\frac{3 \arctan \left(\frac{\sqrt{b^{2}}\left(x+\frac{1+a}{b}-\frac{1}{b}\right)}{\sqrt{-b^{2}\left(x+\frac{1+a}{b}\right)^{2}+2 b\left(x+\frac{1+a}{b}\right)}}\right) a^{2}}{b^{3} \sqrt{b^{2}}}-\frac{3 \arctan \left(\frac{\sqrt{b^{2}}\left(x+\frac{1+a}{b}-\frac{1}{b}\right)}{\sqrt{-b^{2}\left(x+\frac{1+a}{b}\right)^{2}+2 b\left(x+\frac{1+a}{b}\right)}}\right) a}{b^{3} \sqrt{b^{2}}}
\end{aligned}
$$

Problem 230: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{1-(b x+a)^{2}}}{(b x+a+1) x^{2}} d x
$$

Optimal(type 3, 80 leaves, 4 steps):

$$
\frac{2 b \operatorname{arctanh}\left(\frac{\sqrt{1-a} \sqrt{b x+a+1}}{\sqrt{1+a} \sqrt{-b x-a+1}}\right)}{(1+a) \sqrt{-a^{2}+1}}-\frac{\sqrt{-b x-a+1} \sqrt{b x+a+1}}{(1+a) x}
$$

Result(type 3, 564 leaves):
$-\frac{\left(-b^{2} x^{2}-2 a b x-a^{2}+1\right)^{3 / 2}}{(1+a)\left(-a^{2}+1\right) x}-\frac{2 a b \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{(1+a)\left(-a^{2}+1\right)}+\frac{a^{2} b^{2} \arctan \left(\frac{\sqrt{b^{2}}\left(x+\frac{a}{b}\right)}{\sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}\right)}{(1+a)\left(-a^{2}+1\right) \sqrt{b^{2}}}$


$$
\begin{aligned}
& +\frac{b \sqrt{-b^{2}\left(x+\frac{1+a}{b}\right)^{2}+2 b\left(x+\frac{1+a}{b}\right)}}{(1+a)^{2}}+\frac{b^{2} \arctan \left(\frac{b}{\sqrt{-b^{2}\left(x+\frac{1+a}{b}\right)^{2}+2 b\left(x+\frac{1+a}{b}\right)}}\right)}{(1+a)^{2} \sqrt{b^{2}}}-\frac{b \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{(1+a)^{2}} \\
& +\frac{b^{2} a \arctan \left(\frac{\sqrt{b^{2}}\left(x+\frac{a}{b}\right)}{\sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}\right)}{(1+a)^{2} \sqrt{b^{2}}}+\frac{b \sqrt{-a^{2}+1} \ln \left(\frac{-2 a^{2}+2-2 a b x+2 \sqrt{-a^{2}+1} \sqrt{-b^{2} x^{2}-2 a b x-a^{2}+1}}{x}\right)}{(1+a)^{2}}
\end{aligned}
$$

Problem 231: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(1-(b x+a)^{2}\right)^{3 / 2}}{(b x+a+1)^{3} x^{3}} \mathrm{~d} x
$$

Optimal(type 3, 174 leaves, 6 steps):

$$
\begin{aligned}
& -\frac{3(3-2 a) b^{2} \operatorname{arctanh}\left(\frac{\sqrt{1-a} \sqrt{b x+a+1}}{\sqrt{1+a} \sqrt{-b x-a+1}}\right)}{(1+a)^{3} \sqrt{-a^{2}+1}}+\frac{(3-2 a) b(-b x-a+1)^{3 / 2}}{2(1-a)(1+a)^{2} x \sqrt{b x+a+1}}-\frac{(-b x-a+1)^{5 / 2}}{2\left(-a^{2}+1\right) x^{2} \sqrt{b x+a+1}} \\
& \quad+\frac{3(3-2 a) b^{2} \sqrt{-b x-a+1}}{(1-a)(1+a)^{3} \sqrt{b x+a+1}}
\end{aligned}
$$

Result(type ?, 2847 leaves): Display of huge result suppressed!
Problem 232: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(1-(b x+a)^{2}\right)^{3 / 2}}{(b x+a+1)^{3} x^{4}} \mathrm{~d} x
$$

Optimal(type 3, 223 leaves, 8 steps):

$$
\begin{aligned}
& \frac{\left(6 a^{2}-18 a+11\right) b^{3} \operatorname{arctanh}\left(\frac{\sqrt{1-a} \sqrt{b x+a+1}}{\sqrt{1+a} \sqrt{-b x-a+1}}\right)}{(1-a)(1+a)^{4} \sqrt{-a^{2}+1}}-\frac{\left(2 a^{2}-51 a+52\right) b^{3} \sqrt{-b x-a+1}}{6(1-a)(1+a)^{4} \sqrt{b x+a+1}}-\frac{(1-a) \sqrt{-b x-a+1}}{3(1+a) x^{3} \sqrt{b x+a+1}} \\
& \quad+\frac{7 b \sqrt{-b x-a+1}}{6(1+a)^{2} x^{2} \sqrt{b x+a+1}}-\frac{(19-16 a) b^{2} \sqrt{-b x-a+1}}{6(1-a)(1+a)^{3} x \sqrt{b x+a+1}}
\end{aligned}
$$

Result(type ?, 4211 leaves): Display of huge result suppressed!
Problem 233: Unable to integrate problem.

$$
\int \mathrm{e}^{n \operatorname{arctanh}(b x+a)} x^{3} \mathrm{~d} x
$$

Optimal(type 5, 180 leaves, 4 steps):
$-\frac{x^{2}(-b x-a+1)^{1-\frac{n}{2}}(b x+a+1)^{1+\frac{n}{2}}}{4 b^{2}}-\frac{(-b x-a+1)^{1-\frac{n}{2}}(b x+a+1)^{1+\frac{n}{2}}\left(6+18 a^{2}-10 a n+n^{2}-2 b(6 a-n) x\right)}{24 b^{4}}$

$$
+\frac{2^{-2+\frac{n}{2}}\left(24 a^{3}-36 a^{2} n+12 a\left(n^{2}+2\right)-n\left(n^{2}+8\right)\right)(-b x-a+1)^{1-\frac{n}{2}} \operatorname{hypergeom}\left(\left[-\frac{n}{2}, 1-\frac{n}{2}\right],\left[2-\frac{n}{2}\right],-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right)}{3 b^{4}(2-n)}
$$

Result(type 8, 15 leaves):

$$
\int \mathrm{e}^{n \operatorname{arctanh}(b x+a)} x^{3} \mathrm{~d} x
$$

Problem 234: Unable to integrate problem.

$$
\int \mathrm{e}^{n \operatorname{arctanh}(b x+a)} x \mathrm{~d} x
$$

Optimal(type 5, 96 leaves, 3 steps):

$$
-\frac{(-b x-a+1)^{1-\frac{n}{2}}(b x+a+1)^{1+\frac{n}{2}}}{2 b^{2}}+\frac{2^{\frac{n}{2}}(2 a-n)(-b x-a+1)^{1-\frac{n}{2}} \text { hypergeom }\left(\left[-\frac{n}{2}, 1-\frac{n}{2}\right],\left[2-\frac{n}{2}\right],-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right)}{b^{2}(2-n)}
$$

Result(type 8, 13 leaves):

$$
\int \mathrm{e}^{n \operatorname{arctanh}(b x+a)} x \mathrm{~d} x
$$

Problem 235: Unable to integrate problem.

$$
\int \mathrm{e}^{n \operatorname{arctanh}(b x+a)} \mathrm{d} x
$$

Optimal(type 5, 59 leaves, 2 steps):

$$
-\frac{2^{1+\frac{n}{2}}(-b x-a+1)^{1-\frac{n}{2}} \text { hypergeom }\left(\left[-\frac{n}{2}, 1-\frac{n}{2}\right],\left[2-\frac{n}{2}\right],-\frac{b x}{2}-\frac{a}{2}+\frac{1}{2}\right)}{b(2-n)}
$$

Result(type 8, 11 leaves):

$$
\int \mathrm{e}^{n \operatorname{arctanh}(b x+a)} \mathrm{d} x
$$

Problem 236: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(b x+a)}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 5, 86 leaves, 2 steps):

$$
-\frac{4 b(-b x-a+1)^{1-\frac{n}{2}}(b x+a+1)^{-1+\frac{n}{2}} \operatorname{hypergeom}\left(\left[2,1-\frac{n}{2}\right],\left[2-\frac{n}{2}\right], \frac{(1+a)(-b x-a+1)}{(1-a)(b x+a+1)}\right)}{(1-a)^{2}(2-n)}
$$

Result(type 8, 15 leaves):

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(b x+a)}}{x^{2}} \mathrm{~d} x
$$

Problem 239: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1) x^{6}}{\sqrt{-x^{2} a^{2}+1}\left(-a^{2} c x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Optimal(type 3, 117 leaves, 6 steps):

$$
\frac{x^{5}(a x+1)}{5 a^{2} c^{3}\left(-x^{2} a^{2}+1\right)^{5 / 2}}-\frac{x^{3}(6 a x+5)}{15 a^{4} c^{3}\left(-x^{2} a^{2}+1\right)^{3 / 2}}-\frac{\arcsin (a x)}{a^{7} c^{3}}+\frac{x(8 a x+5)}{5 a^{6} c^{3} \sqrt{-x^{2} a^{2}+1}}+\frac{16 \sqrt{-x^{2} a^{2}+1}}{5 a^{7} c^{3}}
$$

Result(type 3, 261 leaves):

$$
\begin{aligned}
&-\frac{\arctan \left(\frac{\sqrt{a^{2}} x}{\sqrt{-x^{2} a^{2}+1}}\right)}{c^{3} a^{6} \sqrt{a^{2}}}+\frac{\sqrt{-x^{2} a^{2}+1}}{a^{7} c^{3}}-\frac{\sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)}}{20 c^{3} a^{10}\left(x-\frac{1}{a}\right)^{3}}-\frac{23 \sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)}}{60 c^{3} a^{9}\left(x-\frac{1}{a}\right)^{2}} \\
&-\frac{493 \sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2}-2 a\left(x-\frac{1}{a}\right)}}{240 c^{3} a^{8}\left(x-\frac{1}{a}\right)}-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}}{24 c^{3} a^{9}\left(x+\frac{1}{a}\right)^{2}}+\frac{25 \sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2 a\left(x+\frac{1}{a}\right)}}{48 c^{3} a^{8}\left(x+\frac{1}{a}\right)}
\end{aligned}
$$

Problem 254: Unable to integrate problem.

$$
\int \frac{(a x+1) x^{m}}{\sqrt{-x^{2} a^{2}+1}\left(-a^{2} c x^{2}+c\right)} \mathrm{d} x
$$

Optimal(type 5, 72 leaves, 4 steps):

$$
\frac{x^{1+m} \text { hypergeom }\left(\left[\frac{3}{2}, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right], x^{2} a^{2}\right)}{c(1+m)}+\frac{a x^{2+m} \text { hypergeom }\left(\left[\frac{3}{2}, 1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right], x^{2} a^{2}\right)}{c(2+m)}
$$

Result(type 8, 36 leaves):

$$
\int \frac{(a x+1) x^{m}}{\sqrt{-x^{2} a^{2}+1}\left(-a^{2} c x^{2}+c\right)} \mathrm{d} x
$$

Problem 255: Result more than twice size of optimal antiderivative.

$$
\int(a x+1)\left(-x^{2} a^{2}+1\right) x^{m} \mathrm{~d} x
$$

Optimal(type 3, 54 leaves, 3 steps):

$$
\frac{x^{1+m}}{1+m}+\frac{a x^{2+m}}{2+m}-\frac{a^{2} x^{3+m}}{3+m}-\frac{a^{3} x^{4+m}}{4+m}
$$

Result(type 3, 141 leaves):
$-\frac{1}{(4+m)(3+m)(2+m)(1+m)}\left(x^{1+m}\left(a^{3} m^{3} x^{3}+6 a^{3} m^{2} x^{3}+11 a^{3} m x^{3}+a^{2} m^{3} x^{2}+6 a^{3} x^{3}+7 a^{2} m^{2} x^{2}+14 a^{2} m x^{2}-a m^{3} x+8 x^{2} a^{2}-8 a m^{2} x\right.\right.$
$\left.\left.-19 a m x-m^{3}-12 a x-9 m^{2}-26 m-24\right)\right)$

Problem 257: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1) x^{m}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 5, 24 leaves, 2 steps):

$$
\frac{x^{1+m} \text { hypergeom }([1,1+m],[2+m], a x)}{1+m}
$$

Result(type 5, 99 leaves):

$$
-\frac{\left(-a^{2}\right)^{-\frac{m}{2}}\left(-\frac{2 x^{m}\left(-a^{2}\right)^{\frac{m}{2}}(-m-2)}{(2+m) m}-x^{m}\left(-a^{2}\right)^{\frac{m}{2}} \operatorname{LerchPhi}\left(x^{2} a^{2}, 1, \frac{m}{2}\right)\right)}{2 a}+\frac{x^{1+m}\left(\frac{1}{2}+\frac{m}{2}\right) \operatorname{LerchPhi}\left(x^{2} a^{2}, 1, \frac{1}{2}+\frac{m}{2}\right)}{1+m}
$$

Problem 258: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1) x^{m}}{\left(-x^{2} a^{2}+1\right)^{2}} \mathrm{~d} x
$$

Optimal(type 5, 66 leaves, 6 steps):

$$
\frac{x^{1+m} \text { hypergeom }\left(\left[2, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right], x^{2} a^{2}\right)}{1+m}+\frac{a x^{2+m} \text { hypergeom }\left(\left[2,1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right], x^{2} a^{2}\right)}{2+m}
$$

Result(type 5, 176 leaves):
$-\frac{\left(-a^{2}\right)^{-\frac{m}{2}}\left(\frac{x^{m}\left(-a^{2}\right)^{\frac{m}{2}}(-m-2)}{(2+m)\left(-x^{2} a^{2}+1\right)}+\frac{x^{m}\left(-a^{2}\right)^{\frac{m}{2}} m \operatorname{LerchPhi}\left(x^{2} a^{2}, 1, \frac{m}{2}\right)}{2}\right)}{2 a}$
$+\frac{\left(-a^{2}\right)^{-\frac{1}{2}-\frac{m}{2}}\left(-\frac{2 x^{1+m}\left(-a^{2}\right)^{\frac{1}{2}+\frac{m}{2}}(-1-m)}{(1+m)\left(-2 x^{2} a^{2}+2\right)}+\frac{2 x^{1+m}\left(-a^{2}\right)^{\frac{1}{2}+\frac{m}{2}}\left(-\frac{m^{2}}{4}+\frac{1}{4}\right) \operatorname{LerchPhi}\left(x^{2} a^{2}, 1, \frac{1}{2}+\frac{m}{2}\right)}{1+m}\right)}{2}$

Problem 259: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1) x^{m}}{\left(-x^{2} a^{2}+1\right)^{3}} \mathrm{~d} x
$$

Optimal(type 5, 66 leaves, 6 steps):

$$
\frac{x^{1+m} \text { hypergeom }\left(\left[3, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right], x^{2} a^{2}\right)}{1+m}+\frac{a x^{2+m} \text { hypergeom }\left(\left[3,1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right], x^{2} a^{2}\right)}{2+m}
$$

Result(type 5, 223 leaves):

$$
\begin{aligned}
& \frac{1}{4}\left(-a^{2}\right)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{x^{1+m}\left(-a^{2}\right)^{\frac{1}{2}+\frac{m}{2}}\left(a^{2} m^{2} x^{2}-2 a^{2} m x^{2}-3 x^{2} a^{2}-m^{2}+4 m+5\right)}{2(1+m)\left(-x^{2} a^{2}+1\right)^{2}}\right. \\
&\left.\left.+\frac{4 x^{1+m}\left(-a^{2}\right)^{\frac{1}{2}+\frac{m}{2}}\left(\frac{1}{16} m^{3}-\frac{3}{16} m^{2}-\frac{1}{16} m+\frac{3}{16}\right) \operatorname{LerchPhi}\left(x^{2} a^{2}, 1, \frac{1}{2}+\frac{m}{2}\right)}{1+m}\right)\right) \\
&-\frac{\left(-a^{2}\right)^{-\frac{m}{2}}\left(-\frac{x^{m}\left(-a^{2}\right)^{\frac{m}{2}}\left(a^{2} m x^{2}-m+2\right)}{2\left(-x^{2} a^{2}+1\right)^{2}}-\frac{x^{m}\left(-a^{2}\right)^{\frac{m}{2}}(m-2) m \operatorname{LerchPhi}\left(x^{2} a^{2}, 1, \frac{m}{2}\right)}{4}\right)}{4 a}
\end{aligned}
$$

$4 a$

Problem 260: Unable to integrate problem.

$$
\int \frac{(a x+1) x^{m}}{\sqrt{-x^{2} a^{2}+1}\left(-a^{2} c x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 122 leaves, 7 steps):

$$
\frac{x^{1+m} \text { hypergeom }\left(\left[2, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right], x^{2} a^{2}\right) \sqrt{-x^{2} a^{2}+1}}{c(1+m) \sqrt{-a^{2} c x^{2}+c}}+\frac{a x^{2+m} \text { hypergeom }\left(\left[2,1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right], x^{2} a^{2}\right) \sqrt{-x^{2} a^{2}+1}}{c(2+m) \sqrt{-a^{2} c x^{2}+c}}
$$

Result(type 8, 36 leaves):

$$
\int \frac{(a x+1) x^{m}}{\sqrt{-x^{2} a^{2}+1}\left(-a^{2} c x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Problem 261: Unable to integrate problem.

$$
\int \frac{(a x+1) x^{m}\left(-a^{2} c x^{2}+c\right)^{p}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Optimal(type 5, 128 leaves, 5 steps):

$$
\frac{x^{1+m}\left(-a^{2} c x^{2}+c\right)^{p} \text { hypergeom }\left(\left[\frac{1}{2}-p, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right], x^{2} a^{2}\right)}{(1+m)\left(-x^{2} a^{2}+1\right)^{p}}+\frac{a x^{2+m}\left(-a^{2} c x^{2}+c\right)^{p} \text { hypergeom }\left(\left[1+\frac{m}{2}, \frac{1}{2}-p\right],\left[2+\frac{m}{2}\right], x^{2} a^{2}\right)}{(2+m)\left(-x^{2} a^{2}+1\right)^{p}}
$$

Result(type 8, 36 leaves):

$$
\int \frac{(a x+1) x^{m}\left(-a^{2} c x^{2}+c\right)^{p}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Problem 263: Unable to integrate problem.

$$
\int \frac{(a x+1) x^{3}\left(-a^{2} c x^{2}+c\right)^{p}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Optimal(type 5, 124 leaves, 7 steps):

$$
\frac{\left(-x^{2} a^{2}+1\right)^{3 / 2}\left(-a^{2} c x^{2}+c\right)^{p}}{a^{4}(3+2 p)}+\frac{a x^{5}\left(-a^{2} c x^{2}+c\right)^{p} \text { hypergeom }\left(\left[\frac{5}{2}, \frac{1}{2}-p\right],\left[\frac{7}{2}\right], x^{2} a^{2}\right)}{5\left(-x^{2} a^{2}+1\right)^{p}}-\frac{\left(-a^{2} c x^{2}+c\right)^{p} \sqrt{-x^{2} a^{2}+1}}{a^{4}(1+2 p)}
$$

Result(type 8, 36 leaves):

$$
\int \frac{(a x+1) x^{3}\left(-a^{2} c x^{2}+c\right)^{p}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Problem 264: Unable to integrate problem.

$$
\int \frac{(a x+1) x\left(-a^{2} c x^{2}+c\right)^{p}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Optimal(type 5, 88 leaves, 5 steps):

$$
\frac{a x^{3}\left(-a^{2} c x^{2}+c\right)^{p} \text { hypergeom }\left(\left[\frac{3}{2}, \frac{1}{2}-p\right],\left[\frac{5}{2}\right], x^{2} a^{2}\right)}{3\left(-x^{2} a^{2}+1\right)^{p}}-\frac{\left(-a^{2} c x^{2}+c\right)^{p} \sqrt{-x^{2} a^{2}+1}}{a^{2}(1+2 p)}
$$

Result(type 8, 34 leaves):

$$
\int \frac{(a x+1) x\left(-a^{2} c x^{2}+c\right)^{p}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Problem 265: Unable to integrate problem.

$$
\int \frac{(a x+1)\left(-a^{2} c x^{2}+c\right)^{p}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Optimal(type 5, 74 leaves, 3 steps):

$$
-\frac{2^{\frac{3}{2}+p}(-a x+1)^{\frac{1}{2}+p}\left(-a^{2} c x^{2}+c\right)^{p} \operatorname{hypergeom}\left(\left[\frac{1}{2}+p,-\frac{1}{2}-p\right],\left[\frac{3}{2}+p\right],-\frac{a x}{2}+\frac{1}{2}\right)}{a(1+2 p)\left(-x^{2} a^{2}+1\right)^{p}}
$$

Result(type 8, 33 leaves):

$$
\int \frac{(a x+1)\left(-a^{2} c x^{2}+c\right)^{p}}{\sqrt{-x^{2} a^{2}+1}} \mathrm{~d} x
$$

Problem 282: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2} \sqrt{-a^{2} c x^{2}+c}}{\left(-x^{2} a^{2}+1\right) x^{3}} \mathrm{~d} x
$$

Optimal(type 3, 64 leaves, 6 steps):

$$
-\frac{3 a^{2} \operatorname{arctanh}\left(\frac{\sqrt{-a^{2} c x^{2}+c}}{\sqrt{c}}\right) \sqrt{c}}{2}-\frac{\sqrt{-a^{2} c x^{2}+c}}{2 x^{2}}-\frac{2 a \sqrt{-a^{2} c x^{2}+c}}{x}
$$

Result(type 3, 238 leaves):

$$
\begin{aligned}
& -\frac{\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{2 c x^{2}}-\frac{3 \sqrt{c} \ln \left(\frac{2 c+2 \sqrt{c} \sqrt{-a^{2} c x^{2}+c}}{x}\right) a^{2}}{2}+\frac{3 \sqrt{-a^{2} c x^{2}+c} a^{2}}{2}-\frac{2 a\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{c x}-2 a^{3} x \sqrt{-a^{2} c x^{2}+c} \\
& -\frac{2 a^{3} c \arctan \left(\frac{\sqrt{a^{2} c x}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{\sqrt{a^{2} c}}-2 a^{2} \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}+\frac{2 a^{3} c \arctan \left(\frac{\sqrt{a^{2} c} x}{\left.\sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right.}\right) a c}\right)}{\sqrt{a^{2} c}}
\end{aligned}
$$

Problem 283: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2} x^{2}\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{-x^{2} a^{2}+1} d x
$$

Optimal(type 3, 112 leaves, 7 steps):

$$
-\frac{2 x^{2}\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{5 a}-\frac{x^{3}\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{6}-\frac{(45 a x+32)\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{120 a^{3}}+\frac{3 c^{3 / 2} \arctan \left(\frac{a x \sqrt{c}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{16 a^{3}}+\frac{3 c x \sqrt{-a^{2} c x^{2}+c}}{16 a^{2}}
$$

Result(type 3, 243 leaves):

$$
\begin{aligned}
& \frac{x\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{6 a^{2} c}-\frac{13 x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{24 a^{2}}-\frac{13 c x \sqrt{-a^{2} c x^{2}+c}}{16 a^{2}}-\frac{13 c^{2} \arctan \left(\frac{\sqrt{a^{2} c} x}{\sqrt{-a^{2} c x^{2}+c}}\right)}{16 a^{2} \sqrt{a^{2} c}}+\frac{2\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{5 a^{3} c} \\
& \quad-\frac{2\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{3 / 2}}{3 a^{3}}+\frac{c \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c x}}{a^{2}}+\frac{c^{2} \arctan \left(\frac{\sqrt{a^{2} c} x}{\left.\sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}\right)}\right.}{a^{2} \sqrt{a^{2} c}}
\end{aligned}
$$

Problem 284: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2} x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 3, 91 leaves, 6 steps):

$$
-\frac{x^{2}\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{5}-\frac{(15 a x+14)\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{30 a^{2}}+\frac{c^{3 / 2} \arctan \left(\frac{a x \sqrt{c}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{4 a^{2}}+\frac{c x \sqrt{-a^{2} c x^{2}+c}}{4 a}
$$

Result(type 3, 221 leaves):

$$
\begin{aligned}
& \frac{\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{5 a^{2} c}-\frac{x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{2 a}-\frac{3 c x \sqrt{-a^{2} c x^{2}+c}}{4 a}-\frac{3 c^{2} \arctan \left(\frac{\sqrt{a^{2} c x}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{4 a \sqrt{a^{2} c}}-\frac{2\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{3 / 2}}{3 a^{2}} \\
& \quad+\frac{c \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c} x}{a}+\frac{c^{2} \arctan \left(\frac{\sqrt{a^{2} c x}}{\sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}}\right)}{a \sqrt{a^{2} c}}
\end{aligned}
$$

Problem 285: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2}\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 3, 87 leaves, 6 steps):

$$
-\frac{5\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{12 a}-\frac{(a x+1)\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{4 a}+\frac{5 c^{3 / 2} \arctan \left(\frac{a x \sqrt{c}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{8 a}+\frac{5 c x \sqrt{-a^{2} c x^{2}+c}}{8}
$$

Result(type 3, 185 leaves):

$$
\begin{aligned}
& -\frac{x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{4}-\frac{3 c x \sqrt{-a^{2} c x^{2}+c}}{8}-\frac{3 c^{2} \arctan \left(\frac{\sqrt{a^{2} c x}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{8 \sqrt{a^{2} c}}-\frac{2\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{3 / 2}}{3 a} \\
& \quad+c \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c} x+\frac{c^{2} \arctan \left(\frac{\sqrt{a^{2} c} x}{\sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}}\right)}{\sqrt{a^{2} c}}
\end{aligned}
$$

Problem 286: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2}\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{\left(-x^{2} a^{2}+1\right) x^{4}} \mathrm{~d} x
$$

Optimal(type 3, 97 leaves, 9 steps):

$$
-\frac{\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{3 x^{3}}-a^{3} c^{3 / 2} \arctan \left(\frac{a x \sqrt{c}}{\sqrt{-a^{2} c x^{2}+c}}\right)+a^{3} c^{3 / 2} \operatorname{arctanh}\left(\frac{\sqrt{-a^{2} c x^{2}+c}}{\sqrt{c}}\right)-\frac{a c(a x+1) \sqrt{-a^{2} c x^{2}+c}}{x^{2}}
$$

Result(type 3, 338 leaves):

$$
\begin{aligned}
& -\frac{\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{3 c x^{3}}-\frac{4 a^{2}\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{3 c x}-\frac{4 a^{4} x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{3}-2 a^{4} c x \sqrt{-a^{2} c x^{2}+c}-\frac{2 a^{4} c^{2} \arctan \left(\frac{\sqrt{a^{2} c} x}{\sqrt{-a^{2} c x^{2}+c}}\right)}{\sqrt{a^{2} c}}-\frac{a\left(-a^{2} c x^{2}+c\right)^{5} / 2}{c x^{2}} \\
& \quad-\frac{a^{3}\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{3}+a^{3} c^{3 / 2} \ln \left(\frac{2 c+2 \sqrt{c} \sqrt{-a^{2} c x^{2}+c}}{x}\right)-a^{3} c \sqrt{-a^{2} c x^{2}+c}-\frac{2 a^{3}\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{3 / 2}}{3}
\end{aligned}
$$

$$
+a^{4} c \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c} x+\frac{a^{4} c^{2} \text { arctan } \sqrt{\sqrt{\sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}}} \sqrt{\sqrt{a^{2} c}}}{}
$$

Problem 287: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2} x^{2}\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 3, 134 leaves, 8 steps):

$$
\begin{aligned}
& \frac{11 c x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{192 a^{2}}-\frac{2 x^{2}\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{7 a}-\frac{x^{3}\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{8}-\frac{(385 a x+192)\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{1680 a^{3}}+\frac{11 c^{5 / 2} \arctan \left(\frac{a x \sqrt{c}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{128 a^{3}} \\
& \quad+\frac{11 c^{2} x \sqrt{-a^{2} c x^{2}+c}}{128 a^{2}}
\end{aligned}
$$

Result(type 3, 305 leaves):

$$
\begin{aligned}
& \frac{x\left(-a^{2} c x^{2}+c\right)^{7 / 2}}{8 a^{2} c}-\frac{17 x\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{48 a^{2}}-\frac{85 c x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{192 a^{2}}-\frac{85 c^{2} x \sqrt{-a^{2} c x^{2}+c}}{128 a^{2}}-\frac{85 c^{3} \arctan \left(\frac{\sqrt{a^{2} c} x}{\sqrt{-a^{2} c x^{2}+c}}\right)}{128 a^{2} \sqrt{a^{2} c}}+\frac{2\left(-a^{2} c x^{2}+c\right)^{7} / 2}{7 a^{3} c} \\
& -\frac{2\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{5 / 2}}{5 a^{3}}+\frac{c\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{3 / 2} x}{2 a^{2}}+\frac{3 c^{2} \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c x}}{4 a^{2}} \\
& \quad 3 c^{3} \arctan \left(\frac{\sqrt{a^{2} c} x}{4 a^{2} \sqrt{a^{2} c}}\right.
\end{aligned}
$$

Problem 288: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2} x\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 3, 113 leaves, 7 steps):

$$
\frac{c x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{12 a}-\frac{x^{2}\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{7}-\frac{(35 a x+27)\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{105 a^{2}}+\frac{c^{5 / 2} \arctan \left(\frac{a x \sqrt{c}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{8 a^{2}}+\frac{c^{2} x \sqrt{-a^{2} c x^{2}+c}}{8 a}
$$

Result(type 3, 283 leaves):
$\frac{\left(-a^{2} c x^{2}+c\right)^{7 / 2}}{7 a^{2} c}-\frac{x\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{3 a}-\frac{5 c x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{12 a}-\frac{5 c^{2} x \sqrt{-a^{2} c x^{2}+c}}{8 a}-\frac{5 c^{3} \arctan \left(\frac{\sqrt{a^{2} c} x}{\sqrt{-a^{2} c x^{2}+c}}\right)}{8 a \sqrt{a^{2} c}}$

$$
-\frac{2\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{5 / 2}}{5 a^{2}}+\frac{c\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{3 / 2} x}{2 a}+\frac{3 c^{2} \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c x}}{4 a}
$$

$$
+\frac{3 c^{3} \arctan \left(\frac{\sqrt{a^{2} c} x}{\sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}}\right)}{4 a \sqrt{a^{2} c}}
$$

Problem 289: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2}\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 3, 106 leaves, 7 steps):

$$
\frac{7 c x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{24}-\frac{7\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{30 a}-\frac{(a x+1)\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{6 a}+\frac{7 c^{5 / 2} \arctan \left(\frac{a x \sqrt{c}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{16 a}+\frac{7 c^{2} x \sqrt{-a^{2} c x^{2}+c}}{16}
$$

Result(type 3, 241 leaves):

$$
\begin{aligned}
& -\frac{x\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{6}-\frac{5 c x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{24}-\frac{5 c^{2} x \sqrt{-a^{2} c x^{2}+c}}{16}-\frac{5 c^{3} \arctan \left(\frac{\sqrt{a^{2} c} x}{\sqrt{-a^{2} c x^{2}+c}}\right)}{16 \sqrt{a^{2} c}}-\frac{2\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right)^{a c}\right)^{5 / 2}}{5 a} \\
& \quad+\frac{c\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{3 / 2} x}{2}+\frac{3 c^{2} \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c x}}{4}+\frac{3 c^{3} \arctan \left(\frac{\sqrt{a^{2} c} x}{\left.\sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}\right)}\right.}{4 \sqrt{a^{2} c}}
\end{aligned}
$$

Problem 290: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2}\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{\left(-x^{2} a^{2}+1\right) x^{3}} \mathrm{~d} x
$$

Optimal(type 3, 125 leaves, 10 steps):
$-\frac{a c(a x+12)\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{6 x}-\frac{\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{2 x^{2}}-3 a^{2} c^{5 / 2} \arctan \left(\frac{a x \sqrt{c}}{\sqrt{-a^{2} c x^{2}+c}}\right)+\frac{a^{2} c^{5 / 2} \operatorname{arctanh}\left(\frac{\sqrt{-a^{2} c x^{2}+c}}{\sqrt{c}}\right)}{2}$

$$
-\frac{a^{2} c^{2}(6 a x+1) \sqrt{-a^{2} c x^{2}+c}}{2}
$$

Result(type 3, 398 leaves):

$$
\begin{aligned}
& -\frac{\left(-a^{2} c x^{2}+c\right)^{7 / 2}}{2 c x^{2}}-\frac{a^{2}\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{10}-\frac{a^{2} c\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{6}+\frac{a^{2} c^{5 / 2} \ln \left(\frac{2 c+2 \sqrt{c} \sqrt{-a^{2} c x^{2}+c}}{x}\right)}{2}-\frac{a^{2} c^{2} \sqrt{-a^{2} c x^{2}+c}}{2} \\
& -\frac{2 a\left(-a^{2} c x^{2}+c\right)^{7 / 2}}{c x}-2 a^{3} x\left(-a^{2} c x^{2}+c\right)^{5 / 2}-\frac{5 a^{3} c x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{2}-\frac{15 a^{3} c^{2} x \sqrt{-a^{2} c x^{2}+c}}{4}-\frac{15 a^{3} c^{3} \arctan \left(\frac{\sqrt{a^{2} c} x}{\sqrt{-a^{2} c x^{2}+c}}\right)}{4}+\frac{2 a^{2}\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{5 / 2}}{\sqrt{5}}+\frac{a^{3} c\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{3 / 2} x}{2}+\frac{3 a^{3} c^{2} \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c x}}{4} \\
& -\frac{3 a^{3} c^{3} \arctan \left(\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}{4 \sqrt{a^{2} c}}
\end{aligned}
$$

Problem 291: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2}\left(-a^{2} c x^{2}+c\right)^{7 / 2}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 3, 125 leaves, 8 steps):
$\frac{15 c^{2} x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{64}+\frac{3 c x\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{16}-\frac{9\left(-a^{2} c x^{2}+c\right)^{7 / 2}}{56 a}-\frac{(a x+1)\left(-a^{2} c x^{2}+c\right)^{7 / 2}}{8 a}+\frac{45 c^{7 / 2} \arctan \left(\frac{a x \sqrt{c}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{128 a}$

$$
+\frac{45 c^{3} x \sqrt{-a^{2} c x^{2}+c}}{128}
$$

Result(type 3, 295 leaves):

$$
\begin{aligned}
& -\frac{x\left(-a^{2} c x^{2}+c\right)^{7 / 2}}{8}-\frac{7 c x\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{48}-\frac{35 c^{2} x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{192}-\frac{35 c^{3} x \sqrt{-a^{2} c x^{2}+c}}{128}-\frac{35 c^{4} \arctan \left(\frac{\sqrt{a^{2} c} x}{\sqrt{-a^{2} c x^{2}+c}}\right)}{128 \sqrt{a^{2} c}} \\
& -\frac{2\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{7 / 2}}{7 a}+\frac{c\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{5 / 2} x}{3}+\frac{5 c^{2}\left(-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c\right)^{3 / 2} x}{12} \\
& \quad+\frac{5 c^{3} \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c} x}{8}+\frac{5 c^{4} \arctan \left(\frac{\sqrt{a^{2} c} x}{\left.\sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}\right)}\right.}{8 \sqrt{a^{2} c}}
\end{aligned}
$$

Problem 295: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2} x^{2}}{\left(-x^{2} a^{2}+1\right)\left(-a^{2} c x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 79 leaves, 5 steps):

$$
\frac{(a x+1)^{2}}{3 a^{3}\left(-a^{2} c x^{2}+c\right)^{3 / 2}}+\frac{\arctan \left(\frac{a x \sqrt{c}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{a^{3} c^{3 / 2}}-\frac{5(a x+1)}{3 a^{3} c \sqrt{-a^{2} c x^{2}+c}}
$$

Result(type 3, 165 leaves):

$$
\begin{aligned}
& -\frac{3 x}{a^{2} c \sqrt{-a^{2} c x^{2}+c}}+\frac{\arctan \left(\frac{\sqrt{a^{2} c} x}{\sqrt{-a^{2} c x^{2}+c}}\right)}{a^{2} c \sqrt{a^{2} c}}-\frac{2}{a^{3} c \sqrt{-a^{2} c x^{2}+c}} \\
& \quad+\frac{4 x}{3 a^{4} c\left(x-\frac{1}{a}\right) \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}} \\
& \quad 3 a^{2} c \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}
\end{aligned}
$$

Problem 296: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{2}}{\left(-x^{2} a^{2}+1\right) x\left(-a^{2} c x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 68 leaves, 7 steps):

$$
\frac{2(a x+1)}{3\left(-a^{2} c x^{2}+c\right)^{3 / 2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{-a^{2} c x^{2}+c}}{\sqrt{c}}\right)}{c^{3 / 2}}+\frac{4 a x+3}{3 c \sqrt{-a^{2} c x^{2}+c}}
$$

Result(type 3, 151 leaves):

$$
\frac{1}{c \sqrt{-a^{2} c x^{2}+c}}-\frac{\ln \left(\frac{2 c+2 \sqrt{c} \sqrt{-a^{2} c x^{2}+c}}{x}\right)}{c^{3 / 2}}-\frac{2}{3 c a\left(x-\frac{1}{a}\right) \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}}-\frac{2\left(-2 c\left(x-\frac{1}{a}\right) a^{2}-2 c a\right)}{3 a c^{2} \sqrt{-c\left(x-\frac{1}{a}\right)^{2} a^{2}-2\left(x-\frac{1}{a}\right) a c}}
$$

Problem 298: Unable to integrate problem.

$$
\int \frac{(a x+1)^{2} x^{m}\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Optimal(type 5, 154 leaves, 7 steps):
$-\frac{x^{1+m}\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{4+m}+\frac{c(5+2 m) x^{1+m} \text { hypergeom }\left(\left[-\frac{1}{2}, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right], x^{2} a^{2}\right) \sqrt{-a^{2} c x^{2}+c}}{(1+m)(4+m) \sqrt{-x^{2} a^{2}+1}}$
$+\frac{2 a c x^{2+m} \text { hypergeom }\left(\left[-\frac{1}{2}, 1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right], x^{2} a^{2}\right) \sqrt{-a^{2} c x^{2}+c}}{(2+m) \sqrt{-x^{2} a^{2}+1}}$
Result(type 8, 38 leaves):

$$
\int \frac{(a x+1)^{2} x^{m}\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{-x^{2} a^{2}+1} \mathrm{~d} x
$$

Problem 300: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{3}\left(-a^{2} c x^{2}+c\right)}{\left(-x^{2} a^{2}+1\right)^{3 / 2} x^{2}} \mathrm{~d} x
$$

Optimal(type 3, 60 leaves, 8 steps):
$3 a c \arcsin (a x)-3 a c \operatorname{arctanh}\left(\sqrt{-x^{2} a^{2}+1}\right)-a c \sqrt{-x^{2} a^{2}+1}-\frac{c \sqrt{-x^{2} a^{2}+1}}{x}$
Result(type 3, 121 leaves):

$$
\frac{c a^{3} x^{2}}{\sqrt{-x^{2} a^{2}+1}}-\frac{c a}{\sqrt{-x^{2} a^{2}+1}}-\frac{c}{x \sqrt{-x^{2} a^{2}+1}}+\frac{c a^{2} x}{\sqrt{-x^{2} a^{2}+1}}-3 c a \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2} a^{2}+1}}\right)+\frac{3 c a^{2} \arctan \left(\frac{\sqrt{a^{2}} x}{\sqrt{-x^{2} a^{2}+1}}\right)}{\sqrt{a^{2}}}
$$

Problem 301: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a x+1)^{3}\left(-a^{2} c x^{2}+c\right)}{\left(-x^{2} a^{2}+1\right)^{3 / 2} x^{5}} \mathrm{~d} x
$$

Optimal(type 3, 99 leaves, 8 steps):

$$
-\frac{15 a^{4} c \operatorname{arctanh}\left(\sqrt{-x^{2} a^{2}+1}\right)}{8}-\frac{c \sqrt{-x^{2} a^{2}+1}}{4 x^{4}}-\frac{a c \sqrt{-x^{2} a^{2}+1}}{x^{3}}-\frac{15 a^{2} c \sqrt{-x^{2} a^{2}+1}}{8 x^{2}}-\frac{3 a^{3} c \sqrt{-x^{2} a^{2}+1}}{x}
$$

Result(type 3, 230 leaves):


Problem 307: Unable to integrate problem.

$$
\int \frac{(a x+1)^{3} x^{m} \sqrt{-a^{2} c x^{2}+c}}{\left(-x^{2} a^{2}+1\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 126 leaves, 5 steps):

$$
-\frac{3 x^{1+m} \sqrt{-a^{2} c x^{2}+c}}{(1+m) \sqrt{-x^{2} a^{2}+1}}-\frac{a x^{2}+m \sqrt{-a^{2} c x^{2}+c}}{(2+m) \sqrt{-x^{2} a^{2}+1}}+\frac{4 x^{1+m} \text { hypergeom }([1,1+m],[2+m], a x) \sqrt{-a^{2} c x^{2}+c}}{(1+m) \sqrt{-x^{2} a^{2}+1}}
$$

Result(type 8, 38 leaves):

$$
\int \frac{(a x+1)^{3} x^{m} \sqrt{-a^{2} c x^{2}+c}}{\left(-x^{2} a^{2}+1\right)^{3 / 2}} \mathrm{~d} x
$$

[^4]$$
\int \frac{(a x+1)^{3} x\left(-a^{2} c x^{2}+c\right)^{p}}{\left(-x^{2} a^{2}+1\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 122 leaves, 5 steps):

$$
\frac{32^{\frac{3}{2}+p}(-a x+1)^{-\frac{1}{2}+p}\left(-a^{2} c x^{2}+c\right)^{p} \text { hypergeom }\left(\left[-\frac{1}{2}+p,-\frac{3}{2}-p\right],\left[\frac{1}{2}+p\right],-\frac{a x}{2}+\frac{1}{2}\right)}{a^{2}\left(-2 p^{2}-p+1\right)\left(-x^{2} a^{2}+1\right)^{p}}-\frac{(a x+1)^{3}\left(-a^{2} c x^{2}+c\right)^{p}}{2 a^{2}(1+p) \sqrt{-x^{2} a^{2}+1}}
$$

Result(type 8, 36 leaves):

$$
\int \frac{(a x+1)^{3} x\left(-a^{2} c x^{2}+c\right)^{p}}{\left(-x^{2} a^{2}+1\right)^{3 / 2}} \mathrm{~d} x
$$

Problem 309: Unable to integrate problem.

$$
\int \frac{(a x+1)^{3}\left(-a^{2} c x^{2}+c\right)^{p}}{\left(-x^{2} a^{2}+1\right)^{3 / 2} x^{3}} \mathrm{~d} x
$$

Optimal(type 5, 178 leaves, 8 steps):

$$
\begin{aligned}
& \frac{a^{3}(7-6 p) x\left(-a^{2} c x^{2}+c\right)^{p} \text { hypergeom }\left(\left[\frac{1}{2}, \frac{3}{2}-p\right],\left[\frac{3}{2}\right], x^{2} a^{2}\right)}{\left(-x^{2} a^{2}+1\right)^{p}}-\frac{\left(-a^{2} c x^{2}+c\right)^{p}}{2 x^{2} \sqrt{-x^{2} a^{2}+1}}-\frac{3 a\left(-a^{2} c x^{2}+c\right)^{p}}{x \sqrt{-x^{2} a^{2}+1}} \\
& +\frac{a^{2}(9-2 p)\left(-a^{2} c x^{2}+c\right)^{p} \text { hypergeom }\left(\left[1,-\frac{1}{2}+p\right],\left[\frac{1}{2}+p\right],-x^{2} a^{2}+1\right)}{2(1-2 p) \sqrt{-x^{2} a^{2}+1}}
\end{aligned}
$$

Result(type 8, 38 leaves):

$$
\int \frac{(a x+1)^{3}\left(-a^{2} c x^{2}+c\right)^{p}}{\left(-x^{2} a^{2}+1\right)^{3 / 2} x^{3}} \mathrm{~d} x
$$

Problem 313: Unable to integrate problem.

$$
\int \frac{(a x+1)^{4}\left(-a^{2} c x^{2}+c\right)^{p}}{\left(-x^{2} a^{2}+1\right)^{2}} \mathrm{~d} x
$$

Optimal(type 5, 61 leaves, 3 steps):

$$
\frac{2^{2+p} c(a x+1)^{1-p}\left(-a^{2} c x^{2}+c\right)^{-1+p} \text { hypergeom }\left([-2-p,-1+p],[p],-\frac{a x}{2}+\frac{1}{2}\right)}{a(1-p)}
$$

Result(type 8, 35 leaves):

$$
\int \frac{(a x+1)^{4}\left(-a^{2} c x^{2}+c\right)^{p}}{\left(-x^{2} a^{2}+1\right)^{2}} \mathrm{~d} x
$$

Problem 320: Unable to integrate problem.

$$
\int \frac{\left(-x^{2} a^{2}+1\right)^{p} \sqrt{-x^{2} a^{2}+1}}{a x+1} \mathrm{~d} x
$$

Optimal(type 5, 47 leaves, 2 steps):

$$
-\frac{2^{\frac{1}{2}+p}(-a x+1)^{\frac{3}{2}+p} \text { hypergeom }\left(\left[\frac{3}{2}+p, \frac{1}{2}-p\right],\left[\frac{5}{2}+p\right],-\frac{a x}{2}+\frac{1}{2}\right)}{a(3+2 p)}
$$

Result(type 8, 34 leaves):

$$
\int \frac{\left(-x^{2} a^{2}+1\right)^{p} \sqrt{-x^{2} a^{2}+1}}{a x+1} \mathrm{~d} x
$$

Problem 321: Unable to integrate problem.

$$
\int \frac{\left(-x^{2} a^{2}+1\right)^{p} \sqrt{-x^{2} a^{2}+1}}{(a x+1) x^{2}} \mathrm{~d} x
$$

Optimal(type 5, 66 leaves, 5 steps):

$$
-\frac{\text { hypergeom }\left(\left[-\frac{1}{2}, \frac{1}{2}-p\right],\left[\frac{1}{2}\right], x^{2} a^{2}\right)}{x}+\frac{a\left(-x^{2} a^{2}+1\right)^{\frac{1}{2}+p} \text { hypergeom }\left(\left[1, \frac{1}{2}+p\right],\left[\frac{3}{2}+p\right],-x^{2} a^{2}+1\right)}{1+2 p}
$$

Result(type 8, 37 leaves):

$$
\int \frac{\left(-x^{2} a^{2}+1\right)^{p} \sqrt{-x^{2} a^{2}+1}}{(a x+1) x^{2}} \mathrm{~d} x
$$

Problem 322: Unable to integrate problem.

$$
\int \frac{x^{3}\left(-a^{2} c x^{2}+c\right)^{p} \sqrt{-x^{2} a^{2}+1}}{a x+1} \mathrm{~d} x
$$

Optimal(type 5, 124 leaves, 7 steps):

$$
\frac{\left(-x^{2} a^{2}+1\right)^{3 / 2}\left(-a^{2} c x^{2}+c\right)^{p}}{a^{4}(3+2 p)}-\frac{a x^{5}\left(-a^{2} c x^{2}+c\right)^{p} \text { hypergeom }\left(\left[\frac{5}{2}, \frac{1}{2}-p\right],\left[\frac{7}{2}\right], x^{2} a^{2}\right)}{5\left(-x^{2} a^{2}+1\right)^{p}}-\frac{\left(-a^{2} c x^{2}+c\right)^{p} \sqrt{-x^{2} a^{2}+1}}{a^{4}(1+2 p)}
$$

Result(type 8, 38 leaves):

$$
\int \frac{x^{3}\left(-a^{2} c x^{2}+c\right)^{p} \sqrt{-x^{2} a^{2}+1}}{a x+1} \mathrm{~d} x
$$

Problem 327: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(-a^{2} c x^{2}+c\right)^{5 / 2}\left(-x^{2} a^{2}+1\right)}{(a x+1)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 107 leaves, 7 steps):

$$
\frac{7 c x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{24}+\frac{7\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{30 a}+\frac{(-a x+1)\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{6 a}+\frac{7 c^{5 / 2} \arctan \left(\frac{a x \sqrt{c}}{\sqrt{-a^{2} c x^{2}+c}}\right)}{16 a}+\frac{7 c^{2} x \sqrt{-a^{2} c x^{2}+c}}{16}
$$

Result(type 3, 225 leaves):

$$
\begin{aligned}
& -\frac{x\left(-a^{2} c x^{2}+c\right)^{5 / 2}}{6}-\frac{5 c x\left(-a^{2} c x^{2}+c\right)^{3 / 2}}{24}-\frac{5 c^{2} x \sqrt{-a^{2} c x^{2}+c}}{16}-\frac{5 c^{3} \arctan \left(\frac{\sqrt{a^{2} c} x}{\sqrt{-a^{2} c x^{2}+c}}\right)}{16 \sqrt{a^{2} c}}+\frac{2\left(-c\left(x+\frac{1}{a}\right)^{2} a^{2}+2\left(x+\frac{1}{a}\right) a c\right)^{5 / 2}}{5 a} \\
& \quad+\frac{c\left(-c\left(x+\frac{1}{a}\right)^{2} a^{2}+2\left(x+\frac{1}{a}\right) a c\right)^{3 / 2} x}{2}+\frac{3 c^{2} \sqrt{-c\left(x+\frac{1}{a}\right)^{2} a^{2}+2\left(x+\frac{1}{a}\right) a c x}}{4}+\frac{3 c^{3} \arctan \left(\frac{\sqrt{a^{2} c} x}{\left.\sqrt{-c\left(x+\frac{1}{a}\right)^{2} a^{2}+2\left(x+\frac{1}{a}\right) a c}\right)}\right.}{4 \sqrt{a^{2} c}}
\end{aligned}
$$

Problem 334: Unable to integrate problem.

$$
\int \frac{x^{m} \sqrt{-a^{2} c x^{2}+c}\left(-x^{2} a^{2}+1\right)^{3 / 2}}{(a x+1)^{3}} \mathrm{~d} x
$$

Optimal(type 5, 126 leaves, 5 steps):

$$
-\frac{3 x^{1+m} \sqrt{-a^{2} c x^{2}+c}}{(1+m) \sqrt{-x^{2} a^{2}+1}}+\frac{a x^{2}+m \sqrt{-a^{2} c x^{2}+c}}{(2+m) \sqrt{-x^{2} a^{2}+1}}+\frac{4 x^{1+m} \text { hypergeom }([1,1+m],[2+m],-a x) \sqrt{-a^{2} c x^{2}+c}}{(1+m) \sqrt{-x^{2} a^{2}+1}}
$$

Result(type 8, 38 leaves):

$$
\int \frac{x^{m} \sqrt{-a^{2} c x^{2}+c}\left(-x^{2} a^{2}+1\right)^{3 / 2}}{(a x+1)^{3}} \mathrm{~d} x
$$

[^5]$$
\int \frac{\left(-a^{2} c x^{2}+c\right)^{p}\left(-x^{2} a^{2}+1\right)^{3 / 2}}{(a x+1)^{3}} \mathrm{~d} x
$$

Optimal(type 5, 74 leaves, 3 steps):

$$
-\frac{2^{-\frac{1}{2}+p}(-a x+1)^{\frac{5}{2}+p}\left(-a^{2} c x^{2}+c\right)^{p} \text { hypergeom }\left(\left[\frac{5}{2}+p, \frac{3}{2}-p\right],\left[\frac{7}{2}+p\right],-\frac{a x}{2}+\frac{1}{2}\right)}{a(5+2 p)\left(-x^{2} a^{2}+1\right)^{p}}
$$

Result(type 8, 35 leaves):

$$
\int \frac{\left(-a^{2} c x^{2}+c\right)^{p}\left(-x^{2} a^{2}+1\right)^{3 / 2}}{(a x+1)^{3}} \mathrm{~d} x
$$

Problem 337: Unable to integrate problem.

$$
\int \frac{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}}{\left(-a^{2} c x^{2}+c\right)^{5 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 91 leaves, 5 steps):

$$
-\frac{\left(-x^{2} a^{2}+1\right)^{1 / 4} \operatorname{arctanh}\left(\frac{\sqrt{-a x+1} \sqrt{2}}{2}\right) \sqrt{2}}{2 a c\left(-a^{2} c x^{2}+c\right)^{1 / 4}}+\frac{\left(-x^{2} a^{2}+1\right)^{1 / 4}}{a c\left(-a^{2} c x^{2}+c\right)^{1 / 4} \sqrt{-a x+1}}
$$

Result(type 8, 36 leaves):

$$
\int \frac{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}}{\left(-a^{2} c x^{2}+c\right)^{5 / 4}} d x
$$

Problem 338: Unable to integrate problem.

$$
\int \frac{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}}{x^{2}\left(-a^{2} c x^{2}+c\right)^{5 / 4}} d x
$$

Optimal(type 3, 169 leaves, 9 steps):
$-\frac{a\left(-x^{2} a^{2}+1\right)^{1 / 4} \operatorname{arctanh}(\sqrt{-a x+1})}{c\left(-a^{2} c x^{2}+c\right)^{1 / 4}}-\frac{a\left(-x^{2} a^{2}+1\right)^{1 / 4} \operatorname{arctanh}\left(\frac{\sqrt{-a x+1} \sqrt{2}}{2}\right) \sqrt{2}}{2 c\left(-a^{2} c x^{2}+c\right)^{1 / 4}}+\frac{2 a\left(-x^{2} a^{2}+1\right)^{1 / 4}}{c\left(-a^{2} c x^{2}+c\right)^{1 / 4} \sqrt{-a x+1}}$

$$
-\frac{\left(-x^{2} a^{2}+1\right)^{1 / 4}}{c x\left(-a^{2} c x^{2}+c\right)^{1 / 4} \sqrt{-a x+1}}
$$

Result(type 8, 39 leaves):

$$
\int \frac{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}}}{x^{2}\left(-a^{2} c x^{2}+c\right)^{5 / 4}} d x
$$

Problem 339: Unable to integrate problem.

$$
\int \frac{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}} x^{3}}{\left(-a^{2} c x^{2}+c\right)^{9 / 8}} \mathrm{~d} x
$$

Optimal(type 5, 162 leaves, 5 steps):

$$
\begin{aligned}
& -\frac{4 x^{2}(a x+1)^{1 / 8}\left(-x^{2} a^{2}+1\right)^{1 / 8}}{7 a^{2} c(-a x+1)^{3 / 8}\left(-a^{2} c x^{2}+c\right)^{1 / 8}}+\frac{8(-a x+6)(a x+1)^{1 / 8}\left(-x^{2} a^{2}+1\right)^{1 / 8}}{21 a^{4} c(-a x+1)^{3 / 8}\left(-a^{2} c x^{2}+c\right)^{1 / 8}} \\
& +\frac{642^{1 / 8}(-a x+1)^{5 / 8}\left(-x^{2} a^{2}+1\right)^{1 / 8} \operatorname{hypergeom}\left(\left[\frac{5}{8}, \frac{7}{8}\right],\left[\frac{13}{8}\right],-\frac{a x}{2}+\frac{1}{2}\right)}{105 a^{4} c\left(-a^{2} c x^{2}+c\right)^{1 / 8}}
\end{aligned}
$$

Result(type 8, 39 leaves):

$$
\int \frac{\sqrt{\frac{a x+1}{\sqrt{-x^{2} a^{2}+1}}} x^{3}}{\left(-a^{2} c x^{2}+c\right)^{9 / 8}} \mathrm{~d} x
$$

Problem 340: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)} x}{-a^{2} c x^{2}+c} \mathrm{~d} x
$$

Optimal(type 5, 82 leaves, 3 steps):

$$
-\frac{(a x+1)^{\frac{n}{2}}}{a^{2} \operatorname{cn}(-a x+1)^{\frac{n}{2}}}+\frac{2^{1+\frac{n}{2}} \text { hypergeom }\left(\left[-\frac{n}{2},-\frac{n}{2}\right],\left[1-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right)}{a^{2} \operatorname{cn}(-a x+1)^{\frac{n}{2}}}
$$

Result(type 8, 24 leaves):

$$
\int \frac{\mathrm{e}^{n} \operatorname{arctanh}(a x) x}{-a^{2} c x^{2}+c} \mathrm{~d} x
$$

Problem 341: Unable to integrate problem.

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)} x^{3} \sqrt{-a^{2} c x^{2}+c} \mathrm{~d} x
$$

Optimal(type 5, 206 leaves, 5 steps):
$-\frac{x^{2}(-a x+1)^{\frac{3}{2}-\frac{n}{2}}(a x+1)^{\frac{3}{2}+\frac{n}{2}} \sqrt{-a^{2} c x^{2}+c}}{5 a^{2} \sqrt{-x^{2} a^{2}+1}}-\frac{(-a x+1)^{\frac{3}{2}-\frac{n}{2}}(a x+1)^{\frac{3}{2}+\frac{n}{2}}\left(3 a n x+n^{2}+8\right) \sqrt{-a^{2} c x^{2}+c}}{60 a^{4} \sqrt{-x^{2} a^{2}+1}}$
$-2^{-\frac{1}{2}+\frac{n}{2}} n\left(n^{2}+11\right)(-a x+1)^{\frac{3}{2}-\frac{n}{2}}$ hypergeom $\left(\left[\frac{3}{2}-\frac{n}{2},-\frac{1}{2}-\frac{n}{2}\right],\left[\frac{5}{2}-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right) \sqrt{-a^{2} c x^{2}+c}$

$$
15 a^{4}(3-n) \sqrt{-x^{2} a^{2}+1}
$$

Result(type 8, 26 leaves):

$$
\int \mathrm{e}^{n \operatorname{arctanh}(a x)} x^{3} \sqrt{-a^{2} c x^{2}+c} \mathrm{~d} x
$$

Problem 342: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)} \sqrt{-a^{2} c x^{2}+c}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 5, 218 leaves, 6 steps):

$$
\begin{aligned}
& -\frac{(-a x+1)^{\frac{1}{2}-\frac{n}{2}}(a x+1)^{\frac{1}{2}+\frac{n}{2}} \sqrt{-a^{2} c x^{2}+c}}{x \sqrt{-x^{2} a^{2}+1}}-\frac{2 a n(-a x+1)^{\frac{1}{2}-\frac{n}{2}}(a x+1)^{-\frac{1}{2}+\frac{n}{2}} \text { hypergeom }\left(\left[1, \frac{1}{2}-\frac{n}{2}\right],\left[\frac{3}{2}-\frac{n}{2}\right], \frac{-a x+1}{a x+1}\right) \sqrt{-a^{2} c x^{2}+c}}{(1-n) \sqrt{-x^{2} a^{2}+1}} \\
& \quad+\frac{2^{\frac{1}{2}+\frac{n}{2}} a(-a x+1)^{\frac{1}{2}-\frac{n}{2}} \text { hypergeom }\left(\left[\frac{1}{2}-\frac{n}{2}, \frac{1}{2}-\frac{n}{2}\right],\left[\frac{3}{2}-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right) \sqrt{-a^{2} c x^{2}+c}}{}
\end{aligned}
$$

Result(type 8, 26 leaves):

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)} \sqrt{-a^{2} c x^{2}+c}}{x^{2}} \mathrm{~d} x
$$

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{\sqrt{-a^{2} c x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 5, 80 leaves, 3 steps):
$-\frac{2^{\frac{1}{2}+\frac{n}{2}}(-a x+1)^{\frac{1}{2}-\frac{n}{2}} \text { hypergeom }\left(\left[\frac{1}{2}-\frac{n}{2}, \frac{1}{2}-\frac{n}{2}\right],\left[\frac{3}{2}-\frac{n}{2}\right],-\frac{a x}{2}+\frac{1}{2}\right) \sqrt{-x^{2} a^{2}+1}}{a(1-n) \sqrt{-a^{2} c x^{2}+c}}$
Result(type 8, 23 leaves):

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{\sqrt{-a^{2} c x^{2}+c}} \mathrm{~d} x
$$

Problem 344: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{x^{3} \sqrt{-a^{2} c x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 5, 202 leaves, 6 steps):

$$
\begin{aligned}
& -\frac{(-a x+1)^{\frac{1}{2}-\frac{n}{2}}(a x+1)^{\frac{1}{2}+\frac{n}{2}} \sqrt{-x^{2} a^{2}+1}}{2 x^{2} \sqrt{-a^{2} c x^{2}+c}}-\frac{a n(-a x+1)^{\frac{1}{2}-\frac{n}{2}}(a x+1)^{\frac{1}{2}+\frac{n}{2}} \sqrt{-x^{2} a^{2}+1}}{2 x \sqrt{-a^{2} c x^{2}+c}} \\
& -\frac{a^{2}\left(n^{2}+1\right)(-a x+1)^{\frac{1}{2}-\frac{n}{2}}(a x+1)^{-\frac{1}{2}+\frac{n}{2}} \operatorname{hypergeom}\left(\left[1, \frac{1}{2}-\frac{n}{2}\right],\left[\frac{3}{2}-\frac{n}{2}\right], \frac{-a x+1}{a x+1}\right) \sqrt{-x^{2} a^{2}+1}}{(1-n) \sqrt{-a^{2} c x^{2}+c}}
\end{aligned}
$$

Result(type 8, 26 leaves):

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{x^{3} \sqrt{-a^{2} c x^{2}+c}} \mathrm{~d} x
$$

Problem 347: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{x^{2}\left(-a^{2} c x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 279 leaves, 7 steps):
$\frac{a(2+n)(-a x+1)^{-\frac{1}{2}-\frac{n}{2}}(a x+1)^{-\frac{1}{2}+\frac{n}{2}} \sqrt{-x^{2} a^{2}+1}}{c(1+n) \sqrt{-a^{2} c x^{2}+c}}-\frac{(-a x+1)^{-\frac{1}{2}-\frac{n}{2}}(a x+1)^{-\frac{1}{2}+\frac{n}{2}} \sqrt{-x^{2} a^{2}+1}}{c x \sqrt{-a^{2} c x^{2}+c}}$

$$
\begin{aligned}
& -\frac{a\left(n^{2}+2 n+2\right)(-a x+1)^{\frac{1}{2}-\frac{n}{2}}(a x+1)^{-\frac{1}{2}+\frac{n}{2}} \sqrt{-x^{2} a^{2}+1}}{c\left(-n^{2}+1\right) \sqrt{-a^{2} c x^{2}+c}} \\
& +\frac{2 a n(-a x+1)^{\frac{1}{2}-\frac{n}{2}}(a x+1)^{-\frac{1}{2}+\frac{n}{2}} \operatorname{hypergeom}\left(\left[1,-\frac{1}{2}+\frac{n}{2}\right],\left[\frac{1}{2}+\frac{n}{2}\right], \frac{a x+1}{-a x+1}\right) \sqrt{-x^{2} a^{2}+1}}{c(1-n) \sqrt{-a^{2} c x^{2}+c}}
\end{aligned}
$$

Result(type 8, 26 leaves):

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)}}{x^{2}\left(-a^{2} c x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Problem 348: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)} x^{m}}{-a^{2} c x^{2}+c} \mathrm{~d} x
$$

Optimal(type 6, 38 leaves, 2 steps):

$$
\frac{x^{1+m} \text { AppellF } 1\left(1+m, 1+\frac{n}{2}, 1-\frac{n}{2}, 2+m, a x,-a x\right)}{c(1+m)}
$$

Result(type 8, 26 leaves):

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)} x^{m}}{-a^{2} c x^{2}+c} \mathrm{~d} x
$$

Problem 349: Unable to integrate problem.

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)} x^{m}}{\left(-a^{2} c x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Optimal(type 6, 38 leaves, 2 steps):

$$
\frac{x^{1+m} \text { AppellF1 }\left(1+m, 2+\frac{n}{2}, 2-\frac{n}{2}, 2+m, a x,-a x\right)}{c^{2}(1+m)}
$$

Result(type 8, 26 leaves):

$$
\int \frac{\mathrm{e}^{n \operatorname{arctanh}(a x)} x^{m}}{\left(-a^{2} c x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Test results for the 97 problems in "7.3.7 Inverse hyperbolic tangent functions.txt"
Problem 4: Result more than twice size of optimal antiderivative.

$$
\int x^{4} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right) \mathrm{d} x
$$

Optimal (type 3, 67 leaves, 4 steps):

$$
\frac{2 d\left(e x^{2}+d\right)^{3 / 2}}{15 e^{5 / 2}}-\frac{\left(e x^{2}+d\right)^{5 / 2}}{25 e^{5 / 2}}+\frac{x^{5} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right)}{5}-\frac{d^{2} \sqrt{e x^{2}+d}}{5 e^{5 / 2}}
$$

Result(type 3, 175 leaves):
$\frac{x^{5} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right)}{5}+\frac{e^{3 / 2}\left(\frac{x^{6} \sqrt{e x^{2}+d}}{7 e}-\frac{6 d\left(\frac{x^{4} \sqrt{e x^{2}+d}}{5 e}-\frac{4 d\left(\frac{x^{2} \sqrt{e x^{2}+d}}{3 e}-\frac{2 d \sqrt{e x^{2}+d}}{3 e^{2}}\right.}{3 e}\right)}{5 d}\right)}{5 e}$

$$
-\frac{\sqrt{e}\left(\frac{x^{4}\left(e x^{2}+d\right)^{3 / 2}}{7 e}-\frac{4 d\left(\frac{x^{2}\left(e x^{2}+d\right)^{3 / 2}}{5 e}-\frac{2 d\left(e x^{2}+d\right)^{3 / 2}}{15 e^{2}}\right)}{7 e}\right)}{5 d}
$$

Problem 6: Unable to integrate problem.

$$
\int x^{9 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right) \mathrm{d} x
$$

Optimal(type 4, 177 leaves, 6 steps):

$$
\frac{2 x^{11 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right)}{11}+\frac{36 d x^{5} / 2 \sqrt{e x^{2}+d}}{847 e^{3 / 2}}-\frac{4 x^{9 / 2} \sqrt{e x^{2}+d}}{121 \sqrt{e}}-\frac{60 d^{2} \sqrt{x} \sqrt{e x^{2}+d}}{847 e^{5 / 2}}
$$

$$
+\frac{30 d^{11 / 4} \sqrt{\cos \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right)^{2}} \operatorname{EllipticF}\left(\sin \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{d}+\sqrt{e} x) \sqrt{\frac{e x^{2}+d}{(\sqrt{d}+\sqrt{e} x)^{2}}}}{847 \cos \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right) e^{11 / 4} \sqrt{e x^{2}+d}}
$$

Result(type 8, 21 leaves):

$$
\int x^{9 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right) \mathrm{d} x
$$

Problem 7: Unable to integrate problem.

$$
\int x^{5 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right) \mathrm{d} x
$$

Optimal(type 4, 157 leaves, 5 steps):
$\frac{2 x^{7 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right)}{7}-\frac{4 x^{5} / 2 \sqrt{e x^{2}+d}}{49 \sqrt{e}}+\frac{20 d \sqrt{x} \sqrt{e x^{2}+d}}{147 e^{3 / 2}}$

$$
-\frac{10 d^{7 / 4} \sqrt{\cos \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right)^{2}} \operatorname{EllipticF}\left(\sin \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{d}+\sqrt{e} x) \sqrt{\frac{e x^{2}+d}{(\sqrt{d}+\sqrt{e} x)^{2}}}}{147 \cos \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right) e^{7 / 4} \sqrt{e x^{2}+d}}
$$

Result(type 8, 21 leaves):

$$
\int x^{5 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right) \mathrm{d} x
$$

Problem 8: Unable to integrate problem.

$$
\int \sqrt{x} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right) d x
$$

Optimal(type 4, 139 leaves, 4 steps):
$\frac{2 x^{3 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right)}{3}-\frac{4 \sqrt{x} \sqrt{e x^{2}+d}}{9 \sqrt{e}}$

$$
+\frac{2 d^{3} / 4 \sqrt{\cos \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right)^{2}} \operatorname{EllipticF}\left(\sin \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{d}+\sqrt{e} x) \sqrt{\frac{e x^{2}+d}{(\sqrt{d}+\sqrt{e} x)^{2}}}}{9 \cos \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right) e^{3 / 4} \sqrt{e x^{2}+d}}
$$

Result(type 8, 21 leaves):

$$
\int \sqrt{x} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right) \mathrm{d} x
$$

Problem 9: Unable to integrate problem.

$$
\int \frac{\operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right)}{x^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 122 leaves, 3 steps):

$$
-\frac{2 \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right)}{\sqrt{x}}+\frac{2 e^{1 / 4} \sqrt{\cos \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right)^{2}} \operatorname{EllipticF}\left(\sin \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{d}+\sqrt{e x}) \sqrt{\frac{e x^{2}+d}{(\sqrt{d}+\sqrt{e} x)^{2}}}}{\cos \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right) d^{1 / 4} \sqrt{e x^{2}+d}}
$$

Result(type 8, 21 leaves):

$$
\int \frac{\operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right)}{x^{3 / 2}} d x
$$

Problem 10: Unable to integrate problem.

$$
\int x^{7 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right) \mathrm{d} x
$$

Optimal(type 4, 289 leaves, 7 steps):

$$
\begin{aligned}
& \frac{2 x^{9 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right)}{9}+\frac{28 d x^{3} / 2 \sqrt{e x^{2}+d}}{405 e^{3 / 2}}-\frac{4 x^{7 / 2} \sqrt{e x^{2}+d}}{81 \sqrt{e}}-\frac{28 d^{2} \sqrt{x} \sqrt{e x^{2}+d}}{135 e^{2}(\sqrt{d}+\sqrt{e} x)} \\
& +\frac{28 d^{9 / 4} \sqrt{\cos \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right)^{2}} \operatorname{EllipticE}\left(\sin \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{d}+\sqrt{e} x) \sqrt{\frac{e x^{2}+d}{(\sqrt{d}+\sqrt{e} x)^{2}}}}{135 \cos \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right) e^{9 / 4} \sqrt{e x^{2}+d}} \\
& -\frac{14 d^{9 / 4} \sqrt{\cos \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right)^{2}} \operatorname{EllipticF}\left(\sin \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{d}+\sqrt{e} x) \sqrt{\frac{e x^{2}+d}{(\sqrt{d}+\sqrt{e} x)^{2}}}}{135 \cos \left(2 \arctan \left(\frac{e^{1 / 4} \sqrt{x}}{d^{1 / 4}}\right)\right) e^{9 / 4} \sqrt{e x^{2}+d}}
\end{aligned}
$$

Result(type 8, 21 leaves):

$$
\int x^{7 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e x^{2}+d}}\right) \mathrm{d} x
$$

Problem 11: Result more than twice size of optimal antiderivative.

$$
\int x^{-1+n} \operatorname{arctanh}\left(a+b x^{n}\right) \mathrm{d} x
$$

Optimal(type 3, 45 leaves, 4 steps):

$$
\frac{\left(a+b x^{n}\right) \operatorname{arctanh}\left(a+b x^{n}\right)}{n b}+\frac{\ln \left(1-\left(a+b x^{n}\right)^{2}\right)}{2 n b}
$$

Result(type 3, 120 leaves):

$$
\frac{x^{n} \ln \left(a+b x^{n}+1\right)}{2 n}-\frac{x^{n} \ln \left(1-a-b x^{n}\right)}{2 n}-\frac{\ln \left(x^{n}+\frac{-1+a}{b}\right) a}{2 n b}+\frac{\ln \left(x^{n}+\frac{1+a}{b}\right) a}{2 n b}+\frac{\ln \left(x^{n}+\frac{-1+a}{b}\right)}{2 n b}
$$

Problem 30: Unable to integrate problem.

$$
\int \frac{x^{m}}{\operatorname{arctanh}(\tanh (b x+a))^{3}} \mathrm{~d} x
$$

Optimal(type 5, 90 leaves, 3 steps):

$$
-\frac{x^{m}}{2 b \operatorname{arctanh}(\tanh (b x+a))^{2}}-\frac{m x^{m-1}}{2 b^{2} \operatorname{arctanh}(\tanh (b x+a))}-\frac{m x^{m-1} \operatorname{hypergeom}\left([1, m-1],[m], \frac{b x}{b x-\operatorname{arctanh}(\tanh (b x+a))}\right)}{2 b^{2}(b x-\operatorname{arctanh}(\tanh (b x+a)))}
$$

Result(type 8, 838 leaves):
$-\left(2 \mathrm{I}\left(4 \mathrm{I} \mathrm{e}^{m \ln (x)} x b+\pi m \operatorname{csgn}\left(\frac{\mathrm{I}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right) \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right) \mathrm{e}^{m \ln (x)}\right.\right.$

$$
-\pi m \operatorname{csgn}\left(\frac{\mathrm{I}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right)^{2} \mathrm{e}^{m \ln (x)}+\pi m \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{b x+a}\right)^{2} \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right) \mathrm{e}^{m \ln (x)}
$$

$$
-2 \pi m \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{b x+a}\right) \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right)^{2} \mathrm{e}^{m \ln (x)}+\pi m \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right)^{3} \mathrm{e}^{m \ln (x)}-\pi m \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right)^{2} \mathrm{e}^{m \ln (x)}
$$

$$
\left.\left.+\pi m \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right)^{3} \mathrm{e}^{m \ln (x)}+4 \mathrm{I} m \mathrm{e}^{m \ln (x)} \ln \left(\mathrm{e}^{b x+a}\right)\right)\right) /\left(\left(\pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right) \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right)\right.\right.
$$

$$
-\pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right)^{2}+\pi \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{b x+a}\right)^{2} \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right)-2 \pi \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{b x+a}\right) \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right)^{2}
$$

$$
\begin{aligned}
& \left.\left.+\pi \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right)^{3}-\pi \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right)^{2}+\pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right)^{3}+4 \mathrm{I} \ln \left(\mathrm{e}^{b x+a}\right)\right)^{2} x b^{2}\right)+\int\left(-2 \mathrm{Im} \mathrm{e}^{m \ln (x)}(m\right. \\
& -1)) /\left(b ^ { 2 } x ^ { 2 } \left(-\pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right) \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right)+\pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right)^{2}\right.\right. \\
& -\pi \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{b x+a}\right)^{2} \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right)+2 \pi \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{b x+a}\right) \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right)^{2}-\pi \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right)^{3}+\pi \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right)^{2} \\
& \left.\left.-\pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{b x+a}\right)^{2}}{\left(\mathrm{e}^{b x+a}\right)^{2}+1}\right)^{3}-4 \mathrm{I} \ln \left(\mathrm{e}^{b x+a}\right)\right)\right) \mathrm{d} x
\end{aligned}
$$

Problem 52: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{5 / 2}}{\operatorname{arctanh}(\tanh (b x+a))^{2}} \mathrm{~d} x
$$

Optimal(type 3, 90 leaves, 4 steps):

$$
\frac{5 x^{3 / 2}}{3 b^{2}}-\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{b x-\operatorname{arctanh}(\tanh (b x+a))}}\right)(b x-\operatorname{arctanh}(\tanh (b x+a)))^{3 / 2}}{b^{7 / 2}}-\frac{x^{5 / 2}}{b \operatorname{arctanh}(\tanh (b x+a))}
$$

$$
+\frac{5(b x-\operatorname{arctanh}(\tanh (b x+a))) \sqrt{x}}{b^{3}}
$$

Result(type 3, 293 leaves):
$\frac{2 x^{3 / 2}}{3 b^{2}}-\frac{4 a \sqrt{x}}{b^{3}}-\frac{4(\operatorname{arctanh}(\tanh (b x+a))-b x-a) \sqrt{x}}{b^{3}}-\frac{\sqrt{x} a^{2}}{b^{3} \operatorname{arctanh}(\tanh (b x+a))}-\frac{2 \sqrt{x} a(\operatorname{arctanh}(\tanh (b x+a))-b x-a)}{b^{3} \operatorname{arctanh}(\tanh (b x+a))}$
$-\frac{\sqrt{x}(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{2}}{b^{3} \operatorname{arctanh}(\tanh (b x+a))}+\frac{5 \arctan \left(\frac{b \sqrt{x}}{\sqrt{(-b x+\operatorname{arctanh}(\tanh (b x+a))) b}}\right) a^{2}}{b^{3} \sqrt{(-b x+\operatorname{arctanh}(\tanh (b x+a))) b}}$
$+\frac{10 \arctan \left(\frac{b \sqrt{x}}{\sqrt{(-b x+\operatorname{arctanh}(\tanh (b x+a))) b}}\right) a(\operatorname{arctanh}(\tanh (b x+a))-b x-a)}{}$
$+\frac{5 \arctan \left(\frac{b \sqrt{x}}{\sqrt{(-b x+\operatorname{arctanh}(\tanh (b x+a))) b}}\right)(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{2}}{b^{3} \sqrt{(-b x+\operatorname{arctanh}(\tanh (b x+a))) b}}$

Problem 54: Result more than twice size of optimal antiderivative.


Optimal(type 3, 107 leaves, 5 steps):
$\frac{35 x^{3 / 2}}{12 b^{3}}-\frac{35 \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{b x-\operatorname{arctanh}(\tanh (b x+a))}}\right)(b x-\operatorname{arctanh}(\tanh (b x+a)))^{3 / 2}}{4 b^{9 / 2}}-\frac{x^{7 / 2}}{2 b \operatorname{arctanh}(\tanh (b x+a))^{2}}-\frac{7 x^{5} / 2}{4 b^{2} \operatorname{arctanh}(\tanh (b x+a))}$

$$
+\frac{35(b x-\operatorname{arctanh}(\tanh (b x+a))) \sqrt{x}}{4 b^{4}}
$$

Result (type 3, 417 leaves):

$$
\begin{aligned}
& \frac{2 x^{3 / 2}}{3 b^{3}}-\frac{6 a \sqrt{x}}{b^{4}}-\frac{6(\operatorname{arctanh}(\tanh (b x+a))-b x-a) \sqrt{x}}{b^{4}}-\frac{13 a^{2} x^{3} / 2}{4 b^{3} \operatorname{arctanh}(\tanh (b x+a))^{2}}-\frac{13 x^{3} / 2 a(\operatorname{arctanh}(\tanh (b x+a))-b x-a)}{2 b^{3} \operatorname{arctanh}(\tanh (b x+a))^{2}} \\
& -\frac{13 x^{3 / 2}(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{2}}{4 b^{3} \operatorname{arctanh}(\tanh (b x+a))^{2}}-\frac{11 a^{3} \sqrt{x}}{4 b^{4} \operatorname{arctanh}(\tanh (b x+a))^{2}}-\frac{33 \sqrt{x} a^{2}(\operatorname{arctanh}(\tanh (b x+a))-b x-a)}{4 b^{4} \operatorname{arctanh}(\tanh (b x+a))^{2}} \\
& -\frac{33 \sqrt{x} a(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{2}}{4 b^{4} \operatorname{arctanh}(\tanh (b x+a))^{2}}-\frac{11 \sqrt{x}(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{3}}{4 b^{4} \operatorname{arctanh}(\tanh (b x+a))^{2}} \\
& +\frac{35 \arctan \left(\frac{b \sqrt{x}}{\sqrt{(-b x+\operatorname{arctanh}(\tanh (b x+a))) b}}\right) a^{2}}{4 b^{4} \sqrt{(-b x+\operatorname{arctanh}(\tanh (b x+a))) b}}+\frac{35 \arctan \left(\frac{b \sqrt{x}}{\sqrt{(-b x+\operatorname{arctanh}(\tanh (b x+a))) b}}\right) a(\operatorname{arctanh}(\tanh (b x+a))-b x-a)}{\left.2 b^{4} \sqrt{(-b x+\arctan }\right)} \\
& +\frac{35 \arctan \left(\frac{b \sqrt{x}}{\sqrt{(-b x+\operatorname{arctanh}(\tanh (b x+a))) b}}\right)(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{2}}{} \\
& 4 b^{4} \sqrt{(-b x+\operatorname{arctanh}(\tanh (b x+a))) b}
\end{aligned}
$$

Problem 55: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{5 / 2}}{\operatorname{arctanh}(\tanh (b x+a))^{3}} \mathrm{~d} x
$$

Optimal(type 3, 86 leaves, 4 steps):

$$
-\frac{x^{5 / 2}}{2 b \operatorname{arctanh}(\tanh (b x+a))^{2}}-\frac{5 x^{3 / 2}}{4 b^{2} \operatorname{arctanh}(\tanh (b x+a))}+\frac{15 \sqrt{x}}{4 b^{3}}-\frac{15 \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{b x-\operatorname{arctanh}(\tanh (b x+a))}}\right) \sqrt{b x-\operatorname{arctanh}(\tanh (b x+a))}}{4 b^{7 / 2}}
$$

Result(type 3, 248 leaves):
$\frac{2 \sqrt{x}}{b^{3}}+\frac{9 x^{3 / 2} a}{4 b^{2} \operatorname{arctanh}(\tanh (b x+a))^{2}}+\frac{9 x^{3 / 2}(\operatorname{arctanh}(\tanh (b x+a))-b x-a)}{4 b^{2} \operatorname{arctanh}(\tanh (b x+a))^{2}}+\frac{7 \sqrt{x} a^{2}}{4 b^{3} \operatorname{arctanh}(\tanh (b x+a))^{2}}$


Problem 59: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(\tanh (b x+a))^{3 / 2}}{\sqrt{x}} \mathrm{~d} x
$$

Optimal (type 3, 79 leaves, 3 steps):


$$
-\frac{3(b x-\operatorname{arctanh}(\tanh (b x+a))) \sqrt{x} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{4}
$$

Result (type 3, 164 leaves):
$\frac{\operatorname{arctanh}(\tanh (b x+a))^{3 / 2} \sqrt{x}}{2}+\frac{3 a \sqrt{x} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{4}+\frac{3 \ln (\sqrt{b} \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh (b x+a))}) a^{2}}{4 \sqrt{b}}$

$$
\begin{aligned}
& +\frac{3 a \ln (\sqrt{b} \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh (b x+a))})(\operatorname{arctanh}(\tanh (b x+a))-b x-a)}{2 \sqrt{b}} \\
& +\frac{3(\operatorname{arctanh}(\tanh (b x+a))-b x-a) \sqrt{x} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{4} \\
& +\frac{3 \ln (\sqrt{b} \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh (b x+a))})(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{2}}{4 \sqrt{b}}
\end{aligned}
$$

Problem 63: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arctanh}(\tanh (b x+a))^{5 / 2}}{\sqrt{x}} \mathrm{~d} x
$$

Optimal(type 3, 108 leaves, 4 steps):

```
\(-\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh (b x+a))}}\right)\left(b x-\operatorname{arctanh}(\tanh (b x+a))^{3}\right.}{8 \sqrt{b}}-\frac{5(b x-\operatorname{arctanh}(\tanh (b x+a))) \operatorname{arctanh}(\tanh (b x+a))^{3} / 2 \sqrt{x}}{12}\)
    \(+\frac{\operatorname{arctanh}(\tanh (b x+a))^{5 / 2} \sqrt{x}}{3}+\frac{5(b x-\operatorname{arctanh}(\tanh (b x+a)))^{2} \sqrt{x} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{8}\)
Result(type 3, 285 leaves):
\(\frac{\operatorname{arctanh}(\tanh (b x+a))^{5} / 2 \sqrt{x}}{3}+\frac{5 a \sqrt{x} \operatorname{arctanh}(\tanh (b x+a))^{3 / 2}}{12}+\frac{5 a^{2} \sqrt{x} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{8}+\frac{5 \ln (\sqrt{b} \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh (b x+a))}) a^{3}}{8 \sqrt{b}}\)
    \(+\frac{15 a^{2} \ln (\sqrt{b} \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh (b x+a))})(\operatorname{arctanh}(\tanh (b x+a))-b x-a)}{8 \sqrt{b}}\)
    \(+\frac{5 a(\operatorname{arctanh}(\tanh (b x+a))-b x-a) \sqrt{x} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{4}\)
    \(+\frac{15 a \ln (\sqrt{b} \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh (b x+a))})(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{2}}{8 \sqrt{b}}\)
    \(+\frac{5(\operatorname{arctanh}(\tanh (b x+a))-b x-a) \sqrt{x} \operatorname{arctanh}(\tanh (b x+a))^{3 / 2}}{12}+\frac{5(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{2} \sqrt{x} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{8}\)
    \(+\frac{5 \ln (\sqrt{b} \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh (b x+a))})(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{3}}{8 \sqrt{b}}\)
```

Problem 66: Result more than twice size of optimal antiderivative


Optimal(type 3, 85 leaves, 3 steps):
$\frac{3 \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh (b x+a))}}\right)(b x-\operatorname{arctanh}(\tanh (b x+a)))^{2}}{4 b^{5} / 2}+\frac{x^{3 / 2} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{2 b}$

$$
+\frac{3(b x-\operatorname{arctanh}(\tanh (b x+a))) \sqrt{x} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{4 b^{2}}
$$

Result(type 3, 173 leaves):

$$
\begin{aligned}
& \frac{x^{3 / 2} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{2 b}-\frac{3 a \sqrt{x} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{4 b^{2}}+\frac{3 \ln (\sqrt{b} \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh (b x+a))}) a^{2}}{4 b^{5 / 2}} \\
& \quad+\frac{3 a \ln (\sqrt{b} \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh (b x+a))})(\operatorname{arctanh}(\tanh (b x+a))-b x-a)}{2 b^{5 / 2}}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{3(\operatorname{arctanh}(\tanh (b x+a))-b x-a) \sqrt{x} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{4 b^{2}} \\
& +\frac{3 \ln (\sqrt{b} \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh (b x+a))})(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{2}}{4 b^{5} / 2}
\end{aligned}
$$

Problem 69: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{5 / 2}}{\operatorname{arctanh}(\tanh (b x+a))^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 102 leaves, 4 steps):


$$
+\frac{15(b x-\operatorname{arctanh}(\tanh (b x+a))) \sqrt{x} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{4 b^{3}}
$$

Result(type 3, 260 leaves):


Problem 70: Result more than twice size of optimal antiderivative.


Optimal(type 3, 87 leaves, 4 steps):
$\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh (b x+a))}}\right)(b x-\operatorname{arctanh}(\tanh (b x+a)))}{b^{7 / 2}}-\frac{2 x^{5 / 2}}{3 b \operatorname{arctanh}(\tanh (b x+a))^{3 / 2}}-\frac{10 x^{3 / 2}}{3 b^{2} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}$

$$
+\frac{5 \sqrt{x} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}{b^{3}}
$$

Result(type 3, 179 leaves):
$\frac{x^{5 / 2}}{b \operatorname{arctanh}(\tanh (b x+a))^{3 / 2}}+\frac{5 a x^{3 / 2}}{3 b^{2} \operatorname{arctanh}(\tanh (b x+a))^{3 / 2}}+\frac{5 a \sqrt{x}}{b^{3} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}}-\frac{5 a \ln (\sqrt{b} \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh (b x+a))})}{b^{7 / 2}}$

$$
\begin{aligned}
& +\frac{5(\operatorname{arctanh}(\tanh (b x+a))-b x-a) x^{3 / 2}}{3 b^{2} \operatorname{arctanh}(\tanh (b x+a))^{3 / 2}}+\frac{5(\operatorname{arctanh}(\tanh (b x+a))-b x-a) \sqrt{x}}{b^{3} \sqrt{\operatorname{arctanh}(\tanh (b x+a))}} \\
& -\frac{5(\operatorname{arctanh}(\tanh (b x+a))-b x-a) \ln (\sqrt{b} \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh (b x+a))})}{b^{7 / 2}}
\end{aligned}
$$

Problem 73: Result more than twice size of optimal antiderivative.

$$
\int x^{3} \operatorname{arctanh}(\tanh (b x+a))^{n} \mathrm{~d} x
$$

Optimal(type 3, 121 leaves, 5 steps):

$$
\frac{x^{3} \operatorname{arctanh}(\tanh (b x+a))^{1+n}}{b(1+n)}-\frac{3 x^{2} \operatorname{arctanh}(\tanh (b x+a))^{2+n}}{b^{2}(1+n)(2+n)}+\frac{6 x \operatorname{arctanh}(\tanh (b x+a))^{3+n}}{b^{3}(1+n)(2+n)(3+n)}-\frac{6 \operatorname{arctanh}(\tanh (b x+a))^{4+n}}{b^{4}(1+n)(2+n)(3+n)(4+n)}
$$

Result(type 3, 491 leaves):
$\frac{x^{4} \mathrm{e}^{n \ln (\operatorname{arctanh}(\tanh (b x+a)))}}{4+n}+\frac{n(-b x+\operatorname{arctanh}(\tanh (b x+a))) x^{3} \mathrm{e}^{n \ln (\operatorname{arctanh}(\tanh (b x+a)))}}{b\left(n^{2}+7 n+12\right)}-\frac{6 \mathrm{e}^{n \ln (\operatorname{arctanh}(\tanh (b x+a)))} a^{4}}{b^{4}\left(n^{4}+10 n^{3}+35 n^{2}+50 n+24\right)}$

$$
-\frac{24 \mathrm{e}^{n \ln (\operatorname{arctanh}(\tanh (b x+a)))} a^{3}(\operatorname{arctanh}(\tanh (b x+a))-b x-a)}{b^{4}\left(n^{4}+10 n^{3}+35 n^{2}+50 n+24\right)}-\frac{36 \mathrm{e}^{n \ln (\operatorname{arctanh}(\tanh (b x+a)))} a^{2}(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{2}}{b^{4}\left(n^{4}+10 n^{3}+35 n^{2}+50 n+24\right)}
$$

$$
-\frac{24 \mathrm{e}^{n \ln (\operatorname{arctanh}(\tanh (b x+a)))} a(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{3}}{}-\frac{6 \mathrm{e}^{n \ln (\operatorname{arctanh}(\tanh (b x+a)))}(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{4}}{2}
$$

$$
b^{4}\left(n^{4}+10 n^{3}+35 n^{2}+50 n+24\right) \quad b^{4}\left(n^{4}+10 n^{3}+35 n^{2}+50 n+24\right)
$$

$-\frac{3 n\left(a^{2}+2 a(\operatorname{arctanh}(\tanh (b x+a))-b x-a)+(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{2}\right) x^{2} \mathrm{e}^{n \ln (\operatorname{arctanh}(\tanh (b x+a)))}}{b^{2}\left(n^{3}+9 n^{2}+26 n+24\right)}$
$+\frac{1}{b^{3}\left(n^{4}+10 n^{3}+35 n^{2}+50 n+24\right)}\left(6 n\left(a^{3}+3 a^{2}(\operatorname{arctanh}(\tanh (b x+a))-b x-a)+3 a(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{2}\right.\right.$
$\left.\left.+(\operatorname{arctanh}(\tanh (b x+a))-b x-a)^{3}\right) x \mathrm{e}^{n \ln (\operatorname{arctanh}(\tanh (b x+a)))}\right)$

Problem 75: Unable to integrate problem.

$$
\int x \operatorname{arctanh}(\sinh (x)) \mathrm{d} x
$$

Optimal(type 1, 1 leaves, 8 steps):
0
Result(type 8, 7 leaves):
$\int x \operatorname{arctanh}(\sinh (x)) d x$

Problem 77: Result more than twice size of optimal antiderivative. $\int x^{2} \operatorname{arctanh}(\cosh (x)) d x$
Optimal(type 4, 66 leaves, 10 steps):
 $\left.e^{x}\right)$
Result(type 4, 500 leaves):
$\frac{x^{3} \ln \left(1-\mathrm{e}^{x}\right)}{3}-\frac{x^{3} \ln \left(\mathrm{e}^{x}-1\right)}{3}-2 \operatorname{polylog}\left(4,-\mathrm{e}^{x}\right)+2 \operatorname{polylog}\left(4, \mathrm{e}^{x}\right)+x^{2} \operatorname{polylog}\left(2, \mathrm{e}^{x}\right)-x^{2} \operatorname{polylog}\left(2,-\mathrm{e}^{x}\right)+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{-x}\right) \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{-x}\left(\mathrm{e}^{x}+1\right)^{2}\right)^{2} x^{3}}{12}$
$-\frac{I \pi \operatorname{csgn}\left(I\left(e^{x}+1\right)^{2}\right)^{3} x^{3}}{12}+\frac{I \pi \operatorname{csgn}\left(I\left(e^{x}-1\right)^{2}\right)^{3} x^{3}}{12}+\frac{I \pi \operatorname{csgn}\left(I\left(e^{x}+1\right)^{2}\right) \operatorname{csgn}\left(I e^{-x}\left(e^{x}+1\right)^{2}\right)^{2} x^{3}}{12}-\frac{I \pi x^{3}}{6}$
$+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{Ie}^{-x}\right) \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{x}-1\right)^{2}\right) \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{-x}\left(\mathrm{e}^{x}-1\right)^{2}\right) x^{3}}{12}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{x}-1\right)\right) \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{x}-1\right)^{2}\right)^{2} x^{3}}{6}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{-x}\right) \operatorname{csgn}\left(\mathrm{Ie}^{-x}\left(\mathrm{e}^{x}-1\right)^{2}\right)^{2} x^{3}}{12}$
$-\frac{I \pi \operatorname{csgn}\left(I e^{-x}\right) \operatorname{csgn}\left(I\left(e^{x}+1\right)^{2}\right) \operatorname{csgn}\left(I e^{-x}\left(e^{x}+1\right)^{2}\right) x^{3}}{12}+\frac{I \pi \operatorname{csgn}\left(I\left(e^{x}-1\right)\right)^{2} \operatorname{csgn}\left(I\left(e^{x}-1\right)^{2}\right) x^{3}}{12}+\frac{I \pi \operatorname{csgn}\left(I\left(e^{x}+1\right)\right) \operatorname{csgn}\left(I\left(e^{x}+1\right)^{2}\right)^{2} x^{3}}{6}$
$-\frac{I \pi \operatorname{csgn}\left(I\left(e^{x}-1\right)^{2}\right) \operatorname{csgn}\left(I e^{-x}\left(e^{x}-1\right)^{2}\right)^{2} x^{3}}{12}+\frac{I \pi \operatorname{csgn}\left(I e^{-x}\left(e^{x}-1\right)^{2}\right)^{2} x^{3}}{6}-\frac{I \pi \operatorname{csgn}\left(I\left(e^{x}+1\right)\right)^{2} \operatorname{csgn}\left(I\left(e^{x}+1\right)^{2}\right) x^{3}}{12}$
$-\frac{I \pi \operatorname{csgn}\left(I e^{-x}\left(e^{x}+1\right)^{2}\right)^{3} x^{3}}{12}+2 x \operatorname{polylog}\left(3,-e^{x}\right)-2 x \operatorname{polylog}\left(3, e^{x}\right)-\frac{I \pi \operatorname{csgn}\left(I e^{-x}\left(e^{x}-1\right)^{2}\right)^{3} x^{3}}{12}$

Problem 78: Result more than twice size of optimal antiderivative.
$\int x^{3} \operatorname{arctanh}(1+d+d \tanh (b x+a)) \mathrm{d} x$
Optimal(type 4, 136 leaves, 8 steps):

$$
\begin{aligned}
\frac{b x^{5}}{20} & +\frac{x^{4} \operatorname{arctanh}(1+d+d \tanh (b x+a))}{4}-\frac{x^{4} \ln \left(1+(1+d) \mathrm{e}^{2 b x+2 a}\right)}{8}-\frac{x^{3} \operatorname{polylog}\left(2,-(1+d) \mathrm{e}^{2 b x+2 a}\right)}{4 b}+\frac{3 x^{2} \operatorname{polylog}\left(3,-(1+d) \mathrm{e}^{2 b x+2 a}\right)}{8 b^{2}} \\
& -\frac{3 x \operatorname{polylog}\left(4,-(1+d) \mathrm{e}^{2 b x+2 a}\right)}{8 b^{3}}+\frac{3 \operatorname{polylog}\left(5,-(1+d) \mathrm{e}^{2 b x+2 a}\right)}{16 b^{4}}
\end{aligned}
$$

Result (type 4, 1778 leaves):

$$
\begin{aligned}
& -\frac{\mathrm{I} \pi x^{4}}{8}-\frac{d \operatorname{poly} \log \left(2,(-d-1) \mathrm{e}^{2 b x+2 a}\right) a^{3}}{4 b^{4}(1+d)}+\frac{x^{4} \ln \left(\mathrm{e}^{2 b x+2 a} d+\mathrm{e}^{2 b x+2 a}+1\right)}{8}+\frac{3 d \operatorname{polylog}\left(3,(-d-1) \mathrm{e}^{2 b x+2 a}\right) x^{2}}{8 b^{2}(1+d)} \\
& \quad-\frac{3 d \operatorname{poly} \log \left(4,(-d-1) \mathrm{e}^{2 b x+2 a}\right) x}{8 b^{3}(1+d)}+\frac{d a^{3} \ln \left(1+\mathrm{e}^{b x+a} \sqrt{-d-1}\right) x}{2 b^{3}(1+d)}+\frac{d a^{3} \ln \left(1-\mathrm{e}^{b x+a} \sqrt{-d-1}\right) x}{2 b^{3}(1+d)}-\frac{d \ln \left(1+(1+d) \mathrm{e}^{2 b x+2 a}\right) x a^{3}}{2 b^{3}(1+d)}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{a^{4} \ln \left(1+\mathrm{e}^{b x+a} \sqrt{-d-1}\right)}{2 b^{4}(1+d)}-\frac{3 \ln \left(1+(1+d) \mathrm{e}^{2 b x+2 a}\right) a^{4}}{8 b^{4}(1+d)}+\frac{a^{4} \ln \left(1-\mathrm{e}^{b x+a} \sqrt{-d-1}\right)}{2 b^{4}(1+d)}+\frac{a^{3} \operatorname{dilog}\left(1+\mathrm{e}^{b x+a} \sqrt{-d-1}\right)}{2 b^{4}(1+d)} \\
& +\frac{a^{3} \operatorname{dilog}\left(1-\mathrm{e}^{b x+a} \sqrt{-d-1}\right)}{2 b^{4}(1+d)}-\frac{d \ln \left(1+(1+d) \mathrm{e}^{2 b x+2 a}\right) x^{4}}{8(1+d)}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{Ie}^{2 b x+2 a}\right)^{3} x^{4}}{16}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{Ie} \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{3} x^{4}}{16} \\
& +\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2} x^{4}}{8}-\frac{\operatorname{polylog}\left(2,(-d-1) \mathrm{e}^{2 b x+2 a}\right) x^{3}}{4 b(1+d)}-\frac{\operatorname{polylog}\left(2,(-d-1) \mathrm{e}^{2 b x+2 a}\right) a^{3}}{4 b^{4}(1+d)}+\frac{3 \operatorname{polylog}\left(3,(-d-1) \mathrm{e}^{2 b x+2 a}\right) x^{2}}{8 b^{2}(1+d)} \\
& -\frac{3 \operatorname{poly} \log \left(4,(-d-1) \mathrm{e}^{2 b x+2 a}\right) x}{8 b^{3}(1+d)}-\frac{a^{4} \ln \left(\mathrm{e}^{2 b x+2 a} d+\mathrm{e}^{2 b x+2 a}+1\right)}{8 b^{4}(1+d)}+\frac{3 d \operatorname{polylog}\left(5,(-d-1) \mathrm{e}^{2 b x+2 a}\right)}{16 b^{4}(1+d)} \\
& -\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d+\mathrm{e}^{2 b x+2 a}+1\right)}{\mathrm{e}^{2 b x+2 a}+1}\right)^{3} x^{4}}{16}-\frac{x^{4} \ln \left(\mathrm{e}^{b x+a}\right)}{4}-\frac{\ln (d) x^{4}}{8}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\mathrm{e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{2 b x+2 a}\right) \operatorname{csgn}\left(\frac{\mathrm{I} \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right) x^{4}}{16} \\
& -\underline{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\mathrm{e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d+\mathrm{e}^{2 b x+2 a}+1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d+\mathrm{e}^{2 b x+2 a}+1\right)}{\mathrm{e}^{2 b x+2 a}+1}\right) x^{4}} \\
& +\frac{\mathrm{I} \pi \operatorname{csgn}(\mathrm{I} d) \operatorname{csgn}\left(\frac{\mathrm{Ie}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right) x^{4}}{16}+\frac{d a^{4} \ln \left(1+\mathrm{e}^{b x+a} \sqrt{-d-1}\right)}{2 b^{4}(1+d)}+\frac{d a^{4} \ln \left(1-\mathrm{e}^{b x+a} \sqrt{-d-1}\right)}{2 b^{4}(1+d)} \\
& +\frac{d a^{3} \operatorname{dilog}\left(1+\mathrm{e}^{b x+a} \sqrt{-d-1}\right)}{2 b^{4}(1+d)}+\frac{d a^{3} \operatorname{dilog}\left(1-\mathrm{e}^{b x+a} \sqrt{-d-1}\right)}{2 b^{4}(1+d)}+\frac{a^{3} \ln \left(1+\mathrm{e}^{b x+a} \sqrt{-d-1}\right) x}{2 b^{3}(1+d)}-\frac{\ln \left(1+(1+d) \mathrm{e}^{2 b x+2 a}\right) x a^{3}}{2 b^{3}(1+d)} \\
& +\frac{a^{3} \ln \left(1-\mathrm{e}^{b x+a} \sqrt{-d-1}\right) x}{2 b^{3}(1+d)}-\frac{3 d \ln \left(1+(1+d) \mathrm{e}^{2 b x+2 a}\right) a^{4}}{8 b^{4}(1+d)}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{3} x^{4}}{16} \\
& +\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\mathrm{e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d+\mathrm{e}^{2 b x+2 a}+1\right)}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2} x^{4}}{16}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{Ie}^{b x+a}\right)^{2} \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{2 b x+2 a}\right) x^{4}}{16} \\
& +\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d+\mathrm{e}^{2 b x+2 a}+1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d+\mathrm{e}^{2 b x+2 a}+1\right)}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2} x^{4}}{16}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{b x+a}\right) \operatorname{csgn}\left(\mathrm{Ie}^{2 b x+2 a}\right)^{2} x^{4}}{8} \\
& -\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{Ie}^{2 b x+2 a}\right) \operatorname{csgn}\left(\frac{\mathrm{I} \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2} x^{4}}{16}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I} \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2} x^{4}}{16}-\frac{\mathrm{I} \pi \operatorname{csgn}(\mathrm{I} d) \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2} x^{4}}{16}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\mathrm{e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2} x^{4}}{16}+\frac{b x^{5}}{20}-\frac{\ln \left(1+(1+d) \mathrm{e}^{2 b x+2 a}\right) x^{4}}{8(1+d)}+\frac{3 \operatorname{polylog}\left(5,(-d-1) \mathrm{e}^{2 b x+2 a}\right)}{16 b^{4}(1+d)} \\
& -\frac{d a^{4} \ln \left(\mathrm{e}^{2 b x+2 a} d+\mathrm{e}^{2 b x+2 a}+1\right)}{8 b^{4}(1+d)}-\frac{d \operatorname{poly} \log \left(2,(-d-1) \mathrm{e}^{2 b x+2 a}\right) x^{3}}{4 b(1+d)}
\end{aligned}
$$

Problem 79: Result more than twice size of optimal antiderivative.

$$
\int-x^{2} \operatorname{arctanh}(-1+d+d \tanh (b x+a)) \mathrm{d} x
$$

Optimal(type 4, 120 leaves, 7 steps):
$\frac{b x^{4}}{12}-\frac{x^{3} \operatorname{arctanh}(-1+d+d \tanh (b x+a))}{3}-\frac{x^{3} \ln \left(1+(1-d) \mathrm{e}^{2 b x+2 a}\right)}{6}-\frac{x^{2} \operatorname{polylog}\left(2,-(1-d) \mathrm{e}^{2 b x+2 a}\right)}{4 b}+\frac{x \operatorname{polylog}\left(3,-(1-d) \mathrm{e}^{2 b x+2 a}\right)}{4 b^{2}}$
$-\frac{\operatorname{poly} \log \left(4,-(1-d) \mathrm{e}^{2 b x+2 a}\right)}{8 b^{3}}$
Result(type 4, 1721 leaves):

$$
\begin{aligned}
& -\frac{x^{3} \ln \left(\mathrm{e}^{b x+a}\right)}{3}-\frac{\ln (d) x^{3}}{6}+\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}\left(\frac{\mathrm{I}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{3}}{12}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}-1\right)}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2} x^{3}}{6}+\frac{\ln \left(1+(1-d) \mathrm{e}^{2 b x+2 a}\right) x^{3}}{6(d-1)} \\
& +\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\mathrm{e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}-1\right)}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2} x^{3}}{12} \\
& +\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{\left.2 b x+2 a d-\mathrm{e}^{2 b x+2 a}-1\right)}\right.}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2} x^{3}}{12}+\frac{d a^{3} \ln \left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}-1\right)}{6 b^{3}(d-1)}
\end{aligned}
$$

$$
+\frac{x^{3} \ln \left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}-1\right)}{6}+\frac{d \ln \left(1+(1-d) \mathrm{e}^{2 b x+2 a}\right) a^{3}}{3 b^{3}(d-1)}+\frac{\mathrm{I} \pi x^{3}}{6}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{3} x^{3}}{12}+\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{2 b x+2 a}\right)^{3}}{12}
$$

$$
-\frac{d \operatorname{polylog}\left(4,(d-1) \mathrm{e}^{2 b x+2 a}\right)}{8 b^{3}(d-1)}+\frac{a^{2} \operatorname{dilog}\left(1-\mathrm{e}^{b x+a} \sqrt{d-1}\right)}{2 b^{3}(d-1)}+\frac{a^{2} \operatorname{dilog}\left(1+\mathrm{e}^{b x+a} \sqrt{d-1}\right)}{2 b^{3}(d-1)}+\frac{\operatorname{poly} \log \left(2,(d-1) \mathrm{e}^{2 b x+2 a}\right) x^{2}}{4 b(d-1)}
$$

$$
-\frac{\operatorname{poly} \log \left(2,(d-1) \mathrm{e}^{2 b x+2 a}\right) a^{2}}{4 b^{3}(d-1)}-\frac{\operatorname{polylog}\left(3,(d-1) \mathrm{e}^{2 b x+2 a}\right) x}{4 b^{2}(d-1)}+\frac{a^{3} \ln \left(1-\mathrm{e}^{b x+a} \sqrt{d-1}\right)}{2 b^{3}(d-1)}+\frac{a^{3} \ln \left(1+\mathrm{e}^{b x+a} \sqrt{d-1}\right)}{2 b^{3}(d-1)}
$$

$$
+\frac{d \operatorname{poly} \log \left(3,(d-1) \mathrm{e}^{2 b x+2 a}\right) x}{4 b^{2}(d-1)}+\frac{a^{2} \ln \left(1-\mathrm{e}^{b x+a} \sqrt{d-1}\right) x}{2 b^{2}(d-1)}+\frac{a^{2} \ln \left(1+\mathrm{e}^{b x+a} \sqrt{d-1}\right) x}{2 b^{2}(d-1)}-\frac{d a^{3} \ln \left(1-\mathrm{e}^{b x+a} \sqrt{d-1}\right)}{2 b^{3}(d-1)}
$$

$$
-\frac{d a^{3} \ln \left(1+\mathrm{e}^{b x+a} \sqrt{d-1}\right)}{2 b^{3}(d-1)}-\frac{d a^{2} \operatorname{dilog}\left(1-\mathrm{e}^{b x+a} \sqrt{d-1}\right)}{2 b^{3}(d-1)}-\frac{d a^{2} \operatorname{dilog}\left(1+\mathrm{e}^{b x+a} \sqrt{d-1}\right)}{2 b^{3}(d-1)}-\frac{d \operatorname{polylog}\left(2,(d-1) \mathrm{e}^{2 b x+2 a}\right) x^{2}}{4 b(d-1)}
$$

$$
\begin{aligned}
& +\frac{d \operatorname{poly} \log \left(2,(d-1) \mathrm{e}^{2 b x+2 a}\right) a^{2}}{4 b^{3}(d-1)}-\frac{\ln \left(1+(1-d) \mathrm{e}^{2 b x+2 a}\right) a^{2} x}{2 b^{2}(d-1)}-\frac{d a^{2} \ln \left(1-\mathrm{e}^{b x+a} \sqrt{d-1}\right) x}{2 b^{2}(d-1)}-\frac{d a^{2} \ln \left(1+\mathrm{e}^{b x+a} \sqrt{d-1}\right) x}{2 b^{2}(d-1)} \\
& +\frac{\operatorname{poly} \log \left(4,(d-1) \mathrm{e}^{2 b x+2 a}\right)}{8 b^{3}(d-1)}+\frac{d \ln \left(1+(1-d) \mathrm{e}^{2 b x+2 a}\right) a^{2} x}{2 b^{2}(d-1)} \\
& -\underline{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\mathrm{e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}-1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}-1\right)}{\mathrm{e}^{2 b x+2 a}+1}\right) x^{3}} \\
& 12 \\
& +\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}(\mathrm{I} d) \operatorname{csgn}\left(\frac{\mathrm{Ie}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)}{12}+\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}\left(\frac{\mathrm{I}}{\mathrm{e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\mathrm{I}^{2 b x+2 a}\right) \operatorname{csgn}\left(\frac{\mathrm{I} \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)}{12} \\
& -\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}\left(\mathrm{Ie}^{2 b x+2 a}\right) \operatorname{csgn}\left(\frac{\mathrm{I} \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2}}{12}-\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}\left(\frac{\mathrm{I} \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2}}{12}-\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}(\mathrm{I} d) \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2}}{12} \\
& -\frac{a^{3} \ln \left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}-1\right)}{6 b^{3}(d-1)}-\frac{\ln \left(1+(1-d) \mathrm{e}^{2 b x+2 a}\right) a^{3}}{3 b^{3}(d-1)}-\frac{d \ln \left(1+(1-d) \mathrm{e}^{2 b x+2 a}\right) x^{3}}{6(d-1)} \\
& +\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}-1\right)}{\mathrm{e}^{2 b x+2 a}+1}\right)^{3} x^{3}}{12}+\frac{b x^{4}}{12}-\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}\left(\frac{\mathrm{I}}{\mathrm{e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(\frac{\mathrm{I} \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2}}{12} \\
& +\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}\left(\mathrm{Ie}^{b x+a}\right)^{2} \operatorname{csgn}\left(\mathrm{Ie}^{2 b x+2 a}\right)}{12}-\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}\left(\mathrm{Ie}^{b x+a}\right) \operatorname{csgn}\left(\mathrm{Ie}^{2 b x+2 a}\right)^{2}}{6}
\end{aligned}
$$

Problem 80: Result more than twice size of optimal antiderivative.

$$
\int x^{2} \operatorname{arctanh}(c+d \operatorname{coth}(b x+a)) \mathrm{d} x
$$

Optimal(type 4, 277 leaves, 11 steps):
$\frac{x^{3} \operatorname{arctanh}(c+d \operatorname{coth}(b x+a))}{3}+\frac{x^{3} \ln \left(1-\frac{(1-c-d) \mathrm{e}^{2 b x+2 a}}{1-c+d}\right)}{6}-\frac{x^{3} \ln \left(1-\frac{(1+c+d) \mathrm{e}^{2 b x+2 a}}{1+c-d}\right)}{6}+\frac{x^{2} \operatorname{poly} \log \left(2, \frac{(1-c-d) \mathrm{e}^{2 b x+2 a}}{1-c+d}\right)}{4 b}$

$$
\begin{aligned}
& -\frac{x^{2} \operatorname{polylog}\left(2, \frac{(1+c+d) \mathrm{e}^{2 b x+2 a}}{1+c-d}\right)}{4 b}-\frac{x \operatorname{polylog}\left(3, \frac{(1-c-d) \mathrm{e}^{2 b x+2 a}}{1-c+d}\right.}{4 b^{2}} \\
& +\frac{\operatorname{poly} \log \left(4, \frac{(1-c-d) \mathrm{e}^{2 b x+2 a}}{1-c+d}\right)}{8 b^{3}}-\frac{\operatorname{polylog}\left(4, \frac{(1+c+d) \mathrm{e}^{2 b x+2 a}}{1+c-d}\right)}{8 b^{3}}
\end{aligned}
$$

Result(type ?, 5293 leaves): Display of huge result suppressed!
Problem 82: Result more than twice size of optimal antiderivative.

Optimal(type 4, 118 leaves, 7 steps):
$\frac{b x^{4}}{12}-\frac{x^{3} \operatorname{arctanh}(-1+d+d \operatorname{coth}(b x+a))}{3}-\frac{x^{3} \ln \left(1-(1-d) \mathrm{e}^{2 b x+2 a}\right)}{6}-\frac{x^{2} \operatorname{polylog}\left(2,(1-d) \mathrm{e}^{2 b x+2 a}\right)}{4 b}+\frac{x \operatorname{poly} \log \left(3,(1-d) \mathrm{e}^{2 b x+2 a}\right)}{4 b^{2}}$
$-\frac{\operatorname{poly} \log \left(4,(1-d) \mathrm{e}^{2 b x+2 a}\right)}{8 b^{3}}$
Result(type 4, 1749 leaves):
$-\frac{x^{3} \ln \left(\mathrm{e}^{b x+a}\right)}{3}-\frac{\ln (d) x^{3}}{6}+\frac{\ln \left(1+(d-1) \mathrm{e}^{2 b x+2 a}\right) x^{3}}{6(d-1)}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}-1}\right)^{3} x^{3}}{12}+\frac{\mathrm{I} \pi x^{3}}{6}+\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{2 b x+2 a}\right)^{3}}{12}$
$-\frac{\ln \left(1+(d-1) \mathrm{e}^{2 b x+2 a}\right) a^{2} x}{2 b^{2}(d-1)}+\frac{a^{2} \ln \left(1+\mathrm{e}^{b x+a} \sqrt{1-d}\right) x}{2 b^{2}(d-1)}+\frac{a^{2} \ln \left(1-\mathrm{e}^{b x+a} \sqrt{1-d}\right) x}{2 b^{2}(d-1)}-\frac{d a^{3} \ln \left(1+\mathrm{e}^{b x+a} \sqrt{1-d}\right)}{2 b^{3}(d-1)}$
$-\frac{d a^{3} \ln \left(1-\mathrm{e}^{b x+a} \sqrt{1-d}\right)}{2 b^{3}(d-1)}-\frac{d a^{2} \operatorname{dilog}\left(1+\mathrm{e}^{b x+a} \sqrt{1-d}\right)}{2 b^{3}(d-1)}-\frac{d a^{2} \operatorname{dilog}\left(1-\mathrm{e}^{b x+a} \sqrt{1-d}\right)}{2 b^{3}(d-1)}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{Ie}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}-1}\right)^{3} x^{3}}{12}$
$-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}+1\right)}{\mathrm{e}^{2 b x+2 a}-1}\right)^{2} x^{3}}{6}-\frac{d \operatorname{polylog}\left(4,(1-d) \mathrm{e}^{2 b x+2 a}\right)}{8 b^{3}(d-1)}+\frac{\operatorname{polylog}\left(2,(1-d) \mathrm{e}^{2 b x+2 a}\right) x^{2}}{4 b(d-1)}$
$-\frac{\operatorname{poly} \log \left(2,(1-d) \mathrm{e}^{2 b x+2 a}\right) a^{2}}{4 b^{3}(d-1)}-\frac{\operatorname{poly} \log \left(3,(1-d) \mathrm{e}^{2 b x+2 a}\right) x}{4 b^{2}(d-1)}+\frac{a^{2} \operatorname{dilog}\left(1+\mathrm{e}^{b x+a} \sqrt{1-d}\right)}{2 b^{3}(d-1)}+\frac{a^{2} \operatorname{dilog}\left(1-\mathrm{e}^{b x+a} \sqrt{1-d}\right)}{2 b^{3}(d-1)}$
$-\frac{\ln \left(1+(d-1) \mathrm{e}^{2 b x+2 a}\right) a^{3}}{3 b^{3}(d-1)}+\frac{a^{3} \ln \left(1+\mathrm{e}^{b x+a} \sqrt{1-d}\right)}{2 b^{3}(d-1)}-\frac{d \ln \left(1+(d-1) \mathrm{e}^{2 b x+2 a}\right) x^{3}}{6(d-1)}+\frac{a^{3} \ln \left(1-\mathrm{e}^{b x+a} \sqrt{1-d}\right)}{2 b^{3}(d-1)}$
$-\frac{a^{3} \ln \left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}+1\right)}{6 b^{3}(d-1)}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}+1\right)}{\mathrm{e}^{2 b x+2 a}-1}\right)^{3} x^{3}}{12}+\frac{b x^{4}}{12}+\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}\left(\mathrm{I} \mathrm{e}^{b x+a}\right)^{2} \operatorname{csgn}\left(\mathrm{Ie}^{2 b x+2 a}\right)}{12}$
$-\frac{\mathrm{I} \pi x^{3} \operatorname{csgn}\left(\mathrm{I}^{b x+a}\right) \operatorname{csgn}\left(\mathrm{I}^{2 b x+2 a}\right)^{2}}{6}+\frac{d \ln \left(1+(d-1) \mathrm{e}^{2 b x+2 a}\right) a^{3}}{3 b^{3}(d-1)}+\frac{\operatorname{polylog}\left(4,(1-d) \mathrm{e}^{2 b x+2 a}\right)}{8 b^{3}(d-1)}+\frac{d \ln \left(1+(d-1) \mathrm{e}^{2 b x+2 a}\right) a^{2} x}{2 b^{2}(d-1)}$
$-\frac{d a^{2} \ln \left(1+\mathrm{e}^{b x+a} \sqrt{1-d}\right) x}{2 b^{2}(d-1)}-\frac{d a^{2} \ln \left(1-\mathrm{e}^{b x+a} \sqrt{1-d}\right) x}{2 b^{2}(d-1)}+\frac{\mathrm{I} \pi \operatorname{csgn}(\mathrm{I} d) \operatorname{csgn}\left(\frac{\mathrm{I} \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}-1}\right) \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}-1}\right) x^{3}}{12}$
$+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{Ie}^{2 b x+2 a}\right) \operatorname{csgn}\left(\frac{\mathrm{I}}{\mathrm{e}^{2 b x+2 a}-1}\right) \operatorname{csgn}\left(\frac{\mathrm{I} \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}-1}\right) x^{3}}{12}$

Problem 84: Result more than twice size of optimal antiderivative.

$$
\int \operatorname{arctanh}(\tan (b x+a)) \mathrm{d} x
$$

Optimal(type 4, 64 leaves, 6 steps):

$$
\mathrm{I} x \arctan \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)+x \operatorname{arctanh}(\tan (b x+a))-\frac{\mathrm{I} \operatorname{poly} \log \left(2,-\mathrm{I} \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{4 b}+\frac{\mathrm{I} \operatorname{poly} \log \left(2, \mathrm{Ie}^{2 \mathrm{I}(b x+a)}\right)}{4 b}
$$

Result(type 4, 515 leaves):
$\underline{\arctan (\tan (b x+a)) \operatorname{arctanh}(\tan (b x+a))}+\arctan (\tan (b x+a)) \ln \left(\frac{\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \tan (b x+a)}{\sqrt{1+\tan (b x+a)^{2}}}}{\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}}\right)$
$b$

$$
2 b
$$

$$
\left.-\frac{\mathrm{I} \operatorname{dilog}\left(\frac{\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \tan (b x+a)}{\sqrt{1+\tan (b x+a)^{2}}}}{\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}}\right.}{2 h}\right)
$$

$$
\frac{\arctan (\tan (b x+a)) \ln \left(\frac{-\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{Itan}(b x+a)}{\sqrt{1+\tan (b x+a)^{2}}}}{2 b}\right)}{2-\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}}
$$

$$
\begin{aligned}
& -\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\mathrm{e}^{2 b x+2 a}-1}\right) \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}+1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}+1\right)}{\mathrm{e}^{2 b x+2 a}-1}\right) x^{3}}{12}+\frac{x^{3} \ln \left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}+1\right)}{6} \\
& -\frac{d \operatorname{poly} \log \left(2,(1-d) \mathrm{e}^{2 b x+2 a}\right) x^{2}}{4 b(d-1)}+\frac{d \operatorname{poly} \log \left(2,(1-d) \mathrm{e}^{2 b x+2 a}\right) a^{2}}{4 b^{3}(d-1)}+\frac{d \operatorname{poly} \log \left(3,(1-d) \mathrm{e}^{2 b x+2 a}\right) x}{4 b^{2}(d-1)}+\frac{d a^{3} \ln \left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}+1\right)}{6 b^{3}(d-1)} \\
& +\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}+1\right)\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}+1\right)}{\mathrm{e}^{2 b x+2 a}-1}\right)^{2} x^{3}}{12}-\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I} \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}-1}\right) \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}-1}\right)^{2} x^{3}}{12} \\
& -\frac{\mathrm{I} \pi \operatorname{csgn}\left(\mathrm{I}^{2 b x+2 a}\right) \operatorname{csgn}\left(\frac{\mathrm{I} \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}-1}\right)^{2} x^{3}}{12}+\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\mathrm{e}^{2 b x+2 a}-1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}\left(\mathrm{e}^{2 b x+2 a} d-\mathrm{e}^{2 b x+2 a}+1\right)}{\mathrm{e}^{2 b x+2 a}-1}\right)^{2} x^{3}}{12} \\
& -\frac{\mathrm{I} \pi \operatorname{csgn}\left(\frac{\mathrm{I}}{\mathrm{e}^{2 b x+2 a}-1}\right) \operatorname{csgn}\left(\frac{\mathrm{I}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}-1}\right)^{2} x^{3}}{12}-\frac{\mathrm{I} \pi \operatorname{csgn}(\mathrm{I} d) \operatorname{csgn}\left(\frac{\mathrm{I} d \mathrm{e}^{2 b x+2 a}}{\mathrm{e}^{2 b x+2 a}-1}\right)^{2} x^{3}}{12}
\end{aligned}
$$

$$
\begin{aligned}
& \left.+\frac{\mathrm{I} \operatorname{dilog}\left(\frac{-\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \tan (b x+a)}{\sqrt{1+\tan (b x+a)^{2}}}}{-\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}}\right)}{2 b}+\frac{\arctan (\tan (b x+a)) \ln \left(\frac{-\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \tan (b x+a)}{\sqrt{1+\tan (b x+a)^{2}}}}{2 b}\right)}{-\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}}\right) \\
& \left.-\frac{\mathrm{I} \operatorname{dilog}\left(\frac{-\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{Itan}(b x+a)}{\sqrt{1+\tan (b x+a)^{2}}}}{-\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}}\right.}{2 b}\right) \\
& \frac{\arctan (\tan (b x+a)) \ln \left(\frac{\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{Itan}(b x+a)}{\sqrt{1+\tan (b x+a)^{2}}}}{2 b}\right)}{\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}} \\
& \left.+\frac{\mathrm{I} \operatorname{dilog}\left(\frac{\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \tan (b x+a)}{\sqrt{1+\tan (b x+a)^{2}}}}{\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}}\right.}{2 b}\right)
\end{aligned}
$$

Problem 85: Result more than twice size of optimal antiderivative.

$$
\int-x \operatorname{arctanh}(-1-\mathrm{I} d+d \tan (b x+a)) \mathrm{d} x
$$

Optimal(type 4, 108 leaves, 6 steps):
$\frac{\mathrm{I} b x^{3}}{6}-\frac{x^{2} \operatorname{arctanh}(-1-\mathrm{I} d+d \tan (b x+a))}{2}-\frac{x^{2} \ln \left(1+(1+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{4}+\frac{\mathrm{I} x \operatorname{poly} \log \left(2,-(1+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{4 b}$

$$
-\frac{\operatorname{poly} \log \left(3,-(1+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{8 b^{2}}
$$

Result(type ?, 2357 leaves): Display of huge result suppressed!
Problem 86: Result more than twice size of optimal antiderivative.

$$
\int(f x+e)^{3} \operatorname{arctanh}(\cot (b x+a)) \mathrm{d} x
$$

Optimal(type 4, 251 leaves, 12 steps):

$$
\begin{aligned}
& \frac{\mathrm{I}(f x+e)^{4} \arctan \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{4 f}+\frac{(f x+e)^{4} \operatorname{arctanh}(\cot (b x+a))}{4 f}-\frac{\mathrm{I}(f x+e)^{3} \operatorname{poly} \log \left(2,-\mathrm{Ie}{ }^{2 \mathrm{I}(b x+a)}\right)}{4 b}+\frac{\mathrm{I}(f x+e)^{3} \operatorname{polylog}\left(2, \mathrm{Ie} \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{4 b} \\
& +\frac{3 f(f x+e)^{2} \operatorname{polylog}\left(3,-\mathrm{Ie}^{2 \mathrm{I}(b x+a)}\right)}{8 b^{2}}-\frac{3 f(f x+e)^{2} \operatorname{poly} \log \left(3, \mathrm{I} \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{8 b^{2}}+\frac{3 \mathrm{I} f^{2}(f x+e) \operatorname{poly} \log \left(4,-\mathrm{Ie}^{2 \mathrm{I}(b x+a)}\right)}{8 b^{3}} \\
& \\
& -\frac{3 \mathrm{I} f^{2}(f x+e) \operatorname{polylog}\left(4, \mathrm{I} \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{8 b^{3}}-\frac{3 f^{3} \operatorname{poly} \log \left(5,-\mathrm{Ie}^{2 \mathrm{I}(b x+a)}\right)}{16 b^{4}}+\frac{3 f^{3} \operatorname{polylog}\left(5, \mathrm{I} \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{16 b^{4}}
\end{aligned}
$$

[^6]Problem 87: Result more than twice size of optimal antiderivative.
$\int(f x+e) \operatorname{arctanh}(\cot (b x+a)) \mathrm{d} x$
Optimal(type 4, 133 leaves, 8 steps):
$\frac{\mathrm{I}(f x+e)^{2} \arctan \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{2 f}+\frac{(f x+e)^{2} \operatorname{arctanh}(\cot (b x+a))}{2 f}-\frac{\mathrm{I}(f x+e) \operatorname{polylog}\left(2,-\mathrm{I}{ }^{2 \mathrm{I}(b x+a)}\right)}{4 b}+\frac{\mathrm{I}(f x+e) \operatorname{poly} \log \left(2, \mathrm{I} \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{4 b}$

$$
+\frac{f \text { polylog }\left(3,-\mathrm{I}^{2 \mathrm{I}(b x+a)}\right)}{8 b^{2}}-\frac{f \text { poly } \log \left(3, \mathrm{I}^{2 \mathrm{I}(b x+a)}\right)}{8 b^{2}}
$$

Result(type ?, 2543 leaves): Display of huge result suppressed!
Problem 88: Result more than twice size of optimal antiderivative.

$$
\int \operatorname{arctanh}(\cot (b x+a)) d x
$$

Optimal(type 4, 64 leaves, 6 steps):

$$
\mathrm{I} x \arctan \left(\mathrm{e}^{2 \mathrm{I}(b x+a)}\right)+x \operatorname{arctanh}(\cot (b x+a))-\frac{\mathrm{I} \operatorname{poly} \log \left(2,-\mathrm{Ie}^{2 \mathrm{I}(b x+a)}\right)}{4 b}+\frac{\mathrm{I} \operatorname{poly} \log \left(2, \mathrm{I} \mathrm{e}^{2 \mathrm{I}(b x+a)}\right)}{4 b}
$$

Result(type 4, 764 leaves):


$$
\left.+\frac{\ln \left(\frac{\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \cot (b x+a)}{\sqrt{\cot (b x+a)^{2}+1}}}{\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}}\right) \operatorname{arccot}(\cot (b x+a))}{2 b}+\frac{\mathrm{I} \operatorname{dilog}\left(\frac{\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \cot (b x+a)}{\sqrt{\cot (b x+a)^{2}+1}}}{2 b}\right)}{2}\right)
$$

$$
\left.+\frac{\ln \left(\frac{-\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \cot (b x+a)}{\sqrt{\cot (b x+a)^{2}+1}}}{-\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}}\right) \pi}{4 b}-\frac{\ln \left(\frac{-\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \cot (b x+a)}{\sqrt{\cot (b x+a)^{2}+1}}}{2 b}\right) \operatorname{arccot}(\cot (b x+a))}{-\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}}\right)
$$

$$
\left.-\frac{\mathrm{I} \operatorname{dilog}\left(\frac{-\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \cot (b x+a)}{\sqrt{\cot (b x+a)^{2}+1}}}{-\frac{\sqrt{2}}{2}+\frac{\mathrm{I} \sqrt{2}}{2}}\right)}{2 b}-\frac{\ln \left(\frac{-\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \cot (b x+a)}{\sqrt{\cot (b x+a)^{2}+1}}}{4 b} \pi\right.}{-\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}}\right)
$$



$$
\left.+\frac{\ln \left(\frac{\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \cot (b x+a)}{\sqrt{\cot (b x+a)^{2}+1}}}{\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}}\right) \pi \ln \left(\frac{\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \cot (b x+a)}{\sqrt{\cot (b x+a)^{2}+1}}}{4 b}\right) \operatorname{arccot}(\cot (b x+a))}{\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}}\right)
$$

$$
\left.-\frac{\mathrm{I} \operatorname{dilog}\left(\frac{\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}-\frac{1+\mathrm{I} \cot (b x+a)}{\sqrt{\cot (b x+a)^{2}+1}}}{\frac{\sqrt{2}}{2}-\frac{\mathrm{I} \sqrt{2}}{2}}\right.}{2 b}\right)
$$

Problem 89: Result more than twice size of optimal antiderivative.

$$
\int x^{2} \operatorname{arctanh}(c+d \cot (b x+a)) \mathrm{d} x
$$

Optimal(type 4, 329 leaves, 11 steps):
$\frac{x^{3} \operatorname{arctanh}(c+d \cot (b x+a))}{3}+\frac{x^{3} \ln \left(1-\frac{(1-c-\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}}{1-c+\mathrm{I} d}\right)}{6}-\frac{x^{3} \ln \left(1-\frac{(1+c+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}}{1+c-\mathrm{I} d}\right)}{6}$

$$
\begin{aligned}
& -\frac{\mathrm{I} x^{2} \operatorname{poly} \log \left(2, \frac{(1-c-\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}}{1-c+\mathrm{I} d}\right)}{4 b}+\frac{\mathrm{I} x^{2} \operatorname{poly} \log \left(2, \frac{(1+c+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}}{1+c-\mathrm{I} d}\right)}{4 b}+\frac{x \operatorname{poly} \log \left(3, \frac{(1-c-\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}}{1-c+\mathrm{I} d}\right)}{4 b^{2}} \\
& -\frac{x \operatorname{poly} \log \left(3, \frac{(1+c+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}}{1+c-\mathrm{I} d}\right)}{4 b^{2}}+\frac{\mathrm{I} \operatorname{polylog}\left(4, \frac{(1-c-\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}}{1-c+\mathrm{I} d}\right)}{8 b^{3}}-\frac{\mathrm{I} \operatorname{poly} \log \left(4, \frac{(1+c+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}}{1+c-\mathrm{I} d}\right)}{8 b^{3}}
\end{aligned}
$$

Result(type ?, 6738 leaves): Display of huge result suppressed!
Problem 90: Result more than twice size of optimal antiderivative.
$\int \operatorname{arctanh}(c+d \cot (b x+a)) \mathrm{d} x$
Optimal(type 4, 164 leaves, 7 steps):
$x \operatorname{arctanh}(c+d \cot (b x+a))+\frac{x \ln \left(1-\frac{(1-c-\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}}{1-c+\mathrm{I} d}\right)}{2}-\frac{x \ln \left(1-\frac{(1+c+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}}{1+c-\mathrm{I} d}\right)}{2}-\frac{\mathrm{Ipolylog}\left(2, \frac{(1-c-\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}}{1-c+\mathrm{I} d}\right)}{4 b}$

$$
+\frac{\operatorname{Ipolylog}\left(2, \frac{(1+c+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}}{1+c-\mathrm{I} d}\right)}{4 b}
$$

Result(type 4, 628 leaves):
$-\frac{\operatorname{arctanh}(c+d \cot (b x+a)) \pi}{2 b}+\frac{\operatorname{arctanh}(c+d \cot (b x+a)) \operatorname{arccot}(\cot (b x+a))}{b}$

$$
\begin{aligned}
& -\frac{\arctan \left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right) \ln \left(d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)+c-1\right)}{2 b} \\
& +\frac{\arctan \left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right) \ln \left(d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2 b} \\
& +\frac{\mathrm{I} \ln \left(d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)+c+1\right) \ln \left(\frac{\mathrm{I} d-d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)}{1+c+\mathrm{I} d}\right)}{4 b}
\end{aligned}
$$

$$
-\frac{\mathrm{I} \ln \left(d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)+c+1\right) \ln \left(\frac{\mathrm{I} d+d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)}{\mathrm{I} d-c-1}\right)}{4 b}+\frac{\mathrm{I} \operatorname{dilog}\left(\frac{\mathrm{I} d-d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)}{1+c+\mathrm{I} d}\right)}{4 b}
$$

$$
-\frac{\mathrm{I} \operatorname{dilog}\left(\frac{\mathrm{I} d+d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)}{\mathrm{I} d-c-1}\right)}{4 b}-\frac{\mathrm{I} \ln \left(d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)+c-1\right) \ln \left(\frac{\mathrm{I} d-d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)}{4 b}\right)}{\mathrm{I} d+c-1}
$$

$$
+\frac{\mathrm{I} \ln \left(d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)+c-1\right) \ln \left(\frac{\mathrm{I} d+d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)}{1-c+\mathrm{I} d}\right)}{4 b}-\frac{\mathrm{I} \operatorname{dilog}\left(\frac{\mathrm{I} d-d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)}{\mathrm{I} d+c-1}\right)}{4 b}
$$

$$
+\frac{\mathrm{I} \operatorname{dilog}\left(\frac{\mathrm{I} d+d\left(\frac{c+d \cot (b x+a)}{d}-\frac{c}{d}\right)}{1-c+\mathrm{I} d}\right)}{4 b}
$$

Problem 91: Result more than twice size of optimal antiderivative.

$$
\int x^{2} \operatorname{arctanh}(1+\mathrm{I} d+d \cot (b x+a)) \mathrm{d} x
$$

Optimal(type 4, 136 leaves, 7 steps):
$\frac{\mathrm{I} b x^{4}}{12}+\frac{x^{3} \operatorname{arctanh}(1+\mathrm{I} d+d \cot (b x+a))}{3}-\frac{x^{3} \ln \left(1-(1+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{6}+\frac{\mathrm{I} x^{2} \operatorname{poly} \log \left(2,(1+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{4 b}$
$-\frac{x \operatorname{poly} \log \left(3,(1+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{4 b^{2}}-\frac{\mathrm{I} \text { polylog }\left(4,(1+\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{8 b^{3}}$
Result(type ?, 2455 leaves): Display of huge result suppressed!
Problem 92: Result more than twice size of optimal antiderivative.

$$
\int-x^{2} \operatorname{arctanh}(-1+\mathrm{I} d+d \cot (b x+a)) \mathrm{d} x
$$

Optimal(type 4, 136 leaves, 7 steps):
$\frac{\mathrm{I} b x^{4}}{12}-\frac{x^{3} \operatorname{arctanh}(-1+\mathrm{I} d+d \cot (b x+a))}{3}-\frac{x^{3} \ln \left(1-(1-\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{6}+\frac{\mathrm{I} x^{2} \operatorname{poly} \log \left(2,(1-\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{4 b}$
$-\frac{x \operatorname{poly} \log \left(3,(1-\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{4 b^{2}}-\frac{\mathrm{I} \text { polylog }\left(4,(1-\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{8 b^{3}}$
Result(type ?, 2345 leaves): Display of huge result suppressed!
Problem 93: Result more than twice size of optimal antiderivative.

$$
\int-\operatorname{arctanh}(-1+\mathrm{I} d+d \cot (b x+a)) \mathrm{d} x
$$

Optimal(type 4, 77 leaves, 5 steps):

$$
\frac{\mathrm{I} b x^{2}}{2}-x \operatorname{arctanh}(-1+\mathrm{I} d+d \cot (b x+a))-\frac{x \ln \left(1-(1-\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{2}+\frac{\mathrm{I} \operatorname{polylog}\left(2,(1-\mathrm{I} d) \mathrm{e}^{2 \mathrm{I} a+2 \mathrm{I} b x}\right)}{4 b}
$$

Result(type 4, 334 leaves):
$-\frac{\mathrm{I} \operatorname{arctanh}(-1+\mathrm{I} d+d \cot (b x+a)) \ln (\mathrm{I} d-d \cot (b x+a))}{2 b}+\frac{\mathrm{I} \operatorname{arctanh}(-1+\mathrm{I} d+d \cot (b x+a)) \ln (\mathrm{I} d+d \cot (b x+a))}{2 b}$

$$
\begin{aligned}
& +\frac{\mathrm{I} \operatorname{dilog}\left(\frac{\frac{\mathrm{I}}{2}(-\mathrm{I} d-d \cot (b x+a))}{d}\right)}{4 b}+\frac{\mathrm{I} \ln (\mathrm{I} d-d \cot (b x+a)) \ln \left(\frac{\frac{\mathrm{I}}{2}(-\mathrm{I} d-d \cot (b x+a))}{4 b}\right)}{4}-\frac{\mathrm{I} \operatorname{dilog}\left(\frac{2-\mathrm{I} d-d \cot (b x+a)}{-2 \mathrm{I} d+2}\right)}{4 b} \\
& -\frac{\mathrm{I} \ln (\mathrm{I} d-d \cot (b x+a)) \ln \left(\frac{2-\mathrm{I} d-d \cot (b x+a)}{-2 \mathrm{I} d+2}\right)}{4 b}-\frac{\mathrm{I} \ln (\mathrm{I} d+d \cot (b x+a))^{2}}{8 b}+\frac{\mathrm{I} \ln \left(1-\frac{\mathrm{I} d}{2}-\frac{d \cot (b x+a)}{2}\right) \ln (\mathrm{I} d+d \cot (b x+a))}{4 b} \\
& -\frac{\mathrm{I} \ln \left(1-\frac{\mathrm{I} d}{2}-\frac{d \cot (b x+a)}{2}\right) \ln \left(\frac{\mathrm{I} d}{2}+\frac{d \cot (b x+a)}{2}\right)}{4 b}-\frac{\mathrm{I} \operatorname{dilog}\left(\frac{\mathrm{I} d}{2}+\frac{d \cot (b x+a)}{2}\right)}{4 b}
\end{aligned}
$$

Problem 97: Result more than twice size of optimal antiderivative.

$$
\int x \operatorname{arctanh}\left(a+b f^{d x+c}\right) \mathrm{d} x
$$

Optimal(type 4, 195 leaves, 9 steps):
$-\frac{x^{2} \ln \left(1-a-b f^{d x+c}\right)}{4}+\frac{x^{2} \ln \left(1+a+b f^{d x+c}\right)}{4}+\frac{x^{2} \ln \left(1-\frac{b f^{d x+c}}{1-a}\right)}{4}-\frac{x^{2} \ln \left(1+\frac{b f^{d x+c}}{1+a}\right)}{4}+\frac{x \operatorname{poly} \log \left(2, \frac{b f^{d x+c}}{1-a}\right)}{2 d \ln (f)}-\frac{x \operatorname{poly} \log \left(2,-\frac{b f^{d x+c}}{1+a}\right)}{2 d \ln (f)}$

$$
-\frac{\text { polylog }\left(3, \frac{b f^{d x+c}}{1-a}\right)}{2 d^{2} \ln (f)^{2}}+\frac{\text { polylog }\left(3,-\frac{b f^{d x+c}}{1+a}\right)}{2 d^{2} \ln (f)^{2}}
$$

Result(type 4, 575 leaves):

$$
\begin{aligned}
& \frac{x^{2} \ln \left(1+a+b f^{d x+c}\right)}{4}-\frac{x^{2} \ln \left(1-a-b f^{d x+c}\right)}{4}+\frac{x^{2} \ln \left(1-\frac{b f^{d x+c}}{1-a}\right)}{4}+\frac{\ln \left(1-\frac{b f^{d x+c}}{1-a}\right) x c}{2 d}+\frac{\ln \left(1-\frac{b f^{d x+c}}{1-a}\right) c^{2}}{4 d^{2}}+\frac{x \operatorname{poly} \log \left(2, \frac{b f^{d x+c}}{1-a}\right)}{2 d \ln (f)} \\
& \quad+\frac{\operatorname{poly} \log \left(2, \frac{b f^{d x+c}}{1-a}\right) c}{2 \ln (f) d^{2}}-\frac{\operatorname{poly} \log \left(3, \frac{b f^{d x+c}}{1-a}\right)}{2 d^{2} \ln (f)^{2}}+\frac{c^{2} \ln \left(1-a-b f^{d x+c}\right)}{4 d^{2}}-\frac{c \operatorname{dilog}\left(\frac{b f^{d x+c}+a-1}{-1+a}\right)}{2 \ln (f) d^{2}}-\frac{c \ln \left(\frac{b f^{d x+c}+a-1}{-1+a}\right) x}{2 d} \\
& \quad-\frac{c^{2} \ln \left(\frac{b f^{d x+c}+a-1}{-1+a}\right)}{2 d^{2}}-\frac{\ln \left(1-\frac{b f^{d x+c}}{-1-a}\right) x^{2}}{4}-\frac{\ln \left(1-\frac{b f^{d x+c}}{-1-a}\right) x c}{2 d}-\frac{\ln \left(1-\frac{b f^{d x+c}}{-1-a}\right) c^{2}}{4 d^{2}}-\frac{\operatorname{poly} \log \left(2, \frac{b f^{d x+c}}{-1-a}\right) x}{2 \ln (f) d} \\
& \quad-\frac{\operatorname{poly} \log \left(2, \frac{b f^{d x+c}}{-1-a}\right) c}{2 \ln (f) d^{2}}+\frac{\operatorname{poly} \log \left(3, \frac{b f^{d x+c}}{-1-a}\right)}{2 \ln (f)^{2} d^{2}}-\frac{c^{2} \ln \left(1+a+b f^{d x+c}\right)}{4 d^{2}}+\frac{c \operatorname{dilog}\left(\frac{1+a+b f^{d x+c}}{1+a}\right)}{2 \ln (f) d^{2}}+\frac{c \ln \left(\frac{1+a+b f^{d x+c}}{1+a}\right) x}{2 d} \\
& \quad+\frac{c^{2} \ln \left(\frac{1+a+b f^{d x+c}}{1+a}\right)}{2 d^{2}}
\end{aligned}
$$

## Summary of Integration Test Results

698 integration problems


A - 409 optimal antiderivatives
B - 175 more than twice size of optimal antiderivatives
C - O unnecessarily complex antiderivatives
D - 113 unable to integrate problems
E - 1 integration timeouts


[^0]:    Problem 6: Result more than twice size of optimal antiderivative.

[^1]:    Problem 38: Unable to integrate problem.

[^2]:    Problem 40: Unable to integrate problem.

[^3]:    Problem 160: Result more than twice size of optimal antiderivative.

[^4]:    Problem 308: Unable to integrate problem.

[^5]:    Problem 335: Unable to integrate problem.

[^6]:    Result(type ?, 7428 leaves): Display of huge result suppressed!

